Carrots or Sticks? Short-Time Work vs. Layoff Taxes*

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Abstract

While unemployment insurance systems are widely used to insure workers against income losses after separations, it is well known that they can inefficiently increase separations in the labor market. There are two distinct policy instruments commonly used by governments that can counter this known problem: layoff taxes and short-time work schemes. This study provides a search-and-matching model to evaluate which of the two is the better policy tool. We show that if only a few firms are financially constrained, layoff taxes are better because they do not distort working hours in the economy. With a large share of financially constrained firms, short-time work emerges as the superior tool, as layoff taxes cannot deter separations in financially constrained firms. Additionally, short-time work can help provide insurance against income losses to risk-averse workers that constrained firms cannot afford to provide in their wage contracts. Calibrating the model to the US economy, we find that short-time work is the superior policy instrument if at least 27.5 percent of firms in the economy are financially constrained. Finally, we show that combining both instruments can dominate either policy in isolation when sufficiently many firms of both types are present in the economy.

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1 Introduction

It has long been understood that unemployment insurance systems can inefficiently increase separations in the labor market. To counter this inefficiency, governments commonly have two main policy instruments at their disposal. On the one hand, there are layoff taxes that, in effect, punish firms for laying off workers. On the other hand, there are the short-time work schemes that rewards the retention of endangered workers through subsidizing and enabling reduction in working hours.[1]

This raises the question: which of the two policy instruments should governments use? The existing literature highlights that layoff taxes have many desirable properties. Cahuc and Zylberberg (2008) and Blanchard and Tirole (2008) show that unemployment insurance financed by layoff taxes can implement the social planner allocation in a canonical implicit contract model. Short-time work, on the other hand, has been shown to have the problem of introducing new inefficiencies into the economy through the distortion of working hours (e.g., Burdett and Wright, 1989; Stiepelmann, 2026). However, these results hinge on the absence of financial constraints for firms. This assumption is widespread in the search-and-matching literature, but clearly at odds with reality.[2]

In this study, we relax this assumption in a rich yet analytically tractable Diamond-Mortensen-Pissaridis (DMP) search-and-matching model[3] and allow for a share of firms to be financially constrained. Using the Ramsey policy approach, we then determine the welfare consequences of an optimally set short-time work scheme and optimally set layoff taxes.

Our main result is that layoff taxes are indeed superior when the share of financially constrained firms is small. Short-time work emerges as the superior policy instrument if the share of constrained firms becomes sufficiently large. This is a consequence of two main channels. First, layoff taxes cannot deter separations in financially constrained firms, while short-time work can still operate as effectively as before. Second, with financial constraints, firms lose their ability to insure risk-averse workers against negative income shocks. Short-time work can then partially mitigate this and provide insurance against income shocks in the firms' stead.

Quantitatively, after calibrating the model to the US economy, we find that short-time work

[1]Short-time work schemes are pervasive, e.g., in European countries where they were utilized during the Great Recession and the COVID period. Layoff taxes are implemented, e.g., in the US through an experience-rated unemployment insurance system.

[2]In practice, US experience-rated UI tax rates are capped, potentially to avoid pushing firms into financial distress after mass layoffs (Vroman, 1989). Relatedly, Blanchard and Tirole (2008) show that when firms have "shallow pockets," the optimal layoff tax is lower than the level that would fully internalize the fiscal externality.

[3]See Mortensen and Pissarides (1994).

becomes the superior policy instrument once the share of financially constrained firms reaches 27.5 percent. While this share is not directly observable in empirical data, we discuss how existing empirical measures of financial constraints can be interpreted through the lens of the model and find that values of this magnitude are plausible in the US economy. Finally, we show that optimally combining short-time work and layoff takes advantage of the strengths of both instruments and can dominate using either one in isolation.

The backbone of our model is a canonical DMP type search-and-matching model with idiosyncratic productivity shocks, endogenous separations, and generalized Nash bargaining between workers and firms. We augment the standard model with three realistic key ingredients: risk aversion on the worker side, financial constraints on the firm side, and flexibly adjustable working hours. Workers and firms are randomly matched, form expectations over their match productivity, and bargain over wages, working hours, and separation productivity thresholds. Productivity shocks are independent and identically distributed and are realized after firms and workers complete bargaining.

The government implements an unemployment insurance system under which unemployed workers are paid lump-sum benefits, and either a layoff tax or a short-time work scheme. Further, it collects taxes on firms to finance the systems.

When layoff taxes are imposed by the government, firms have to pay lump-sum layoff taxes once worker and firm separate after productivity falls below the separation threshold. To replicate the US experience-rated unemployment insurance system, we assume that layoff taxes are backloaded to the next period, thereby avoiding firm bankruptcies.

The short-time work scheme consists of two main components: eligibility conditions and benefit payments. Workers become eligible for short-time work once they agree to reduce their working hours below a threshold specified by the government. Short-time work benefits compensate for lost income by providing fixed payments for each hour not worked relative to normal working hours. Short-time work effectively acts as a subsidy paid directly to workers. Working hours under short-time work are also determined through bargaining between firms and workers. In this setup, firms and workers have an incentive to reduce working hours inefficiently to attract more government support.

We assume that a share of firms are financially constrained. Specifically, we assume that these firms can never borrow more than the expected discounted value of the firm. These constraints have direct welfare implications. With risk-averse workers and risk-neutral firms, firms would like to offer workers insurance against low-productivity shocks and commit to paying the worker a fixed consumption-equivalent wage, no matter how low or high productivity turns out to be. The worker is then willing to accept a slightly lower wage in return for the insurance. This implies that for sufficiently low-productivity realizations, firms incur ex-post losses. However, if the firm is financially constrained, it cannot incur ex-post losses and loses its ability to fully insure workers. Low enough productivity shocks make the borrowing constraint binding, and the shocks fully pass through to the worker's wage. Workers are not committed to staying in the match and separate into unemployment once they hit their participation constraint (i.e., once the value from separating, becoming unemployed, and looking for a new job is greater than staying). Therefore, the welfare effect of financial constraints is twofold: First, firms cannot provide as much insurance to workers as they would like, and second, workers separate sooner.

We derive closed-form expressions for optimal layoff taxes and optimal short-time work schemes, depending on how many firms are financially constrained. The theoretical results show that the sole purpose of layoff taxes is to offset the fiscal externality of the unemployment insurance system on financially unconstrained firms. However, layoff taxes cannot correct inefficient separations or address the lack of insurance for workers in financially constrained firms. The intuition is straightforward: once financial constraints become binding, firms can no longer absorb shocks, which are instead passed on to workers in the form of reduced income. Even though firms would like to keep workers employed to avoid future layoff taxes, workers separate unitarily when income falls low enough, rendering the layoff tax ineffective.[4]

By contrast, short-time work is particularly effective in supporting financially constrained firms. By supplementing the income of workers who experience reduced hours, it incentivizes workers to remain attached to the firm. In addition, it provides workers with income insurance, when constrained firms can no longer provide it. We proceed to show that short-time work cannot simply be replaced by providing loans to financially constrained firms. This is because whereas short-time work increases the joint surplus of firms and workers by providing insurance to workers, loans merely shift funds across periods and therefore have no bearing on a worker's wage and decision to separate. Unlike short-time work, loans therefore do not prevent inefficient separations.

When the share of financially constrained firms is low, layoff taxes are clearly preferable to short-time work, as the distortionary cost of reduced working hours under short-time work outweighs the cost of insufficient support for financially constrained firms under a layoff tax regime. However, as the prevalence of financially constrained firms increases, the trade-off shifts. Both theoretically and quantitatively, we demonstrate that short-time work becomes the superior policy instrument once a sufficiently large share of firms face

[4] In the US, workers who separate following a substantial deterioration in job conditions remain eligible for unemployment insurance benefits.

financial constraints. Calibrating the model to the US economy and targeting a 45 percent unemployment insurance replacement rate, we find that short-time work is the superior policy instrument if 27.5 percent of firms in the economy are financially constrained. If the planner is allowed to optimize unemployment insurance benefits jointly with either short-time work or layoff taxes, this threshold drops to 24 percent. We discuss how existing empirical measures of financial constraints can be interpreted through the lens of the model. This evidence suggests that values of these magnitudes are plausible in the US economy.

Finally, we study the joint design of short-time work and layoff taxes.[5] Since layoff taxes are most effective at deterring separations among financially unconstrained firms, and short-time work is most suitable for preserving matches at financially constrained firms, their optimal combination can improve upon using either instrument in isolation when sufficiently many firms of both types are present in the economy.

As the share of constrained firms rises, the optimal policy mix shifts from layoff taxes toward short-time work. We find that the welfare gains from combining the two instruments, relative to using the better of the two instruments in isolation, are largest when around 30 percent of firms are financially constrained.

Related Literature Our study contributes to four branches of the economic literature. First, we add to the literature on optimal unemployment insurance. The trade-off between the benefits of unemployment insurance and the adverse effects on job searches and separations has been a recurring theme in the economic literature (e.g., Shavell and Weiss, 1979; Baily, 1978). How UI should be used optimally has been examined in seminal papers such as Hopenhayn and Nicolini (1997) and Chetty (2006). More recently, Landais et al. (2018) show how UI should vary with labor market tightness, and Kroft and Notowidigdo (2016) show evidence that the moral hazard costs of UI are procyclical. We contribute to this literature by evaluating policy tools that can mitigate the adverse externalities of unemployment insurance.

Second, we contribute to the branch of the literature that concerns itself with layoff taxes, often emphasizing their effectiveness in overcoming UI fiscal externalities. Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008) propose frameworks in which layoff taxes can decentralize the planner solution. Closely related to our work, Jung and Kuester (2015) and Michau (2015) take a Ramsey planner approach and examine optimal unemployment insurance with layoff taxes and vacancy subsidies in a DMP. Duggan et al. (2023) show that layoff taxes can act as a stabilizer over the business cycle. Ratner (2013) makes the point that

[5]The joint design of the two instruments is of practical relevance. Thirty US states operate short-time compensation programs alongside the experience-rated unemployment insurance system.

the experience-rated UI system in the US reduces layoffs but also hampers hiring. Similarly, Johnston (2021) finds that increases in layoff taxes lead to less hiring. We contribute to this literature by introducing firm borrowing constraints and showing that they reduce the effectiveness of deterring separations with layoff taxes.

Third, we add to a growing body of research on short-time work, though the literature remains divided on its overall usefulness. Some studies emphasize potential inefficiencies. For example, Burdett and Wright (1989) argue that short-time work encourages inefficient reductions in working hours. Cooper et al. (2017) highlight the risk of subsidizing employment at unproductive firms, while Giupponi and Landais (2023) raise concerns about impeding beneficial worker reallocations, though they also note that short-time work may support firms in maintaining efficient levels of labor hoarding. Cahuc et al. (2021) highlight the potential windfall effects of short-time work.

Other studies focus on the potential advantages of short-time work. Balleer et al. (2016) show that it can introduce valuable flexibility at the intensive margin of employment. Giupponi et al. (2022) argue that short-time work complements unemployment insurance by insuring against different types of labor market shocks. Similarly, Braun and Brügemann (2017) analyze optimal unemployment insurance and short-time work jointly within an implicit contract model.

Despite these advancements, the relationship between short-time work and unemployment insurance remains insufficiently understood, as emphasized in a comprehensive review by Cahuc (2024). Addressing this gap, Stiepelmann (2026) introduces the analysis of optimal short-time work policy and optimal unemployment insurance into a search-and-matching framework. Building on this foundation, our study contributes to this branch of the literature by enhancing the understanding of the interplay between short-time work and unemployment insurance by highlighting the pivotal role of financial constraints.

Finally, we relate to the literature on firms' financial constraints. Drechsel (2023) shows that earnings-based borrowing constraints can substantially alter the transmission of macroeconomic shocks relative to standard collateral constraints. One strand of the literature shows that financial constraints matter for monetary shock transmissions (e.g., Ottonello and Winberry, 2020) or innovation (e.g., Cascaldi-Garcia et al., 2023). In the labor market, fewer financial constraints are empirically shown to lead to higher employment (e.g., Campello et al., 2010; Duygan-Bump et al., 2015; Fonseca and Van Doornik, 2022). We contribute to this literature by incorporating financial constraints into a search-and-matching framework with endogenous employment responses. Unlike Wasmer and Weil (2004) and Iliopulos et al. (2019), who model hiring costs as working capital so that constraints primarily affect vacancy posting, in our setting, financial constraints operate through separations, as constrained firms are less able to sustain unprofitable matches.

2 Model

2.1 Basic Assumptions

Set-Up Time is discrete. There is a unit mass of workers. Workers can be either employed or unemployed. While unemployed, workers search for vacant jobs and receive unemployment benefits b. Unemployed workers and firms with vacancies are matched randomly. The number of contacts is governed by the Cobb-Douglas matching function $m(v,n) = \bar{m} \cdot v^{(1-\gamma)} \cdot (1-n)^{\gamma}$, where v is the mass of vacancies and n is the mass of employed workers. We denote the job-finding rate by f, the vacancy-filling rate by q, and the separation rate by ρ. Let θ denote labor market tightness, i.e. $\theta = \frac{v}{1-n}$.

A worker-firm match produces output y according to the production function

$$y(\epsilon, h) = \epsilon \cdot h^{\alpha} - (\mu_{\epsilon} - \epsilon) \cdot k_f$$

where ϵ is the realization of a productivity shock $\tilde{\epsilon}$ satisfying $\log \tilde{\epsilon} \sim \mathcal{N}(\mu, \sigma^2)$ with cdf $G(\epsilon)$ and pdf $g(\epsilon)$. New productivity shocks arrive with probability λ. h is the number of working hours worked by the worker, and A is total factor productivity. The term $(\mu_{\epsilon} - \epsilon) \cdot k_f$ is a cost shock. While workers are employed they receive a salary $w(\epsilon)$ and suffer disutility $\phi(h) = \frac{h^{(1+\psi)}}{1+\psi}$ from working h hours.

Firm profits go to firm owners, of whom there is a mass of ν_f.

Preferences Workers are risk-averse with a concave flow utility function over consumption, net of work disutility $u(c - \phi(h))$. Because utility is defined over consumption-equivalent units, we introduce notation for production in consumption-equivalent units for use in later expressions and let $z(\epsilon, h) = y(\epsilon, h) - \phi(h)$. Firm owners have the same flow utility function u but draw utility only from their consumption c_f (i.e., $u(c_f)$).

Government Policy The government can choose between two policy regimes: the short-time work (STW) regime and the layoff tax (LT) regime. Since the chosen policy regime has implications for firm and worker value, bargaining, and equilibrium, the remaining model assumptions are described in two separate sections for each respective policy regime in the following.

2.2 The Model with Layoff Taxes

Government Under the LT regime, firms have to pay layoff taxes F once the worker and firm jointly agree to separate. In the following, let ρ denote the probability that firms and workers separate. The layoff tax F and the unemployment insurance benefits b are set by the government. To finance the policy regime, firms pay a lump-sum tax τ whenever their productivity changes. The government budget constraint is

$$n^s \cdot \tau = (1-n) \cdot b - n^s \cdot \rho \cdot F$$

where n^s is the mass of matches that receive a new productivity shock.

Firms There are two types of firms: financially constrained and financially unconstrained firms. Firms are financially constrained with probability p. When constrained, once the productivity shock has realized to ϵ, a firm can borrow no more than its expected value, conditional on its realized productivity. Specifically,

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$

must hold. $\bar{J}$ is the expected firm value once a new shock arrives, and $J^c(\epsilon)$ is the value of a constrained firm with productivity draw ϵ. The constrained (monthly) wage function $w^c(\epsilon)$ will be made explicit in the bargaining section below. The bargained-over hours functions will depend on productivity ϵ but not on the financial constraint. Anticipating this, we save on notation and denote $h^u(\epsilon)$ and $h^c(\epsilon)$ as $h(\epsilon)$. Note that with persistent shocks ($\lambda < 1$), a low realization of productivity will lead to a tighter borrowing constraint.

The value of an unconstrained firm *after* the productivity shock has realized to ϵ is

$$J^u(\epsilon) = y(\epsilon, h(\epsilon)) - w^u(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) J^u(\epsilon)\right]$$

The value of a constrained firm *after* the productivity shock has realized to ϵ is

$$J^c(\epsilon) = y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$

Here, $\bar{J}$ is the expected firm value *before* the shock has realized and *before* it becomes known if the firm is constrained. It is given by

$$\bar{J} = -\tau + (1-p)\left(\int_{\epsilon_s^u}^{\infty} J^u(\epsilon)\, dG(\epsilon)\right) + p\left(\int_{\epsilon_s^c}^{\infty} J^c(\epsilon)\, dG(\epsilon)\right) \quad - \rho \cdot \beta \cdot F$$

Firms separate from a worker once productivity drops below the separation threshold ϵ_s^u or ϵ_s^c for the unconstrained and constrained case, respectively. We denote the probability of separation when a new shock arrives as ρ. When separation occurs, the firm has to pay a layoff tax F. In accordance with the experience rating system in the US, we assume that layoff taxes are backloaded, i.e., they are paid with a one-period delay. This ensures that financial constraints do not bind at the time of separation. The layoff tax is therefore discounted by β.[6]

Firms can freely enter the labor market and post vacancies at cost k_v. The mass of vacancies v must therefore solve

$$\frac{k_v}{q} = \beta \cdot \bar{J}$$

Workers Employed workers can work at financially constrained and unconstrained firms. The value of a worker at an unconstrained firm *after* the productivity shock realizes to ϵ is

$$V^u(\epsilon) = u(w^u(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^u(\epsilon)\right]$$

The value of a worker at a constrained firm *after* the productivity shock realizes to ϵ is

$$V^c(\epsilon) = u(w^c(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) V^c(\epsilon)\right]$$

Here, $\bar{V}$ denotes the expected worker value *before* a new productivity shock has realized. It is given by

$$\bar{V} = (1-p)\left(\int_{\epsilon_s^u}^{\infty} V^u(\epsilon)\, dG(\epsilon)\right) + p\left(\int_{\epsilon_s^c}^{\infty} V^c(\epsilon)\, dG(\epsilon)\right) + \rho \cdot U$$

With probability ρ, a worker becomes unemployed. The value of an unemployed worker U is given by

$$U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1-f) \cdot U\right]$$

Bargaining With financial constraints, bargaining takes place before the productivity shock is realized and before it is known whether the firm is constrained. For each possible state $\epsilon \in \mathbb{R}_+$ worker and firm agree on the total monthly wage functions $w^u(\epsilon)$ and $w^c(\epsilon)$,

[6] In principle, a firm could remain financially constrained in the subsequent period and thus be unable to pay the layoff tax, potentially leading to bankruptcy. We abstract from this possibility and assume that all firms are able to meet their tax obligations when due.

the hours functions $h^u(\epsilon)$ and $h^c(\epsilon)$, on the separation threshold when unconstrained ϵ_s^u, as well as the separation threshold when constrained ϵ_s^c.

Further, there is no commitment to the contract on the workers' side. This means that if the outside option of the worker becomes weakly better than staying with the match, the worker separates unitarily:

$$V^u(\epsilon) < U, \qquad V^c(\epsilon) < U$$

Formally, the bargaining outcome solves

$$\max_{\mathcal{X}} \quad \bar{J}^{1-\eta} \left(\bar{V} - U\right)^{\eta}$$

subject to

$$\begin{aligned} V^u(\epsilon) \geq U, \quad V^c(\epsilon) \geq U & \qquad\qquad \text{(commitment problem)} \\ y(\epsilon, h^c(\epsilon)) - w^c(\epsilon) \geq -\beta\left[\lambda\bar{J} + (1-\lambda)J^c(\epsilon)\right] & \qquad\qquad \text{(financial constraint)} \end{aligned}$$

where the choice set is given by

$$\mathcal{X} = \left\{w^u(\epsilon), w^c(\epsilon), h^u(\epsilon), h^c(\epsilon), \epsilon_s^u, \epsilon_s^c\right\}.$$

where η denotes the bargaining power of workers. Note that firms and workers have to take the commitment problem of the worker and the financial constraints of the firm into account when writing their contracts. Appendix G.1 derives the bargaining outcomes in detail.

Since workers are risk-averse and firms are risk-neutral, the bargaining outcome is that firms insure workers against low-productivity states. Workers will accept a lower average wage in exchange for insurance. As long as the firm is not constrained (either because it is an unconstrained firm or because constraints do not bind) the firm will *fully* insure the worker against productivity risk and the total monthly wage $w(\epsilon)$ will be set to a constant consumption equivalent c^w, equating worker utility across all realizations of ϵ that do not lead to binding constraints. c^w is pinned down by the condition

$$u'(c(\epsilon)) = u'(c^w)$$

Once a firm becomes constrained and the constraint binds, it instead pays the maximum monthly consumption equivalent $c^w(\epsilon)$ it can still afford:

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \beta \cdot \bar{J}$$

The hours function that worker and firm agree on is, as already mentioned, the same for constrained and unconstrained firms. It is pinned down by

$$\alpha \cdot \epsilon \cdot h(\epsilon)^{\alpha-1} = h(\epsilon)^{\psi}$$

in both the constrained and the unconstrained case. The equilibrium, therefore, only contains one general hours function $h(\epsilon)$. Since $h(\epsilon)$ equates the marginal disutility of work and the marginal product of labor, working hours are always set efficiently.

Note that, because under bargaining, each productivity level ϵ will imply a working hours level $h(\epsilon)$, we write $z(\epsilon, h(\epsilon))$ simply as $z(\epsilon)$. The job-destruction equations pin down the bargained-over separation thresholds of constrained and unconstrained matches:

$$\begin{aligned} z(\epsilon_s^u) + [1 - \beta \cdot (1 - \lambda)] \cdot \beta \cdot F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} &= 0 \\ \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1 - \eta} \cdot \frac{k_v}{q} &= 0 \end{aligned}$$

Note that F enters the separation condition for unconstrained firms only. In the unconstrained case, firms fully insure workers' income against idiosyncratic productivity shocks, such that a low idiosyncratic productivity level is not passed on to the worker. As a result, the worker has no incentive to separate unilaterally ($V^u(\epsilon_s^u) \geq U$). The layoff tax makes separations costly for the firm. It therefore lowers the separation threshold ϵ_s^u. However, a sufficiently low-productivity realization in a financially constrained firm causes the constraint to bind, forcing the firm to pass the shock directly through to the worker's income. For low enough productivity levels, the continuation value of employment falls below the worker's outside option, such that $V^c(\epsilon_s^c) \leq U$, and the worker separates unilaterally. Although the firm would prefer to retain the worker in order to avoid paying the backloaded layoff tax, it lacks the financial resources to offer a wage that satisfies the worker's participation constraint. Consequently, the layoff tax does not affect the separation decision.

Labor Markets The separation rate ρ is given by $\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$ where $\rho^u = G(\epsilon_s^u)$ and $\rho^c = G(\epsilon_s^c)$. The steady-state law of motion for employment is

$$n = (1 - \lambda) \cdot n + (1 - \rho) \cdot n^s,$$

where n^s denotes the number of matches that received a new shock

$$n^s = f \cdot (1-n) + \lambda \cdot n$$

The mass of unemployed workers can be expressed as $u = 1 - n$. The mass of workers who work for unconstrained firms n^u and the mass of workers who work at constrained firms n^c are given by:

$$\begin{aligned} n^u &= \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s \\ n^c &= \frac{p}{\lambda} \cdot (1-\rho^c) \cdot n^s \end{aligned}$$

Equilibrium A steady-state equilibrium consists of the working hours function $h(\epsilon)$; the consumption equivalent c^w paid as the monthly wage to workers without binding constraints, the consumption-equivalent function $c^w(\epsilon)$ paid when financial constraints bind; the separation thresholds ϵ_s^u and ϵ_s^c, the productivity level at which the borrowing constraint becomes binding ϵ^p; and labor market flows, i.e. the job-finding rate f, the vacancy-filling rate q, and the separation rates ρ^u and ρ^c as well as n. The exact equations pinning down equilibrium and their derivation are delegated to Sections G and I in the appendix.

2.3 The Model with Short-Time Work

Government Under the STW regime, firms become eligible for short-time work benefits if they set their working hours on short-time work $h_{\text{stw}}(\epsilon)$ below a threshold D. In this case, the worker receives $s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))$ worth of total benefits. $\bar{h}$ is a parameter that reflects the normal level of working hours. In essence, for every hour the worker works less than she would under normal circumstances, she receives compensation s_{stw}. Choosing an hours threshold D is a realistic implementation of what short-time work rules mandate in most economies that have implemented short-time work.[7] Enforcing a reduction in working hours can help screen out firms that are actually productive from receiving subsidies because only unproductive firms will find reducing hours to be optimal (Teichgräber et al., 2022).

In principle, the government chooses the policy parameters D, s_{stw}, and b. However, since the hours functions $h^u(\epsilon)$, $h^c(\epsilon)$, $h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$ will always be strictly decreasing in ϵ,

[7] In practice, this is implemented as a minimum reduction of hours threshold - which in our model is given by $\frac{D-h}{h}$ and is implicitly chosen by the government through D. Most economies with STW have such a minimum hours reduction condition according to Hijzen and Martin (2013).

choosing D is equivalent to setting the eligibility productivity threshold ϵ_{stw} that will induce the same hours threshold D. The resulting government budget constraint is

$$\begin{aligned} n^s \cdot \tau = \frac{1-p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^u}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^u\}} s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\ + \frac{p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\ + (1-n) \cdot b \end{aligned}$$

where n^s is the mass of matches that receive a new productivity shock. ϵ_s^u and ϵ_s^c are the separation thresholds of matches with access to short-time work of unconstrained and constrained matches, respectively. The max operators in the integral limits are necessary because the government can choose an eligibility condition that is so tight that all matches separate without gaining access to short-time work first.

Firms Again, firms can be either financially constrained or unconstrained. With short-time work, they can also currently be on short-time work or not. Firms are financially constrained with probability p. When constrained, once the productivity shock has realized to ϵ, a firm can borrow no more than its expected value, conditional on its realized productivity. Specifically, without access to short-time work

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$

and with short-time work

$$y(\epsilon, h_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c_{\text{stw}}(\epsilon)\right]$$

must hold. $\bar{J}$ is the expected firm value once a new shock arrives, and $J^c(\epsilon)$ is the value of a constrained firm without short-time work, and $J^c_{\text{stw}}(\epsilon)$ is the value of a constrained firm on short-time work. The constrained monthly wage function $w^c(\epsilon)$ will be made explicit in the bargaining section below. As in the model with layoff taxes, the bargained-over hours functions will depend on productivity ϵ but not on the financial constraint. Anticipating this, we save on notation and denote $h^u(\epsilon)$ and $h^c(\epsilon)$ as $h(\epsilon)$ and $h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$ as $h_{\text{stw}}(\epsilon)$. Further we write $z(\epsilon)$ instead of $z(\epsilon, h(\epsilon))$ and $z_{\text{stw}}(\epsilon)$ instead of $z(\epsilon, h_{\text{stw}}(\epsilon))$. The value of an unconstrained firm *without* short-time work *after* the productivity shock has realized to

ϵ is

$$J^u(\epsilon) = y(\epsilon, h(\epsilon)) - w^u(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^u(\epsilon)\right]$$

The value of an unconstrained firm *with* short-time work *after* the productivity shock has realized to ϵ is

$$J^u_{\text{stw}}(\epsilon) = y(\epsilon, h_{\text{stw}}(\epsilon)) - w^u_{\text{stw}}(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^u_{\text{stw}}(\epsilon)\right].$$

The value of a constrained firm *without* short-time work *after* the productivity shock has realized to ϵ is

$$J^c(\epsilon) = y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^c(\epsilon)\right]$$

The value of a constrained firm *with* short-time work *after* the productivity shock has realized to ϵ is

$$J^c_{\text{stw}}(\epsilon) = y(\epsilon, h_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) + \beta \cdot \left[\lambda \bar{J} + (1-\lambda) J^c_{\text{stw}}(\epsilon)\right]$$

Here, $\bar{J}$ is the expected firm value *before* the shock has realized. It is given by

$$\begin{aligned}\bar{J} = -\tau + (1-p) &\left(\int_{\max\{\xi^u_s, \epsilon_{\text{stw}}\}}^{\infty} J^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\epsilon_{\max\{\epsilon^u_s, \text{stw}\}}} J^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\ + p &\left(\int_{\max\{\xi^c_s, \epsilon_{\text{stw}}\}}^{\infty} J^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon^c_s, \epsilon_{\text{stw}}\}} J^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \right)\end{aligned}$$

The max operators in the integral bounds reflect that a sufficiently strict eligibility threshold ϵ_{stw} can exclude certain matches from accessing short-time work, even if their productivity would allow them to survive with such support but not without it. In this case, otherwise, viable matches are denied access to the short-time work scheme, leading to unnecessary separations. Firms can freely enter the labor market and post vacancies at cost k_v. The mass of vacancies v must therefore solve

$$\frac{k_v}{q} = \beta \cdot \bar{J}$$

Workers The value of a worker at an unconstrained firm *without* short-time work *after* the productivity shock has realized to ϵ is

$$V^u(\epsilon) = u(w^u(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda)V^u(\epsilon)\right]$$

The value of a worker at an unconstrained firm *with* short-time work *after* the productivity shock has realized to ϵ is

$$V^u_{\text{stw}}(\epsilon) = u(w^u_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) - \phi(h_{\text{stw}}(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda)V^u_{\text{stw}}(\epsilon)\right]$$

The value of a worker at a constrained firm *without* short-time work *after* the productivity shock has realized to ϵ is

$$V^c(\epsilon) = u(w^c(\epsilon) - \phi(h(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda)V^c(\epsilon)\right]$$

The value of a worker at a constrained firm *with* short-time work *after* after the productivity shock realizes to ϵ is

$$V^c_{\text{stw}}(\epsilon) = u(w^c_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) - \phi(h^c_{\text{stw}}(\epsilon))) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda)V^c_{\text{stw}}(\epsilon)\right]$$

Here, $\bar{V}$ is the expected worker value *before* a new productivity shock has realized. It is given by

$$\begin{aligned}
\bar{V} = &(1-p)\left(\int_{\max_{\{\epsilon_{\text{stw}}, \xi^u_s\}}}^{\infty} V^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^u_s\}} V^u_{\text{stw}}(\epsilon)\, dG(\epsilon)\right) \\
&+ p\left(\int_{\max\{\epsilon_{\text{stw}}, \xi^c_s\}}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^c_s\}} V^c_{\text{stw}}(\epsilon)\, dG(\epsilon)\right) \\
&+ \rho \cdot U
\end{aligned}$$

where ρ is the separation rate. The value of an unemployed worker U is given by

$$U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1-f) \cdot U\right]$$

Bargaining Bargaining takes place before both the realization of productivity shock and whether the firm is constrained or not become known. With short-time work workers and firms bargain about a few more items than in the layoff tax regime.

Further, there is no commitment to the contract on the workers' side. This implies that workers will unilaterally choose to leave the match once the outside option of becoming unemployed offers a higher expected utility than remaining employed at the firm

$$V^u(\epsilon) < U, \qquad V^u_{\text{stw}}(\epsilon) < U$$
$$V^c(\epsilon) < U, \qquad V^c_{\text{stw}}(\epsilon) < U$$

in the case with and without short-time work, respectively. For each possible state $\epsilon \in \mathbb{R}_+$ worker and firm agree on the total monthly wage functions $w^u(\epsilon)$ and $w^c(\epsilon)$, as well as the total monthly wages paid in both cases while on short-time work $w^u_{\text{stw}}(\epsilon)$ and $w^c_{\text{stw}}(\epsilon)$. The hours functions, likewise, are bargained over separately - for times in which the firm has no access to short-time work ($h^u(\epsilon)$ and $h^c(\epsilon)$) and for times on short-time work ($h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$). Lastly, the separation thresholds are also agreed upon for both cases: access to short-time work and no access to short-time work. We denote the separation thresholds without short-time work by ξ^u_s and ξ^c_s. ϵ^u_s and ϵ^c_s denote the separation thresholds while on short-time work.

Formally, the bargaining outcome solves

$$\max_{\mathcal{X}} \quad \bar{J}^{1-\eta} \left(\bar{V} - U\right)^{\eta}$$

subject to

$$\begin{aligned}
V^u(\epsilon) \geq U,\, V^c(\epsilon) \geq U,\, V^u_{\text{stw}}(\epsilon) \geq U,\, V^c_{\text{stw}}(\epsilon) \geq U \quad &\text{(commitment problem)} \\
y(\epsilon, h^c(\epsilon)) - w^c(\epsilon) \geq -\beta\left[\lambda\bar{J} + (1-\lambda)J^c(\epsilon)\right] \quad &\text{(financial constraint)} \\
y(\epsilon, h^c_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) \geq -\beta\left[\lambda\bar{J} + (1-\lambda)J^c_{\text{stw}}(\epsilon)\right] \quad &\text{(financial constraint, STW)}
\end{aligned}$$

where the choice set is given by

$$\mathcal{X} = \{w^u(\epsilon),\, w^c(\epsilon),\, w^u_{\text{stw}}(\epsilon),\, w^c_{\text{stw}}(\epsilon),\, h^u(\epsilon),\, h^c(\epsilon),\, h^u_{\text{stw}}(\epsilon),\, h^c_{\text{stw}}(\epsilon),\, \epsilon^u_s,\, \epsilon^c_s,\, \xi^u_s,\, \xi^c_s\}.$$

Here, η denotes the bargaining power of workers. Firms and workers take both the workers' participation constraints and the firms' financial constraints into account when writing contracts. Appendix H.1 derives the bargaining outcomes.

Since workers are risk-averse and firms are risk-neutral, the bargaining outcome is still that firms insure workers against low-productivity states, while workers will accept a lower average wage in exchange for insurance. Much as in the layoff tax model, as long as the firm is not constrained (either because it is an unconstrained firm or because constraints do not bind) the firm will *fully* insure the worker against productivity risk and the total monthly wage $w(\epsilon)$ will be set to a constant consumption equivalent c^w, equating worker utility across all realizations of ϵ that do not lead to binding constraints. This also includes the state where workers are on STW:

$$u'(c^w(\epsilon)) = u'(c^w_{\text{stw}}(\epsilon)) = u'(c^w)$$

Once a firm becomes constrained and the constraint binds, it instead pays the maximum monthly wage $c^w(\epsilon)$ it can still afford. This will depend on whether the firm has access to short-time work or not:

$$\begin{aligned} c^w(\epsilon) &= z(\epsilon) + \lambda \cdot \frac{k_v}{q} \\ c^w_{\text{stw}}(\epsilon) &= z_{\text{stw}}(\epsilon) + s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \frac{k_v}{q}. \end{aligned}$$

When the firm goes on short-time work, the government compensates the worker for the reduced working hours. This means that when the firm has a binding borrowing constraint and can only afford to pay the worker $c_{\text{stw}}(\epsilon)$ in monthly wages, going on short-time work can still increase the worker's income. The hours function that worker and firm agree on also differs depending on whether the firm is on short-time work or not. The two cases are pinned down by the conditions.

$$\begin{aligned} \alpha \cdot \epsilon \cdot h(\epsilon)^{\alpha-1} &= h(\epsilon)^{\psi} \\ \alpha \cdot \epsilon \cdot h_{\text{stw}}(\epsilon)^{\alpha-1} &= h_{\text{stw}}(\epsilon)^{\psi} + s_{\text{stw}}. \end{aligned}$$

Again, the hours functions turn out to be independent of financial constraints and equilibrium; therefore, they only contain the two general hours functions $h(\epsilon)$ and $h_{\text{stw}}(\epsilon)$. Note that without short-time work, the bargaining outcome for working hours equates to the marginal product of labor and the marginal disutility from labor and thus sets working hours efficiently. With short-time work, working hours take the subsidy s_{stw} into account, and hours are set lower than the efficient number of working hours. This means that through inefficiently low working hours, the short-time work scheme introduces a distortion into the economy. The bargained-over separation thresholds are pinned down by the job-destruction equations:

$$
\begin{aligned}
z(\xi_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} &= 0 \\
z_{\text{stw}}(\epsilon_s^u) + s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} &= 0 \\
\frac{u(c(\xi_s^u)) - u(b)}{u'(c^w)} + (\lambda - f)\frac{\eta}{1-\eta} \cdot \frac{k_v}{q} &= 0 \\
\frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f)\frac{\eta}{1-\eta} \cdot \frac{k_v}{q} &= 0
\end{aligned}
$$

In unconstrained firms, the separation decision is determined as a bargaining outcome: firms and workers separate once their joint surplus becomes negative. Importantly, short-time work benefits are not included in the worker's utility function. The rationale is straightforward—under short-time work, the firm continues to insure the worker's income against idiosyncratic productivity shocks. The worker receives a constant consumption equivalent c^w, regardless of the realization of ϵ. As a result, the worker has no incentive to separate unilaterally.

In constrained firms, the worker's commitment problem can become binding: low-productivity realizations are passed through to the worker's income. If income falls sufficiently, the worker chooses to separate. Short-time work increases the worker's available income in such cases, thereby increasing the worker's incentive to remain with the firm. In essence, short-time work operates by reducing the wage burden on the firm required to sustain this consumption level. In effect, short-time work lowers the cost of providing income insurance, thereby reducing the firm's incentive to initiate separations.

Labor Markets The separation rate ρ is given by $\rho = (1-p)\cdot\rho^u + p\cdot\rho^c$ where $\rho^u = G(\epsilon_s^u) + G(\max\{\xi_s^u, \epsilon_{\text{stw}}\}) - G(\max\{\epsilon_s^u, \epsilon_{\text{stw}}\})$ and $\rho^c = G(\epsilon_c^c) + G(\max\{\xi_s^c, \epsilon_{\text{stw}}\}) - G(\max\{\epsilon_s^c, \epsilon_{\text{stw}}\})$. The remaining worker flows are given by $f = \frac{m}{1-n}$ and $q = \frac{m}{v}$. The steady-state law of motion for employment is

$$
n = (1-\lambda)\cdot n + (1-\rho)\cdot n^s,
$$

where n^s denotes the number of matches that received a new shock

$$
n^s = f \cdot (1-n) + \lambda \cdot n
$$

Unemployment can be denoted as $u = 1 - n$. The number of workers who work for unconstrained firms n^u and the number of workers who work at constrained firms n^c are given by:

$$n^u(p) = \frac{1-p}{\lambda} \cdot (1 - \rho^u) \cdot n^s$$
$$n^c(p) = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

Equilibrium A steady-state equilibrium consists of the working hours functions $h(\epsilon)$, $h_{\text{stw}}(\epsilon)$; the consumption equivalent c^w paid as the monthly wage to workers without binding constraints; the consumption-equivalent wage $c^w(\epsilon)$ and c^w_{stw} paid when financial constraints bind; the separation thresholds ξ^u_s, ξ^c_s, ϵ^u_s and ϵ^c_s; and labor market flows, i.e. the job-finding rate f, the vacancy-filling rate q, and the separation rates ρ^u and ρ^c as well as n. The exact equations pinning down equilibrium and their derivation are delegated to Sections H and I in the appendix.

3 Optimal Policy

3.1 The Ramsey Problem

To show the differences in how short-time work and layoff taxes act against the fiscal externality of UI on separations, we take a Ramsey planner approach. We set up separate Ramsey problems for the layoff tax regime and the short-time work regime and proceed to compare optimal policies and their implications. In both cases, the Ramsey planner maximizes welfare, subject to the equilibrium constraints stated in Sections G and H in the appendix. To make the theoretical results more concise, we henceforth assume that $\beta \to 1$ for the remainder of the paper.

To state the welfare function formally, recall that there is a mass of firm owners v^f. Firm owners have the same preferences over consumption as workers and receive equal shares of firm profits in the economy. Further, let Ω denote the *average* loss of production in consumption-equivalent units due to distortions in working hours:

$$\Omega = \frac{1}{1-\rho}\left((1-p)\cdot\underbrace{\int_{\epsilon_s^u}^{\max\{\epsilon_{\text{stw}},\epsilon_s^u\}}\Omega(\epsilon)\,dG(\epsilon)}_{:=(1-\rho^u)\Omega^u} + p\cdot\underbrace{\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}\Omega(\epsilon)\,dG(\epsilon)}_{:=(1-\rho^c)\Omega^c}\right)$$

where

$$\Omega(\epsilon) = z(\epsilon) - z_{\text{stw}}(\epsilon).$$

Stiepelmann (2026) shows that $\Omega \geq 0$, $\frac{\partial\Omega}{\partial s_{\text{stw}}} \geq 0$ and $\frac{\partial\Omega}{\partial \epsilon_{\text{stw}}} \geq 0$.

The welfare function is given by

$$\begin{aligned} W(\pi) &= n^u \cdot u(c^w) + n^c \cdot u^c + (1-n)\cdot u(b) \\ &\quad + v^f \cdot u((n\cdot(z-\Omega) - n^u\cdot c^w - n^c\cdot e^c - \tau(b) - \theta\cdot(1-n)\cdot k_v)/v^f) \end{aligned} \tag{1}$$

where n^u is the mass of workers employed at firms that do not hit their constraint and n^c is the mass of workers whose employers have a binding borrowing constraint. u^c is the *average* utility of a worker at a firm that has hit its borrowing constraint. z is the *average* production, net of work disutility, and Ω is the *average* loss of production in consumption-equivalent units due to distortions in working hours: Under the layoff tax regime, $\Omega = 0$ will always hold. The *average* total monthly wage of a worker at a firm that has hit its borrowing constraint is e^c. The UI tax $\tau(b) = (1-n)\cdot b$ is used to finance the UI system.

π is the vector of policy parameters the planner can choose, i.e., $\pi = (b, F)$ in the layoff tax regime and $\pi = (b, \epsilon_{\text{stw}}, s_{\text{stw}})$ under the short-time work regime. The problem of the Ramsey planner is given by

$$\max_{\pi} W(\pi) \quad \text{s.t. equilibrium conditions are fulfilled}$$

The full problem is stated in Section I in the appendix. Since the analysis does not focus on distributional effects between firm owners and workers, we set the mass of firm owners ν^f such that the consumption equivalent of workers always equals the consumption of firm owners.

3.2 The Optimal Layoff Tax Regime

Before we turn our attention to how F is set optimally, we first look at how the planner sets b in the presence of a layoff tax. This allows us to illustrate how a layoff tax interacts with the fiscal externalities caused by unemployment insurance.

Setting the unemployment insurance level b must balance a trade-off. On the one hand, the Ramsey planner would like to insure the risk-averse worker against income losses after a separation. If there were no welfare costs to increasing b, the Ramsey planner would like to set b to a level that fully equates the utility of employed and unemployed workers. However, as is well known, unemployment insurance leads to fiscal externalities.

Increasing b raises the value of unemployment, allowing workers to bargain for higher wages. This increases separations in both financially constrained and unconstrained firms. The resulting decline in firm value in turn reduces vacancy posting. On top of that, the lower continuation value tightens firms' borrowing constraints, further limiting their ability to provide income insurance.

The resulting trade-off between more worker insurance and greater fiscal externalities is balanced by the Ramsey planner's first-order condition in Proposition 1.

Proposition 1. *The optimal unemployment insurance benefit b given layoff tax F must fulfill the following first-order condition*

$$
\begin{aligned}
\underbrace{(1-n)\cdot\left(\frac{u'(b)-u'(c^w)}{u'(c^w)}\right)}_{\textit{Income insurance to unemployed, MUIB}} &= \underbrace{\left(-\frac{df}{db}\right)^{ge}\cdot u\cdot L_v(b)}_{\textit{Less vacancy posting, MLV}} + \underbrace{\left(\frac{d\epsilon_s^u}{db}\right)^{ge}\cdot\left(-\frac{\partial n^u(p)}{\partial \epsilon_s^u}\right)\cdot L_s^u(b,F)}_{\textit{More separation in unconstrained firms, } MLS^u} \\
&+ \underbrace{\left(\frac{d\xi_s^c}{db}\right)^{ge}\cdot\left(-\frac{\partial n^c(p)}{\partial \epsilon_s^c}\right)\cdot L_{s,\xi}^c(b)}_{\textit{More separations in constrained firms, } MLS^c} \\
&+ \underbrace{MIE_b^c(b)}_{\textit{Less income insurance to employed}}
\end{aligned}
$$

where L_v denotes the social value of hiring an additional worker, L_s^u and L_s^c denote the social *value of a marginal match in an unconstrained and a constrained firm,*

$$L_v(b) = \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \bar{J} + \frac{1}{f} \cdot b \right)$$
$$L_s^u(b, F) = \lambda \cdot \left(\frac{1}{f} \cdot b - F \right)$$
$$L_s^c(b) = \frac{\lambda}{f} \cdot b$$

and MIE_b denotes the social costs from the insurance loss in unconstrained firms when UI benefits are set more generously.

$$MIE_b^c(b) = n^c(p) \cdot \frac{1}{1-\rho^c} \cdot \left(- \left(\frac{d\theta}{db} \right)^{ge} \right) \cdot \left(\int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right) \cdot k_v$$

PROOF: See Section J.6 in the appendix.

The term labeled $MUIB$ represents the marginal social benefit of a higher level of unemployment benefits b, stemming from improved income insurance for unemployed workers.

The term labeled MLV captures the marginal social loss associated with a decline in hiring induced by an increase in unemployment insurance benefits. The term comprises two components. First, L_v denotes the social value of hiring one additional worker. Second, $\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge} \cdot u$ represents the reduction in the number of new hires. Intuitively, higher unemployment benefits raise the worker's outside options, which puts upward pressure on wages and reduces the value of a worker for the firm. As a result, firms find it less profitable to post vacancies, leading to a decline in job creation.

The MLS terms denote the marginal social loss resulting from increased separations caused by UI benefits. Specifically, MLS^u captures the social cost arising from increased separations in unconstrained firms, while MLS^c captures the corresponding cost in constrained firms. Intuitively, higher unemployment insurance benefits raise workers' outside options, exerting upward pressure on wages. As a result, unconstrained firms want to separate earlier. Additionally, in constrained firms, workers are less willing to accept income reductions and thus enter unemployment sooner.

The term MLS^u consists of two components. First, L_s^u denotes the social value of a marginal match at an unconstrained firm. Second, $\left(\frac{\mathrm{d}\epsilon_s^u}{\mathrm{d}b}\right)^{ge} \cdot \left(-\frac{\partial n^u(p)}{\partial \epsilon_s^u}\right)$ represents the number of additional separations in unconstrained firms.

Similarly, MLS^c consists of L_s^c, the social value of a marginal match at a constrained firm, and $\left(\frac{\mathrm{d}\epsilon_s^c}{\mathrm{d}b}\right)^{ge} \cdot \left(-\frac{\partial n^c(p)}{\partial \epsilon_s^c}\right)$ which denotes the number of additional constrained matches lost due to higher UI benefits.

Finally, $MIE^c_b(b)$ denotes the social loss arising from reduced worker income insurance in financially constrained firms. Higher UI benefits lower the firm's continuation value and thereby tighten its financial constraint, limiting its ability to insure workers against adverse shocks.

Note that, in general, all these loss terms consist of the marginal social effect of b on the respective threshold (or labor market tightness in the case of MIE^c_b) multiplied by the mass of affected workers.

The layoff tax F enters the policy trade-off exclusively through the MLS^u term. Importantly, it does not affect any terms associated with financially *constrained* firms. The intuition is straightforward: once firms become financially constrained, they lose their ability to insure workers against adverse productivity shocks. As described in Section 2, low-productivity is then fully passed through to the worker's income. If the income falls sufficiently, the value of remaining employed falls below the value of unemployment ($V^c(\epsilon) < U$) and the worker will choose to become unemployed. Since the firm cannot sustain a wage that the worker is willing to work for, it cannot prevent the worker from separating. Thus, the firm has no way of avoiding paying layoff taxes, and layoff taxes have no bearing on the separation decision.

As a result, layoff taxes can only mitigate the negative fiscal externalities of UI by combating separations in *unconstrained* firms. This means that when $p = 0$, layoff taxes can act on all separation decisions in the economy. As p increases, the layoff taxes gradually lose their effectiveness until they lose their bite completely once $p = 1$.

On the upside, the planner can choose to completely eliminate inefficient separations in constrained firms. This can be done with the optimal layoff tax stated in Proposition 2.

Proposition 2. *The optimal backloaded layoff tax F is determined by its FOC:*

$$F = T_{UI} \cdot b + BE_{lt}(b) \tag{2}$$

Here, $T_{UI} = \frac{1}{f}$ denotes the expected time a worker spends on the UI system on average.

PROOF: See Section J.5 in the appendix.

The first component captures the fiscal externality of the unemployment insurance system. It reflects the total expected UI benefits a worker receives upon entering unemployment, and thus represents the fiscal cost that each separation imposes on the UI system.

BE_{lt} represents the bargaining effect. The formula reveals that Nash bargaining under risk aversion is not fully efficient: wages cannot perfectly distribute utility between firms and workers. Due to decreasing marginal utility, workers lose more from a wage cut than firms gain. Consequently, wage cuts reduce the joint surplus, leading to fewer vacancies, more separations, and potentially less private insurance.

The bargaining effect captures how layoff tax increases indirectly affect general equilibrium outcomes through wage adjustments. This same logic applies to the bargaining effects in subsequent formulas analyzing STW, though the mechanism is not repeated each time. Quantitatively, the effect is small, accounting for less than 2 percent of the fiscal externality in the case of layoff taxes.

Corollary 1 states that the optimal layoff tax implements the efficient number of separations in unconstrained firms.

Corollary 1. *(i) Optimal layoff taxes fully eliminate socially inefficient separations in* unconstrained *firms.*
(ii) If $p = 0$, then all inefficient separations are eliminated in the economy

PROOF: See Section J.5 in the appendix.

Note that our model extends the insights of Cahuc and Zylberberg (2008) and Blanchard and Tirole (2008) by embedding optimal layoff taxes within a modern search-and-matching framework of the labor market. Both of these earlier studies show that layoff taxes combined with unemployment insurance can decentralize the social planner's allocation in settings that focus solely on the separation margin. By contrast, search-and-matching models feature both a hiring margin and a separation margin. In the absence of financial constraints (i.e., when $p = 0$), our model confirms that the logic underlying layoff taxes - namely, their ability to correct the fiscal externality associated with unemployment insurance - carries over to this richer framework by eliminating inefficient separations. Jung and Kuester (2015) show that the combination of a vacancy subsidy, a layoff tax, and unemployment insurance can decentralize the planner's solution in a search-and-matching model, as the vacancy subsidy additionally operates on the vacancy-posting margin. Again, their results depend on the assumption that firms do not face financial constraints.

3.3 The Optimal Short-Time Work Regime

Under the short-time work regime, the Ramsey planner chooses s_{stw} and ϵ_{stw} (through D). Again, we first turn to how the planner sets the level of UI benefits b in the presence of the short-time work scheme. Setting the unemployment insurance level b must balance the same trade-off as before.

Proposition 3. *The optimal unemployment insurance benefit b given a short-time work scheme $(s_{stw}, \epsilon_{stw})$ must fulfill the following first-order condition:*

$$
\begin{aligned}
\underbrace{(1-n)\cdot\left(\frac{u'(b)-u'(c^w)}{u'(c^w)}\right)}_{\textit{Income insurance to unemployed, MUIB}} &= \underbrace{\left(-\frac{df}{db}\right)^{ge}\cdot u\cdot L_v(b)}_{\textit{Less vacancy posting, MLV}} \\
&+ \underbrace{\left(\frac{d\epsilon^u_s}{db}\right)^{ge}\cdot\left(-\frac{\partial n^u(p)}{\partial\epsilon^u_s}\right)\cdot L^u_s(b,s_{stw})}_{\textit{More separations in unconstr. firms, } MLS^u} \\
&+ \underbrace{\mathbb{1}(\epsilon_{stw}\geq\epsilon^c_s)\cdot\left(\frac{d\epsilon^c_s}{db}\right)^{ge}\cdot\left(-\frac{\partial n^c(p)}{\partial\epsilon^c_s}\right)\cdot L^c_s(b,s_{stw})}_{\textit{More separations in constr. firms with STW, } MLS^c_\epsilon} \\
&+ \underbrace{\mathbb{1}(\epsilon_{stw}\leq\xi^c_s)\cdot\left(\frac{d\xi^c_s}{db}\right)^{ge}\cdot\left(-\frac{\partial n^c(p)}{\partial\xi^c_s}\right)\cdot L^c_{s,\xi}(b)}_{\textit{More separations in constr. firms without STW, } MLS^c_\xi} \\
&+ \underbrace{MIE^c_b}_{\textit{Less income insurance to employed workers}}
\end{aligned}
$$

where L_v denotes the social value of hiring an additional worker

$$L_v = \left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\bar{J}+\frac{1}{f}\cdot b\right)$$

L^u_s, L^c_s and $L^c_{s,\xi}$ denote the social value of a marginal match in a constrained firm, in an unconstrained firm with access to STW, and in an unconstrained firm without access to STW,

$$L^i_s = \left(\frac{\lambda}{f}\cdot b - s_{stw}\cdot\left(\bar{h}-h_{stw}(\xi^i_s)\right)\right)\quad \textit{with}\quad i\in\{u,c\},\quad L^c_{s,\xi}=\frac{\lambda}{f}\cdot b$$

and MIE^c_b denotes the total insurance loss from marginally increasing UI benefits in con-

strained firms

$$MIE_b^c = n^c(p) \cdot \left(-\frac{d\theta}{db}\right)^{ge} \cdot \frac{1}{1-\rho^c} \cdot \left(\int_{\max\{\epsilon_{stw}, \xi_s^c\}}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right.$$
$$\left. + \int_{\epsilon_s^c}^{\max\{\epsilon_{stw}, \epsilon_s^c\}} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{stw}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right) \cdot k_v$$

PROOF: See Section K.6 in the appendix.

As in the layoff tax regime, the marginal social benefit of more generous unemployment benefits ($MUIB$) must equal the associated marginal welfare losses. These losses arise from three sources: fewer vacancies being posted (MLV), increased separations in both unconstrained and constrained firms (MLS^u and MLS^c), and a loss of insurance in constrained firms (MIE^c). Importantly, constrained firms feature two distinct separation margins. Matches may separate with short-time work (STW) support at the threshold ϵ_s^c, or without STW support at ξ_s^c.

However, there is a key difference between the FOCs under the layoff tax regime (Proposition 1) and the FOCs under the short-time work regime. The layoff tax parameter F entered only the MLS^u term, and the layoff tax could only counteract the adverse effect of unemployment benefits on separations in unconstrained firms. By contrast, the short-time work parameters $(s_{\text{stw}}, \epsilon_{\text{stw}})$ appear in all the terms except the MLV term. Unlike layoff taxes, short-time work has an effect not only on separations in unconstrained firms, but can also act on constrained firms, counteracting inefficiently high separations and inefficiently low insurance provision.

The core problem for financially constrained firms is their inability to absorb negative productivity shocks, which forces them to pass the resulting income loss directly onto workers. If the decline in income is large enough, workers may choose to separate. Short-time work functions as a subsidy that partially replaces this lost income, thereby incentivizing workers to remain with the firm. Through this channel, STW not only reduces inefficient separations, captured by the marginal social loss from separations (MLS) terms, but also enhances income insurance by acting on the MIE_b^c term.

A second difference is that the eligibility threshold ϵ_{stw} determines which firms have access to STW, leading to the MLS^c term being split into two parts. If the eligibility condition is strict, i.e., $\epsilon_{\text{stw}} < \xi_s^c$, then some firms that could otherwise survive with STW support are excluded from the system. Naturally, UI benefits also influence the separation threshold without STW support. Moreover, if ϵ_{stw} is set even more strictly to $\epsilon_{\text{stw}} < \epsilon_s^c$, then constrained firms are entirely excluded from the STW system. As a result, unemployment insurance benefits can no longer influence separations under the STW regime, eliminating the corresponding term from the equation.

It is clear from Proposition 3 that short-time work has the advantage of acting on the loss terms of constrained firms. On the downside, as argued in the bargaining paragraph of Section 2.3, firms on short-time work will distort their working hours, in turn leading to welfare losses. The Ramsey planner has to trade off the benefits of short-time work as a tool that can counteract the fiscal externalities of UI in both unconstrained and constrained firms, and the adverse effects of distortions in working hours. This is reflected in how optimal short-time work is set.

We begin by examining how the short-time work eligibility threshold ϵ_{stw} should be determined. The following proposition establishes that there are three candidates for the optimal eligibility threshold. The optimal threshold can be found by comparing the welfare levels induced by these three candidates. As discussed in Section 2.3, the optimal eligibility condition in terms of working hours D is uniquely implied by the optimal ϵ_{stw}.

Proposition 4. *Assume that the Ramsey planner never wants to use short-time work to impede vacancy posting by destroying the efficiency of the match.*[8] *The optimal eligibility threshold $\epsilon^{\star}_{stw}$ is one of three candidates pinned down by the following conditions:*

Candidate 1

Suppose it is optimal to set $\epsilon_{stw} \geq \xi_s^c$ (letting all firms in need access short-time work). If

$$MWL_{\epsilon_{stw}} + BE_{stw,2} \quad > \quad MIE^c_{\epsilon_{stw}}, \tag{3}$$

then the optimal threshold is given by $\epsilon_{stw} = \xi_s^c$.

Otherwise, the optimal threshold solves

$$MWL_{\epsilon_{stw}} + BE_{stw,2} \quad = \quad MIE^c_{\epsilon_{stw}}. \tag{4}$$

Here, $MWL_{\epsilon_{stw}} = n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ denotes the total welfare loss from marginally increasing STW

[8]In principle, this could happen in extreme cases of deviations from the Hosios condition. In this case, too many vacancies could introduce inefficiencies to such an extent that the Ramsey planner would use short-time work to destroy vacancies. Since this is neither realistic nor the focus of our analysis, we exclude this case. The formal corresponding assumption is stated in Section K.5 in the appendix.

benefits, and $MIE^c_{\epsilon_{stw}}$ *denotes the marginal welfare gain from providing income insurance:*

$$MIE^c_{\epsilon_{stw}} = n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{stw}) \cdot \frac{\frac{u(c_{stw}(\epsilon_{stw})) - u(c(\epsilon_{stw}))}{c_{stw}(\epsilon_{stw}) - c(\epsilon_{stw})} - u'(c^w)}{u'(c^w)} \cdot [c_{stw}(\epsilon_{stw}) - c(\epsilon_{stw})] .$$

Candidate 2

Suppose it is optimal to set $\epsilon_{stw} < \epsilon^c_s$ *with* $\xi^u_s < \epsilon^c_s$ *(full exclusion of constrained firms from STW). Then the optimal threshold is given by* $\epsilon_{stw} = \xi^u_s$.

Candidate 3

Let $I = [\max\{\epsilon^c_s, \xi^u_s\}, \xi^c_s]$. *Suppose it is optimal to set* $\epsilon_{stw} \in I$ *(partial exclusion of constrained firms in need of STW). If*

$$\underbrace{MMS^c_{\epsilon_{stw}} \cdot L^c_{s,\epsilon}(\epsilon_{stw})}_{\substack{\text{Social value of preserved matches}\\ \text{in constrained firms}}} \quad > \quad \underbrace{MWL^u_{\epsilon_{stw}}}_{\substack{\text{Social cost of hours distortion}\\ \text{in unconstrained firms}}} + BE_{stw,2} \quad \forall \epsilon_{stw} \in I, \tag{5}$$

then the optimal threshold is given by $\epsilon_{stw} = \xi^c_s$.

If

$$MMS^c_{\epsilon_{stw}} \cdot L^c_{s,\epsilon}(\epsilon_{stw}) \quad < \quad MWL^u_{\epsilon_{stw}} + \ BE_{stw,2} \quad \forall \epsilon_{stw} \in I, \tag{6}$$

then the optimal threshold is given by $\epsilon_{stw} = \max\{\epsilon^c_s, \xi^u_s\}$.

Otherwise, the optimal threshold is given by ϵ_{stw} *that solves*

$$MMS^c_{\epsilon_{stw}} \cdot L^c_{s,\epsilon}(\epsilon_{stw}) \quad = \quad MWL^u_{\epsilon_{stw}} + \ BE_{stw,2} \tag{7}$$

Here, $L^c_s(\epsilon_{stw})$ *denotes the social value of an unconstrained match retained by marginally loosening the participation constraint.* $MMS^c_{\epsilon_{stw}} = n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{stw})$ *denotes the number of constrained matches retained with a looser eligibility condition.* $MWL^u_{s_{stw}} = n^s \cdot \frac{1-p}{\lambda} \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ *denotes the additional welfare cost of distortions in working hours in unconstrained firms when loosening the eligibility condition. Finally,* $BE_{stw,2}$ *captures general equilibrium effects through the generalized Nash bargaining process.*

PROOF: See Section K.5 in the appendix.

To explain the economic logic behind Proposition 4 and why it can only characterize candidates, we first discuss the basic trade-off the Ramsey planner faces. For this, one intuitive additional result is needed, namely, that constrained firms will always separate at even higher productivity levels than their unconstrained counterparts, as stated by the following lemma:

Lemma 1. *Constrained firms separate at higher productivity levels than unconstrained firms:*

$$\epsilon_s^c > \epsilon_s^u \quad \textit{and} \quad \xi_s^c > \xi_s^u$$

PROOF: See Section L in the appendix.

This fact confronts the Ramsey planner with the following trade-off: The planner can set the eligibility threshold ϵ_{stw} leniently enough to allow constrained matches that would otherwise separate access to short-time work, that is, $\epsilon_{stw} \geq \xi_s^c$. However, this generates windfall effects: unconstrained matches with productivity $\epsilon \in [\xi_s^u, \epsilon_{stw}]$ also gain access to short-time work, even though they face no separation risk absent the program. These windfall effects are costly because short-time work distorts working hours below their efficient level, reducing aggregate output.

Alternatively, to avoid windfall effects, the planner could choose to ignore constrained firms and set the eligibility threshold to $\epsilon_{\text{stw}} = \xi_s^u$. At this threshold, unconstrained firms separate without access to short-time work. In this case, constrained firms with $\epsilon \in [\max\{\epsilon_s^c, \xi_s^u\}, \xi_s^c]$ are excluded from the short-time work scheme even though they are in greater need of support than unconstrained firms. Since which of these is the lesser evil depends on the exact parameters, Proposition 4 can only characterize candidates for the optimal eligibility threshold.

With this in mind, we now turn to explaining the rationale behind each of the three candidates for the optimal eligibility threshold. One possibility is that the eligibility condition should be set so loosely that all firms in need of support can access short-time work. In this case, candidate 1 is optimal. To provide access to short-time work to all firms in need, $\epsilon_{\text{stw}} = \xi_s^c$ is sufficient. The question remains whether an even looser eligibility condition can be optimal. Because short-time work can provide income insurance to workers in financially constrained firms even before the match reaches the separation margin, it may be socially efficient to adopt a more generous eligibility condition. However, this effect could be outweighed by the additional distortion in working hours introduced to both constrained and unconstrained firms, in which case the threshold remains at ξ_s^c.

Proposition 4 characterizes the conditions under which either of these cases is optimal. Our numerical results presented in Section 5, indeed, show that quantitatively, the distortion effect dominates and that $\epsilon_{\text{stw}} = \xi_s^c$ will be optimal.

A second possibility is to set the eligibility threshold to include only unconstrained firms that are actually in need of short-time work. In this case, candidate 2 is optimal. The optimal threshold is given by $\epsilon_{\text{stw}} = \xi_s^u$. As argued by Stiepelmann (2026), $\epsilon_{\text{stw}} < \xi_s^u$ cannot be optimal, as this would exclude the most productive firms in need of support from short-time work. Increasing the threshold cannot be optimal either, as this would not impede additional separations but would introduce additional distortions in working hours.

A third possibility is that the eligibility criterion for short-time work should be set sufficiently strict that some financially constrained firms are excluded, even though STW is necessary to retain their workers, but not strict enough to exclude all financially constrained firms. In this case, candidate 3 is optimal. In this scenario, all unconstrained firms that require STW to retain workers can participate in short-time work, along with some that could survive without it. The Ramsey planner must balance the social gains from protecting constrained firms through a looser eligibility threshold against the costs of windfall effects, the additional distortion in working hours imposed on unconstrained firms where STW is not needed to retain workers. This trade-off is captured by inequalities 5 and 6 and depends on the value of p.

A higher p increases the share of workers in financially constrained firms. When p is large, the social value of retaining these workers dominates the costs of distorting working hours in unconstrained firms, as the marginal cost of distorting working hours in unconstrained firms $MWL^u_{\epsilon_{\text{stw}}}$ declines with p while the number of constrained workers retained $MMS^c_{\epsilon_{\text{stw}}}$ rises. In this case, the eligibility threshold is set sufficiently loose to allow all unconstrained firms for which STW is needed to retain workers to participate, i.e., $\epsilon_{\text{stw}} = \xi_s^c$. Conversely, when p is small, the cost of distorting working hours dominate, and the planner effectively ignores STW, setting $\epsilon_{\text{stw}} = \xi_s^u$. For intermediate values of p, the optimal threshold might balance the social benefits of worker retention against the social costs of distorting working hours in unconstrained firms. Our numerical results in section 5 show that either the social value of worker retention or the social cost of working hours distortion dominates for almost all values of p.

Next, we turn to the optimal level of STW benefits s_{stw}, characterized by the following proposition:

Proposition 5. *The optimal short-time work subsidy s_{stw} is pinned down by the first-order condition*

$$\bar{s}_{stw} = \underbrace{\frac{T_{UI}}{T_{STW}} \cdot b}_{\textit{Fiscal Ext.}} + \underbrace{\frac{MIE^c_{s_{stw}}}{MMS_{s_{stw}}}}_{\textit{Insurance}} - \underbrace{\frac{MWL_{s_{stw}}}{MMS_{s_{stw}}}}_{\textit{Distortion}} + \underbrace{BE_{stw,3}}_{\textit{Bargaining Effect}}$$

Here, $T_{UI} = \frac{1}{f}$ denotes the expected duration that a worker spends on the UI system, $T_{STW} = \frac{1}{\lambda}$ denotes the expected duration that a worker spends on the STW system, and $MIE^c_{s_{stw}}$ denotes the marginal welfare gain from increasing insurance provided to workers in unconstrained firms:

$$MIE^c_{s_{stw}} = \frac{n^c(p)}{1-\rho^c} \cdot \int_{\epsilon^c_s}^{\max\{\epsilon^c_s, \epsilon_{stw}\}} \left(\frac{u'(c_{stw}(\epsilon)) - u'(c^w)}{u'(c^w)} \right) (\bar{h} - h_{stw}(\epsilon))\, dG(\epsilon)$$

Further, $MWL_{\epsilon_{stw}} = n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}$ denotes the marginal welfare loss from a distortion in working hours from increasing STW benefits. $MMS_{s_{stw}} = MMS^c_{s_{stw}} + MMS^u_{s_{stw}}$ denotes the marginal number of matches saved with an increase in STW benefits, where

$$MMS^u_{s_{stw}} = n^s \cdot \frac{1-p}{\lambda} \cdot g(\epsilon^u_s) \cdot \frac{\partial \epsilon^u_s}{\partial s_{stw}}$$
$$MMS^c_{s_{stw}} = \mathbb{1}\{\epsilon_{stw} \geq \epsilon^c_s\} \cdot n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon^c_s) \cdot \frac{\partial \epsilon^c_s}{\partial s_{stw}}$$

denote the marginal number of matches saved in constrained and unconstrained firms. The net transfer of firms and workers is weighted by how many matches of constrained and unconstrained firms can be retained with an increase in STW benefits:

$$\bar{s}_{stw} = \frac{MMS^u_{s_{stw}}}{MMS_{s_{stw}}} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^u_s)\right) + \frac{MMS^c_{s_{stw}}}{MMS_{s_{stw}}} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^c_s)\right)$$

PROOF: See Section K.4 in the appendix.

Proposition 5 highlights both the forces that make more generous short-time work attractive to the Ramsey planner and the associated social costs. The most important motive is the fiscal externality correction arising from the unemployment insurance system. Short-time work helps internalize the fiscal externality that UI imposes on separation decisions in both constrained and unconstrained firms. In unconstrained firms, STW reduces inefficient separations by lowering the cost of insuring workers when the match is hit by a low-productivity shock. In financially constrained firms, STW partly compensates workers for the associated income loss and thereby reduces their incentive to separate.

The optimal level of STW benefits depends on the expected duration of time T_{stw} spent in the STW and UI systems. If the time a match spends in STW T_{stw} is relatively long, the Ramsey planner sets lower benefits in order to encourage reallocation toward more productive matches. Conversely, if workers tend to spend a long time in the UI system, higher STW benefits become desirable, as preserving the match and waiting for the firm's recovery may restore productivity.

The second term reflects the insurance motive for short-time work. It shows that short-time work can provide income insurance to workers in financially constrained firms. Here, $MIE^c_{s_{\text{stw}}}$ denotes the marginal insurance effect—the total welfare gain from increasing income insurance through higher STW benefits. As in the standard optimal UI framework, the planner would ideally like to fully insure workers against income shocks.

However, the Ramsey planner faces a trade-off: increasing short-time work benefits improves insurance but simultaneously weakens separation incentives. This can be a problem because with full insurance, labor does not get reallocated even for very low levels of productivity, which is inefficient. The optimal level of short-time work benefits therefore balances the insurance motive against the behavioral distortion arising from inefficient worker retention. This trade-off is reflected in the formula by the fact that $MIE^c_{s_{\text{stw}}}$ is scaled by the marginal number of matches preserved when short-time work benefits increase $MMS_{s_{\text{stw}}}$. If short-time work is highly effective at preventing separations, providing insurance through short-time work becomes more distortionary, since higher benefits lead to a larger increase in inefficient retention. In this case, the optimal level of short-time work benefits is lower.

The main reason why the planner does not want to set short-time work benefits too high is that short-time work distorts firms' choices about working hours, generating a marginal welfare loss captured by the $MWL_{s_{\text{stw}}}$ term. Since the planner trades off the employment stabilization benefits of worker retention against the efficiency costs from distorted hours, the marginal welfare loss $MWL_{s_{\text{stw}}}$ is scaled by the marginal number of matches preserved, $MMS_{s_{\text{stw}}}$.

It is noteworthy how the Ramsey planner sets the *average net* subsidy $\bar{s}_{\text{stw}}$ optimally and then adjusts the actual benefits s_{stw} to achieve this optimal level. Since constrained firms have higher separation thresholds ($\epsilon^c_s > \epsilon^u_s$), it will hold that financially constrained firms reduce working hours less and get a smaller net subsidy: $s_{\text{stw}} \cdot \big(\bar{h} - h_{\text{stw}}(\epsilon^c_s)\big) < s_{\text{stw}} \cdot \big(\bar{h} - h_{\text{stw}}(\epsilon^u_s)\big)$. The net subsidies are weighted according to how many firms of each firm type the Ramsey planner can save with an increase in short-time work benefits ($MMS^i_{s_{\text{stw}}}$). Therefore, optimal short-time work benefits depend on the share of financially constrained firms. As constrained firms receive a smaller net subsidy, an increase in their share requires higher benefits to compensate for the smaller reduction in working hours. At the same time, a higher fraction of constrained firms raises the marginal insurance gain $MIE^{c}_{s_{\text{stw}}}$, further increasing optimal benefits.

3.4 Why Loans Cannot Replace Short-Time Work

The theoretical analysis shows that short-time work does particularly well when a lot of firms are financially constrained. A common critique of using short-time work to address inefficiencies arising from financial constraints is that such constraints could instead be alleviated by providing loans to firms or workers. However, the financial friction we consider cannot be resolved through the provision of loans.

In our model, the financial friction arises from the assumption that financially constrained firms cannot write fully state-contingent contracts that insure them against low-productivity realizations. Therefore, in low-productivity states, firms do not have the resources to pay workers enough to sustain the match. However, a Ramsey planner would prefer to retain some of these matches, as workers may inefficiently separate in order to receive UI benefits. Providing loans does not resolve this inefficiency because loans merely shift funds across periods and therefore do not alter the fundamental surplus generated by the match and thus the financial constraint.

To see this, assume the government offers a loan to the firm. For simplicity, assume that the government loans a sum of L and the firm has to repay it if the it gets productive in the next period, and that it does not have to repay it if the firm is not productive. We assume that the loan is fairly priced so that

$$L = \beta \cdot \lambda \cdot R \cdot L + \beta \cdot 0 \quad \Leftrightarrow \quad R = \frac{1}{\beta \cdot \lambda}.$$

Consider the impact of the loan on the financial constraint:

$$y(\epsilon, h(\epsilon)) + L - w^c(\epsilon) \geq -\beta \cdot \left[\lambda \cdot \left(\bar{J} - R \cdot L\right) + (1 - \lambda) \cdot J^c(\epsilon)\right]$$

Inserting the fairly priced loan into the formula shows that loans do not alter the financial constraint:

$$y(\epsilon, h(\epsilon)) + L - w^c(\epsilon) \geq L - \beta \cdot \left[\lambda \cdot \bar{J} + (1 - \lambda) \cdot J^c(\epsilon)\right]$$

The loan proves ineffective. The reason for this is that loans do not alter the fundamental surplus of firms and workers. This is because the constraint faced by the firm is not a simple liquidity constraint. To sustain employment, the joint surplus of firms and workers must increase. Short-time work achieves this by compensating workers for lost income when hours are reduced, thereby raising the joint surplus of the match.

3.5 Comparing Layoff Taxes and Short-Time Work

Having disentangled how the Ramsey planner uses layoff taxes and short-time work to counteract the fiscal externalities created by unemployment insurance, it is now time to determine which is the superior policy tool. From a theoretical point of view, this cannot be definitively determined. While layoff taxes do not introduce any inefficiencies into the economy and can, in fact, induce the efficient number of separations in unconstrained firms, they cannot counteract fiscal externalities on constrained firms. Short-time work, on the other hand, can act on constrained firms as well. This, though, comes at the price of distorted working hours. To determine which is more effective in practice, we focus on the quantitative analysis of the model in the remainder of the paper.

Corollary 2. *If there are no constrained firms in the economy (i.e., $p = 0$), the optimal layoff tax increases welfare more than the optimal short-time work scheme.*

PROOF: This follows from the fact that layoff taxes introduce no distortions but can fully eliminate inefficient separations in unconstrained firms. All inefficiencies that short-time work can act on but layoff taxes cannot drop out when $p = 0$.

It is also clear that short-time work is the superior policy instrument if all firms are constrained.

Corollary 3. *If there are no unconstrained firms in the economy (i.e., $p = 1$), the optimal short-time work scheme increases welfare more than the optimal layoff tax.*

PROOF: This follows from the fact that the only inefficiency that layoff taxes can act on, i.e., inefficient separations in unconstrained firms, cannot occur anymore. Short-time work can still act against inefficient separations in constrained firms and provide worker insurance.

In all cases with $p \in (0, 1)$ which of the two policy regimes is superior is ambiguous without quantitative analysis, to which we turn next. It is clear, however, that as p increases, layoff taxes gradually lose their advantage over short-time work. In the following sections, we turn to quantitatively assessing the relative performance of the two policy regimes for different values of p.

4 Calibration

We calibrate the model to the US economy at monthly frequency, assuming CRRA utility: $u(c) = \frac{c^{1-\zeta}}{1-\zeta}$. Further, we set the coefficient of relative risk aversion ζ to the standard value of 2. Following Christoffel and Linzert (2010), we set the labor elasticity of the production function α to 0.65. The elasticity of matching with respect to unemployment γ is set to the standard value of 0.65. As customary, we set the worker bargaining power η to 0.65 to satisfy the Hosios condition (Hosios, 1990). Note that 0.65 falls within the range of estimates reported by Petrongolo and Pissarides (2001). The disutility of labor parameter ψ is set to 1.5 to target a Frisch elasticity of labor supply of 0.65, which is within the reasonable set of parameters according to Domeij and Floden (2006).

We set the shock arrival probability λ to 0.07, so that a firm will remain at any drawn productivity level for $1/0.07 = 14.29$ months on average. This choice is informed by the estimate of yearly plant-level autocorrelation of productivity of around 0.4 by Abraham and White (2006).[9]

The location of the productivity shock distribution μ is set to -0.80 to normalize the average wage $\bar{w}$ to 1. Since the effective dispersion of productivity shocks is governed by the cost shock parameter c_f, which is calibrated internally, we set σ to 0.12 following Krause and Lubik (2007).

The remaining parameters are calibrated internally. The calibration targets are shown in Table 1. All targets are met exactly.

[9] In the model, the autocorrelation of productivity is given by $1-\lambda$. Therefore, $\lambda = 0.07$ implies a monthly autocorrelation of 0.93, which implies a yearly autocorrelation of 0.42.

Table 1: Model Calibration Targets

Parameter	**Description**	**Value**
Worker flows		
q	Vacancy-filling rate	0.3381
f	Job-finding rate	0.41
ρ^{eff}	Effective monthly separation rate	0.03
Policy parameters		
b^{rep}	UI replacement rate	0.45
F^{rep}	Layoff tax coverage of UI payments	0.5

Notes: $b^{\text{rep}} \equiv \frac{b}{w}$ denotes the unemployment insurance (UI) replacement rate, i.e. benefits b relative to the average wage w (normalized to 1 in the calibration). The layoff tax coverage target is reported as $\frac{f}{b}F^{\text{cal}}$, i.e., what fraction of expected UI payments to a laid-off worker is covered by the layoff tax F.

For the purpose of calibration, we set p to 0.5. We show that our results concerning the relative effectiveness of short-time work and layoff taxes are robust to even drastic changes in p during the calibration in Appendix C. We target a UI replacement rate $b^{\text{rep}} = \frac{b}{w}$ of 0.45 as reported by Engen and Gruber (2001). Following Jung and Kuester (2015), we target a layoff tax coverage F^{rep} of 0.5, which means that the layoff tax covers 50 percent of the expected UI payments transferred to a worker after a layoff.[10] Given these policy parameter targets to inform the calibration of b and F, we calibrate the remaining parameters (matching efficiency $\bar{m}$, cost shock scale c_f, and vacancy posting cost k_v) to match standard worker flow targets. Note that since ρ is the probability of separation conditional on a new shock arriving, the effective separation rate ρ^{eff} is different from ρ. It is given by $\rho^{\text{eff}} = \frac{n \cdot \lambda \cdot \rho + (1-n) \cdot f \cdot \rho}{n_0}$, where n_0 is the mass of matches at the beginning of a period. It includes the mass of already existing matches n and new matches $(1-n) \cdot f$. Existing matches draw a new shock with probability λ, while all new matches draw a productivity shock and, like every other match, will separate with probability ρ once productivity is realized.

Table 2 summarizes the calibrated and set parameters of the model.

[10] Since the expected duration of unemployment is $1/f$, the expected UI payments per unemployed worker are given by $\frac{b}{f}$, implying that $F^{\text{rep}} = \frac{F \cdot f}{b}$.

Table 2: Model Parameters: Calibrated and Set Values

Parameter	Description	Value
Calibrated Parameters		
$\bar{m}$	Matching efficiency parameter	0.383
c_f	Cost shock parameter	0.407
k_v	Vacancy posting cost	0.086
b	Unemployment benefit level	0.450
$\bar{h}$	Regular working hours level	0.953
F^{cal}	Layoff tax	0.549
Set Parameters		
ζ	Coefficient of relative risk aversion	2
α	Labor elasticity in production function	0.65
γ	Elasticity of matching w.r.t. unemployment	0.65
η	Worker bargaining power	0.65
ψ	Disutility of labor parameter	1.5
λ	Shock arrival probability	0.07
μ	Location of productivity shocks	-0.80
σ	Scale of productivity shocks	0.12
p	Fraction of financially constrained firms	0.5

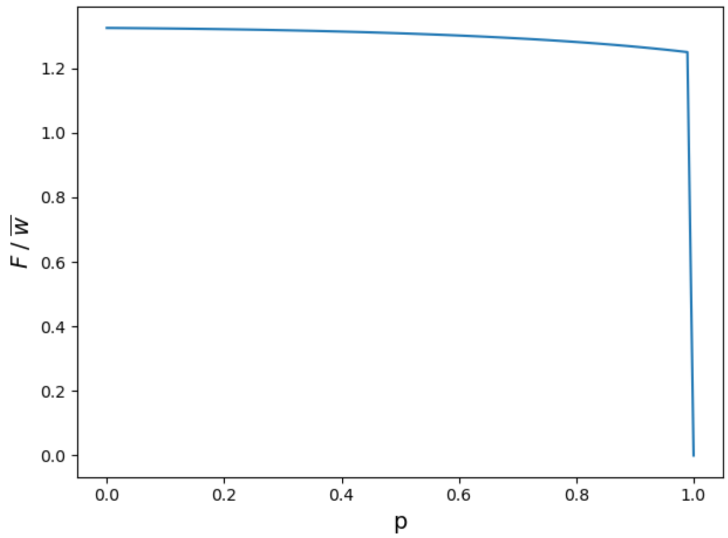
5 Quantitative Results

5.1 Taking UI as given

As a first benchmark, we compute the optimal layoff tax and short-time work parameters chosen by a Ramsey planner who takes the calibrated UI system (i.e. $b = 0.45$) as given. To calculate the Ramsey optimal policy parameters, we solve the Ramsey planner's maximization as described in Section 3.1 numerically. The details are described in Section N in the appendix.

Figure 1 shows the results for the optimal layoff tax F across $p \in [0, 1]$. The result is what the FOC pinning down the optimal layoff tax (Equation 2) leads us to expect. The planner exactly offsets the fiscal externality on unconstrained firms $\frac{\lambda}{f} \cdot b$. Since this does not depend on p in any way, the level of the optimal layoff tax F hardly changes across p. The slight downward slope is explained by general-equilibrium feedback effects in $BE_{\text{lt}}(b)$. At $p = 1$, layoff taxes lose their bite completely as they can no longer target any inefficiencies. To illustrate this, we show the optimal layoff tax to be $F = 0$. In fact, any layoff tax $F \in \mathbb{R}_+$ is optimal.

Figure 1: Optimal Layoff Tax

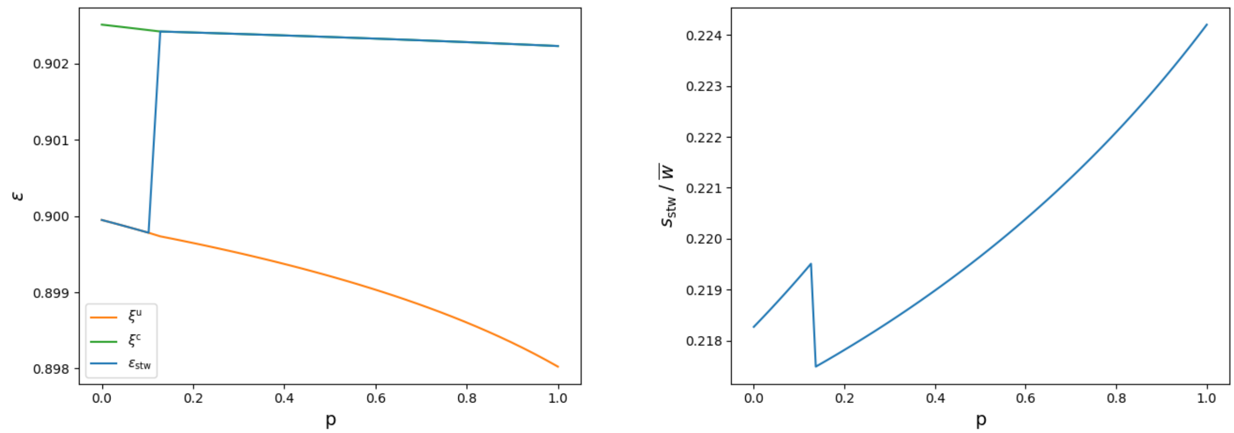


Notes: Optimal layoff tax F chosen by the Ramsey planner for each value of $p \in [0, 1]$, normalized by the average wage $\bar{w}$. The unemployment benefit parameter is fixed at its calibrated value, $b = 0.45$

Figure 2 shows the Ramsey-optimal short-time work parameters across p. The way that the Ramsey planner sets the eligibility threshold reflects the trade-off described in Section 3.3 under Proposition 4: while there are only a few constrained firms in the economy, it is very costly to protect all firms, including constrained ones, from premature separations. Setting the eligibility constraint high enough to allow all constraint firms into short-time work before they hit their separation threshold ξ_s^c will allow many unconstrained firms into short-time work and to collect windfall profits, even though they would not separate without it. As discussed, this introduces inefficient distortion in working hours, and as long as only a few firms are constrained, these distortions outweigh the benefit of protecting these few constrained firms. The optimal eligibility threshold is therefore ξ_s^u. However, as the mass of constrained firms grows along the x-axis of Figure 2, the benefits from protecting constrained firms eventually outweigh the added distortion from protecting excess unconstrained firms. At this point ($p = 0.128$), the eligibility condition jumps to the threshold ξ_s^c at which constrained firms without short-time work separate.

The optimal short-time work benefit s_{stw} is increasing in p except at the kink at $p = 0.128$. This is because, as discussed in the context of Proposition 5 in Section 3.3, the planner targets the *average* net subsidy. As constrained firms have less margin to adjust their hours, this means that s_{stw} has to rise in p as long as ϵ_{stw} does not change much. Additionally, the planner can provide more insurance with higher s_{stw}, which becomes more valuable as p

Figure 2: Optimal Short-time Work



(a) Optimal ϵ_{stw} *(b) Optimal s_{stw}*

Notes: Optimal eligibility threshold ϵ_{stw} and optimal short-time work benefits s_{stw} set by the Ramsey planner across $p \in [0, 1]$, normalized by the average wage $\bar{w}$. ξ_s^u is the separation threshold of an unconstrained firm without access to short-time work. ξ_s^c is the separation threshold of a constrained firm without access to short-time work. The calibrated UI parameter of $b = 0.45$ is taken as given.

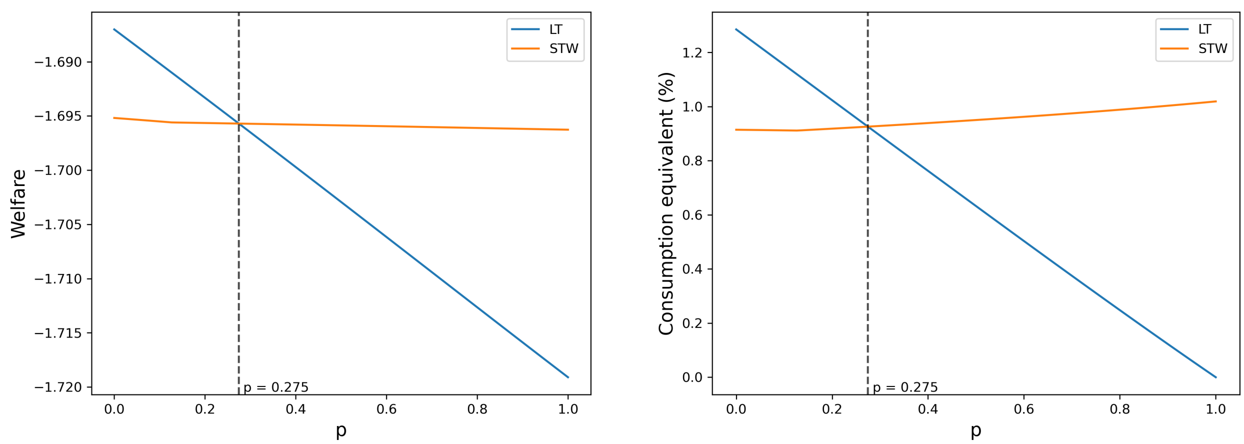
increases, also contributing to the upward slope. However, when ϵ_{stw} jumps to ξ_s^c, many more firms are suddenly on short-time work, introducing a greater distortion in working hours to the economy. To counteract that distortion, the planner reduces the s_{stw} as the more lenient eligibility threshold is introduced, explaining the kink.

Having established how the Ramsey planner sets both layoff taxes and short-time work, it is now time to discuss which is the superior policy tool. Figure 8 shows the respective welfare induced by the optimal layoff tax and the optimal short-time work schemes across p on the left. Unsurprisingly, welfare is decreasing in p in both cases as higher shares of firms become constrained. As expected, when there are no or only a few constrained firms, the layoff tax is the superior policy tool. Crucially, however, the slopes are different for the two policy regimes: as p increases, layoff taxes gradually lose effectiveness, leading to a faster decline in welfare. The two curves have different slopes and intersect at $p = 0.275$. The interpretation is simple: if more than 27.5 percent of firms are financially constrained, short-time work becomes the superior policy instrument; for smaller shares of constrained firms, layoff taxes are the better tool.

For better interpretation, Figure 3 shows the consumption-equivalent variation (CEV). While welfare improvements in terms of consumption-equivalent variation intersect at the exact same point as raw welfare functions, they carry two additional messages. First, the resulting improvements in the respective superior policy scheme are substantial at every p, ranging around 0.95-1.25 percent. Second, as p increases, short-time work becomes not only preferable to layoff taxes, but improvements in terms of consumption-equivalent variation even increase. This stems from the effect that short-time work helps firms provide insurance, which becomes increasingly important for higher p.

Aggregate outcomes of the economies associated with both policy regimes are shown in Figure 11 in Appendix B.

Figure 3: Welfare Comparison



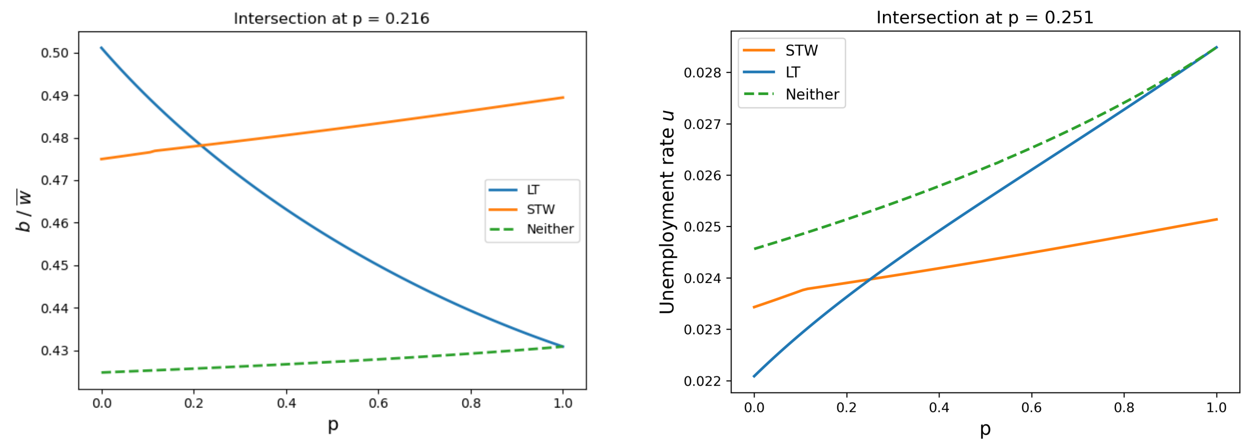
(a) Welfare *(b) Consumption Equivalent*

Notes: Plot (a) on the left shows welfare across $p \in [0, 1]$ of the economy with the optimal layoff tax and the optimal short-time work regimes, respectively. The calibrated UI parameter of $b = 0.45$ is taken as given. Plot (b) on the right shows improvements of optimal short-time work and optimal layoff taxes over the economy without either of these two policies, but with the calibrated UI parameter in terms of consumption-equivalent variation.

5.2 Full Optimization

We now turn to the full optimization problem, where the Ramsey planner optimally chooses either layoff tax or short-time work parameters simultaneously with the unemployment benefit. As a first benchmark, we compute how the Ramsey planner sets the optimal unemployment benefit b absent both layoff taxes and short-time work. Figure 4 shows the result as a dashed green line. Without layoff taxes that could counteract the fiscal externality of the unemployment benefit, the optimal unemployment benefit b is set to a lower level compared to the calibrated value of 0.45, ranging between 0.35 and 0.36. It slightly increases in p, indicating that the planner values higher insurance more than lowering unemployment when faced with more separations and therefore higher unemployment because of more constrained firms.

Figure 4: Optimal Unemployment Benefits



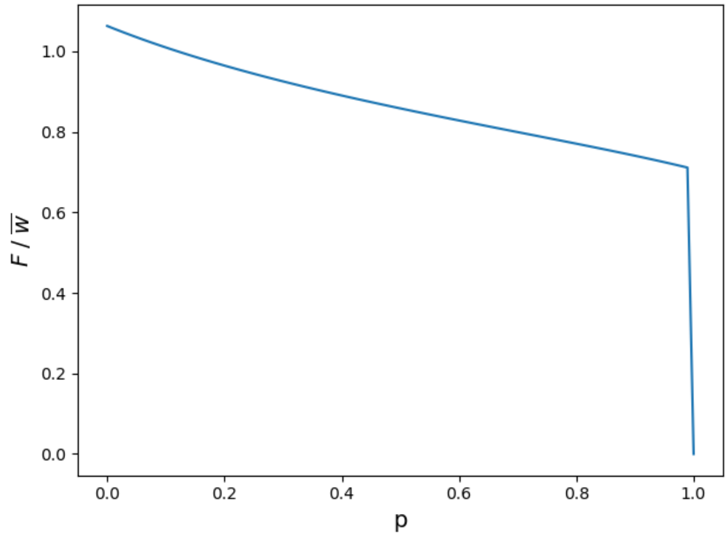
(a) Optimal Unemployment Benefits b *(b) Unemployment Rate u*

Notes: Panel (a) shows the optimal unemployment benefit b chosen by the Ramsey planner for each value of $p \in [0, 1]$, normalized by the average wage $\bar{w}$. The dashed green line corresponds to the case without layoff taxes or short-time work; The blue line allows the planner to set layoff taxes, and the orange line allows the planner to set short-time work. Panel (b) shows the corresponding unemployment rate u in each case.

Having established how unemployment benefits are set in the absence of supplementary policies, we now turn to the problem of jointly optimizing the unemployment benefit b and the layoff tax F. The first thing to note is that b is set considerably higher than without supplementary policies for $p < 1$ as shown by the blue line in Figure 4. This is because the planner now has the layoff tax at her disposal to counteract the fiscal externality of the unemployment benefit, which allows her to set a higher unemployment benefit and thus provide more insurance to workers. The second thing to note is that b sharply decreases with p until it reaches the same level as without supplementary policies at $p = 1$. The reason behind this downward slope is that with more constrained firms, the layoff tax that can only act on unconstrained matches becomes less and less effective at counteracting the fiscal externality of the unemployment benefit. This forces the planner to set a lower unemployment benefit to reduce the externality. Indeed, as the right panel of Figure 4 shows, the unemployment rates associated with the regime with no supplementary policies and the regime with the optimal layoff tax converge as p increases until the two curves meet at $p = 1$ as well.

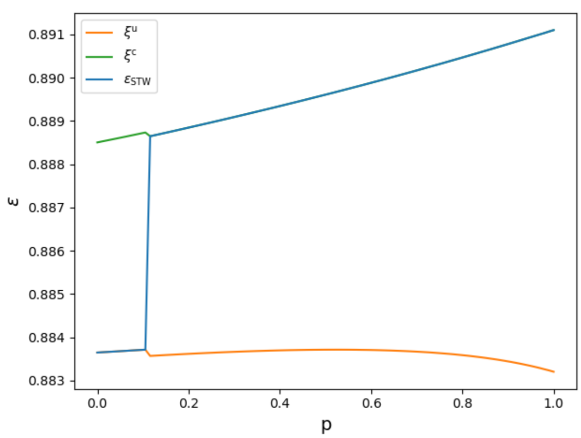
A secondary effect of this is that the optimal layoff tax F shown in Figure 5 also decreases in p as the planner sets lower and lower unemployment benefits and thus has to counteract a smaller and smaller fiscal externality in unconstrained matches. At $p = 1$, the layoff tax loses its bite completely, and the planner sets the same unemployment benefit as without the option to set a layoff tax.

Figure 5: Optimal Layoff Tax

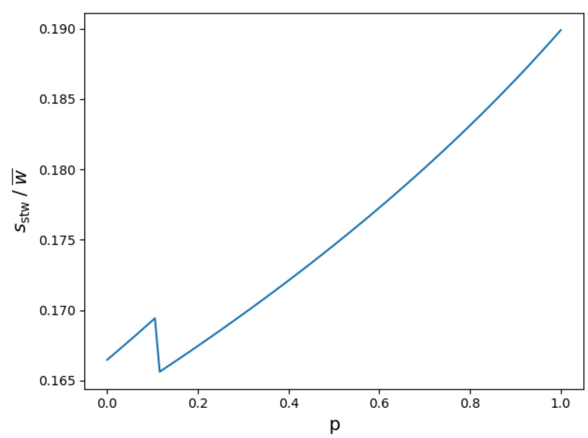


Notes: The optimal layoff tax F across $p \in [0, 1]$, normalized by the average wage $\bar{w}$, when the Ramsey planner optimally chooses the optimal layoff tax F and the optimal unemployment benefit b simultaneously.

Figure 6: Optimal Short-time Work



(a) Optimal ϵ_{stw}



(b) Optimal s_{stw}

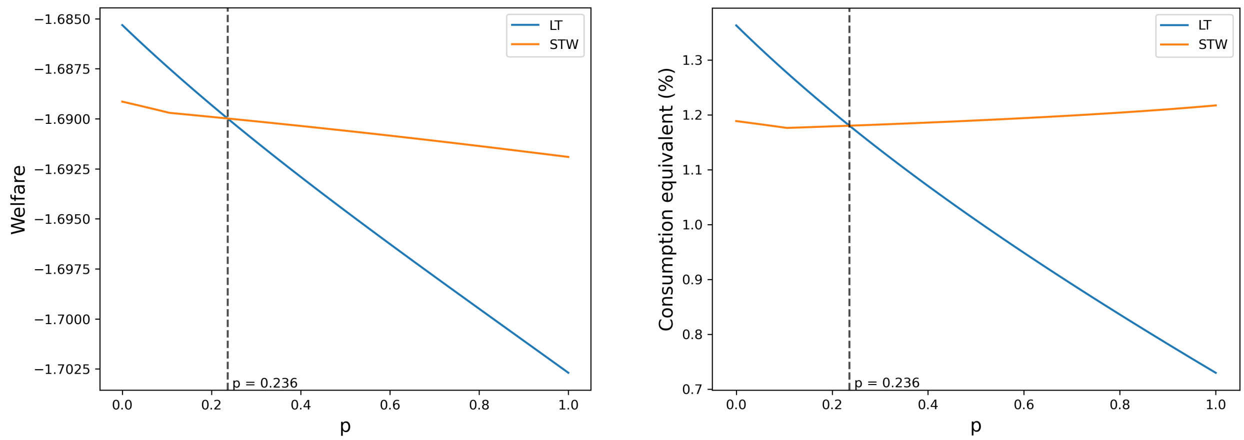
Notes: Optimal STW eligibility threshold ϵ_{stw} and STW benefit s_{stw} chosen by the Ramsey planner for each value of $p \in [0, 1]$, while choosing the optimal unemployment benefit simultaneously. Benefits are normalized by the average wage $\bar{w}$. The thresholds ξ_s^u and ξ_s^c denote the separation thresholds of unconstrained and constrained firms without access to short-time work, respectively.

Next, we turn to the joint optimization of the unemployment benefit b and the short-time work parameters ϵ_{stw} and s_{stw}. As with the layoff tax, the optimal unemployment benefit b with short-time work shown in Figure 4 as an orange line is set at a higher level than without supplementary policies. Unlike with the layoff tax, however, the optimal unemployment benefit b increases with p. The reason for this pattern is that with more constrained firms, more matches are at risk of separation, leading to higher welfare gains from providing insurance through the unemployment benefit. Since short-time work remains effective at counteracting the fiscal externality of the unemployment benefit even with more constrained firms, the planner is not kept from increasing b by increasing the number of separations as with the layoff tax. As the right panel of Figure 4 shows, the unemployment rate under the short-time work regime remains much lower for higher p than in the other regimes as a result.

This dynamic also explains how the optimal short-time work parameters ϵ_{stw} and s_{stw} shown in Figure 6 are set differently from those in the case without optimizing b discussed in the previous section. The qualitative patterns of the optimal eligibility thresholds persist: the planner still initially sets $\epsilon_{\text{stw}} = \xi_s^u$ before transitioning to $\epsilon_{\text{stw}} = \xi_s^c$ as p increases for the same reasons described in Section 5.1. However, there are quantitative differences. Both ξ_s^u and, with it, the eligibility threshold, now increase with p, reflecting that higher unemployment benefits lead to higher separation thresholds and thus firms are in need of support sooner. Meanwhile, the optimal short-time work benefit s_{stw} increases far more steeply in p than in the case where b is held fixed. This sharp increase arises because the higher unemployment benefits amplify the fiscal externality that the planner must counteract, necessitating a much higher generosity of short-time work to keep the average net subsidy at the optimal level.

Finally, we compare the welfare of the optimal layoff tax and the optimal short-time work when the Ramsey planner optimally sets the unemployment benefit b simultaneously. Figure 7 in Appendix F shows the results. As expected, the welfare of both optimal policies is higher than without optimizing b. The improvement over an economy with calibrated UI parameters but without either layoff taxes or short-time work in terms of consumption-equivalent variation is substantial, especially for higher values of p. The improvement over the benchmark economy with only the calibrated unemployment insurance and neither short-time work nor layoff taxes is around 1.2 percent with short-time work instead of around 1 percent, and close to 1.35 percent with layoff taxes instead of around 1.25 percent for $p = 0$. In particular, in the case of full optimization, short-time work becomes the superior policy instrument at an even lower share of financially constrained firms of $p = 0.236$ compared to $p = 0.275$ when b is held fixed.

Figure 7: Welfare Comparison



(a) Welfare *(b) Consumption Equivalent*

Notes: Plot (a) on the left shows welfare across $p \in [0, 1]$ of the economy with the optimal layoff tax and the optimal short-time work regimes, respectively, while the Ramsey planner optimally chooses the optimal unemployment benefit b simultaneously. Plot (b) on the right shows improvements of optimal short-time work and optimal layoff taxes over the economy without either short-time work or layoff taxes, as well as without setting b optimally, but with the calibrated UI parameter in terms of consumption equivalent variation.

5.3 Optimal Joint Design of STW and Layoff Taxes

As discussed in the previous sections, both short-time work and layoff taxes have advantages and disadvantages. Layoff taxes do not introduce distortions, but unlike short-time work, they cannot act on financially constrained firms. Therefore, it is only natural to ask whether both systems can be combined to take full advantage of the strengths of both schemes. Indeed, many states in the US implement both systems simultaneously, lending this question practical relevance.[11] In this section, we characterize the Ramsey-optimal combination of short-time work and layoff taxes and quantitatively assess how much it improves upon using only one of the two systems.

The rationale for combining both policy tools is straightforward: ideally, the planner would like to use layoff taxes to deter inefficient separations in unconstrained firms without introducing any new distortions, and a short-time work program only to target the same inefficiency in constrained firms. However, because the government cannot perfectly distinguish between firm types, any eligibility condition that grants constrained firms access to short-time work will inevitably allow unconstrained firms to participate as well. In these

[11] Layoff taxes are implemented through experience-rated unemployment insurance systems. Short-time work exists under the name short-time compensation in more than half of the states, though it is rarely used according to Krolikowski and Weixel (2020).

unconstrained firms, layoff taxes already prevent inefficient separations, making short-time work redundant. This creates windfall effects that generate costly distortions in working hours without achieving additional efficiency gains.

The Ramsey planner must therefore weigh the benefits from supporting constrained matches against the costs of windfall effects when introducing short-time work on top of layoff taxes. This tradeoff is reflected in how the Ramsey planner sets the policy parameters. Proposition 6 formalizes how the planner sets the optimal layoff tax and the optimal short-time work benefits when optimizing over both systems simultaneously:

Proposition 6. *The optimal layoff tax and STW can be expressed as:*

$$\bar{s}_{stw} = \frac{T_{UI}}{T_{stw}} \cdot b - \omega^u \cdot \lambda \cdot F + \frac{MIE^c_{s_{stw}}}{MMS_{s_{stw}}} - \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} + BE_{stw}$$
$$F = T_{UI} \cdot b - (1-\omega^{\xi}) \cdot T_{stw} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^u_s)\right) + BE_F$$

Here ω^u denotes the fraction of marginal firms that are unconstrained and saved with STW. ω^{ξ} denotes the fraction of marginal firms that separated and did not have access to STW in unconstrained firms.

PROOF: See Section O in the appendix.

Proposition 6 yields the same optimal formulas for STW benefits and layoff taxes as in the single-instrument cases in Propositions 2 and 5, except for one key modification: the optimal STW benefits are reduced by the term $\omega_u \cdot \lambda \cdot F$, which depends on the optimal layoff tax, while the optimal layoff tax is reduced by the term $(1-\omega_{\xi}) \cdot T_{\text{stw}} \cdot s_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon^c_s))$, which depends on the optimal STW benefits. Consequently, when the planner implements both policies simultaneously, both the layoff tax F and the short-time work benefits s_{stw} are set at lower levels than when either policy is implemented in isolation.

The underlying mechanism is straightforward: since unconstrained firms have access to short-time work despite not requiring support, the program generates windfall effects[12] and, with them, unnecessary distortions in working hours. These costs lead the Ramsey planner to reduce optimal STW benefits. At the same time, both experience rating and short-time work discourage separations in unconstrained firms. This dual approach risks over-discouraging separations and thereby hindering the efficient reallocation of workers from less productive

[12] This can be seen by the fact that the fraction of marginal unconstrained firms saved with STW, ω^u, is positive for $p < 1$.

to more productive firms. To prevent costly misallocation, the planner reduces the optimal layoff tax.

As in the model with only short-time work, the Ramsey planner also chooses the eligibility condition for short-time work. To provide additional insight into the interaction between short-time work, layoff taxes, and the share of constrained firms in the economy, Corollary 4 considers two special cases. The first is a loose eligibility condition under which all constrained firms that would otherwise separate can access short-time work. The second is a strict eligibility condition that excludes all constrained firms from short-time work.

Corollary 4. *(i) Suppose the planner has already set a loose eligibility condition that allows all firms that would otherwise separate to enter short-time work. Then* $\omega^{\xi} = 0$ *and* $\bar{s}_{stw}$ *is pinned down by the simplified condition*

$$s_{stw} \cdot (\bar{h} - h_{stw}(\epsilon_s^c)) = \frac{\lambda}{f} \cdot b + \frac{MIE^c_{s_{stw}}}{MMS^c_{s_{stw}}} - \frac{1}{1 - \omega^u(p)} \cdot \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} + BE_{stw}$$

In this case, if $p \to 0$*, then* $s_{stw} = 0$*.*

(ii) Suppose the planner has already set the eligibility condition so strictly that constrained firms cannot access short-time work. Then $\omega^u = 1$ *and* $s_{stw} = 0$*.*

PROOF: See Section O.1 in the appendix

Corollary 4 shows that, in the case of the loose eligibility condition, a formula similar to the one in Proposition 5 pins down s_{stw}. The main difference is the multiplier term $M = 1/(1 - \omega^u(p))$, which reflects windfall effects: $\omega^u(p)$ is the fraction of marginal matches saved by STW that are unconstrained. Saving these matches with STW is unnecessary, as layoff taxes could have discouraged their separations without distorting working hours; the support they receive is therefore a pure windfall.

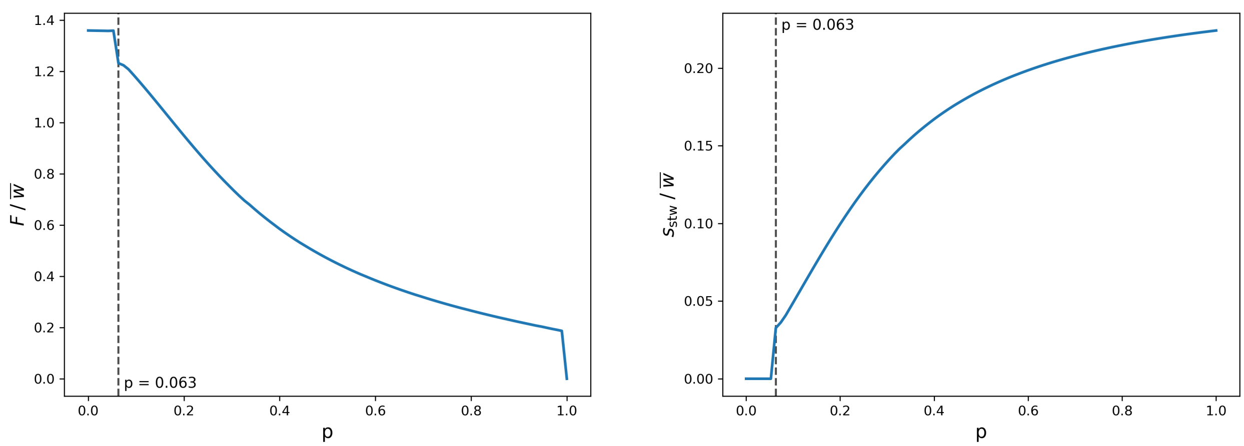
Windfall effects leave the insurance value of STW, $\frac{MIE^c_{s_{\text{stw}}}}{MMS^c_{s_{\text{stw}}}}$, unchanged: insurance gains accrue only to constrained matches and are normalized by the marginal constrained matches saved, $MMS^c_{s_{\text{stw}}}$, making the term invariant to the share of constrained firms p. They do, however, raise the welfare costs of the program: distortion in working hours arise in all matches on the STW program, whether or not STW is needed to save them, and M captures the costs that saving unconstrained matches incurs.

As p falls, more matches are saved unnecessarily: $\omega^u(p)$ rises, the cost from windfall effects increases, and optimal STW benefits decline. In the limiting case where p approaches zero, all firms accessing STW would be unconstrained, leading the planner to set STW benefits to zero.

If, on the other hand, the eligibility condition is set strict enough to screen out all constrained firms from short-time work support, the short-time work scheme is obsolete, as all inefficiencies in constrained firms are better tackled using the layoff tax, and the planner sets s_{stw} to 0.

Given these results, both a combination of short-time work and layoff taxes and using only layoff taxes could be optimal in practice. We therefore turn to numerical evaluations of the optimal policy mix to determine how the planner sets both policy tools simultaneously in the calibrated model.

Figure 8: Optimal Joint Design of Layoff Taxes and STW Benefits



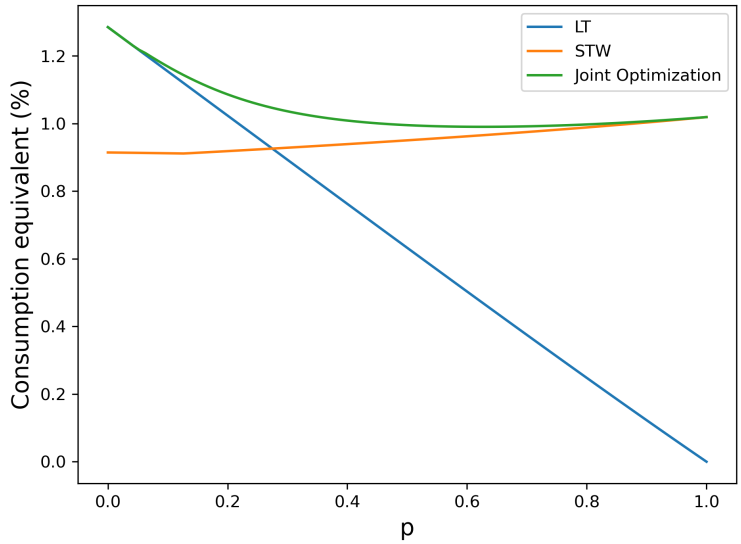
(a) Optimal F *(b) Optimal s_{stw}*

Notes: Optimal layoff tax F and optimal short-time work benefits s_{stw} set by the Ramsey planner across $p \in [0, 1]$. Both are normalized by the average wage $\bar{w}$. The calibrated UI parameter of $b = 0.45$ is taken as given.

Figure 8 shows how the Ramsey planner optimally sets the layoff tax F and the short-time work benefits s_{stw} in the calibrated model. Appendix E discusses the corresponding eligibility condition. There are two key takeaways from Figure 8. First, for small p, there are no measurable welfare gains from combining layoff taxes with STW. The planner chooses to ignore short-time work and implements the optimal layoff tax regime. Only once at least 6.3 percent of firm matches are financially constrained does the benefit of introducing short-time work outweigh the social cost of windfall effects among unconstrained firms. Second, the more important protecting constrained matches becomes as p increases, the more the planner relies on short-time work instead of layoff taxes to impede inefficient separations.

This is evident from the downward slope of F and the upward slope of s_{stw} in p.[13]

Figure 9: Joint Optimal Design Welfare



Notes: The plot shows improvements from optimal short-time work, optimal layoff taxes, and the optimal joint design over the economy without either short-time work or layoff taxes, and without setting b optimally, but with the calibrated UI parameter in terms of consumption equivalent variation.

Figure 9 compares welfare induced by the independently optimal policy regimes with that induced by the joint optimization in terms of CEV. While the joint implementation always weakly dominates using only one of the two policy instruments, gains from implementing the joint policy are the greatest around $p = 0.3$. For p close to 1 or 0, the combination of short-time work and layoff taxes hardly improves upon using the optimal individual policy tools. This highlights that in economies with only a few constrained firms, layoff taxes work well, and in economies with many constrained firms, short-time work is very effective. In economies with a large number of both constrained and unconstrained firms, however, implementing the optimal combination leads to more sizable improvements.

6 Discussion: How Large Is p?

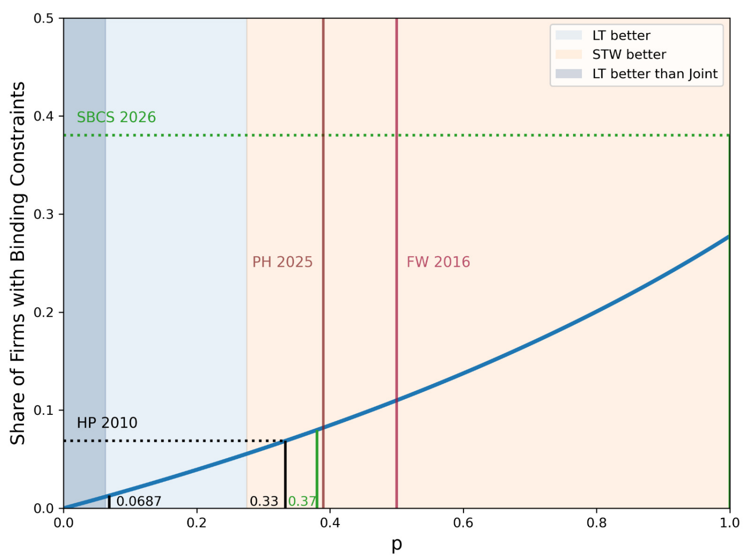
The fact that the relative effectiveness of short-time work and layoff taxes depends on the value of p raises the question of how large p is in practice. However, there are some challenges

[13] Appendix F translates these optimal policies into two measures with direct institutional counterparts, offering a complementary view of the policy instruments: the layoff tax is translated into an Experience Rating Index, as is sometimes used to characterize US state UI systems, and STW benefits are translated into the ratio of STW to UI benefits a worker receives over the spell.

connected with mapping p to moments that are observable from empirical data.

First, the empirical literature uses a variety of definitions, proxies, and data sources to identify financially constrained firms, and there is no single, universally accepted measure. Second, existing methods often rely on observable binding financial constraints, e.g., by observing rejected loan applications. In the model p is the share of worker-firm matches that cannot *ex post* borrow beyond its expected value, i.e. the share of matches without access to full Arrow-Debreu state-contingent contracts. Therefore, measuring p amounts to measuring the share of workers in the economy who cannot be retained by their employers because of insufficient credit in some *potential* states that may never materialize. Taking our model calibration as given, it is possible, however, to back out the value of p implied by an estimate for the share of firms with binding financial constraints. This relationship is shown in Figure 14 in Appendix D. In what follows, we relate p to existing empirical measures of financial constraints among firms, which provides guidance on its plausible magnitude.

Figure 10: Empirical Value of p



Notes: The figure maps the model parameter p to the model-implied share of firms with binding borrowing constraints. The solid blue line gives this mapping. The shaded areas indicate where layoff taxes (LT) and short-time work (STW) are welfare superior, respectively. The dotted horizontal lines correspond to empirical estimates of financially constrained firms from Hadlock and Pierce (2010) and the US Bureau of Labor Statistics (2025), while the solid vertical lines show liquidity-based benchmarks from Pursell and Hussey (2025) and Farrell and Wheat (2016).

Hadlock and Pierce (2010) classify Compustat firms into categories of being affected by financial constraints. Under their preferred specification, 6.9 percent of firms are either financially constrained (FC) or likely financially constrained (LFC). FC firms show clear evidence of binding financing problems, while LFC firms report difficulties obtaining financing but without evidence that these constraints have yet affected investment. If, on the other hand, we interpret FC and LFC firms as being the firms with binding financial constraints, this implies $p = 0.33$. If, on the other hand, we interpret LFC as containing all firms with potentially binding constraints, this would rationalize $p = 0.069$. The Hadlock and Pierce (2010) estimate therefore implies plausible estimates in the interval [0.069,0.33].[14].

It is worth noting that Hadlock and Pierce (2010) draw on Compustat data, which cover only publicly traded firms and thus skew toward larger firms. Hadlock and Pierce (2010) themselves show that smaller firms are more likely to be financially constrained. Indeed, since p is the share of constrained one-on-one matches, a natural interpretation is that these matches exist within larger firms. Under this interpretation, it seems likely that funds sufficient to retain workers can be shifted around within large firms relatively easily and that most constrained matches exist within small firms, making short-time work particularly suitable for protecting matches in small firms. Drawing on the Small Business Credit Survey (2026) and using data from the US Bureau of Labor Statistics (2025), we calculate that 37 percent of small-business workers are employed at financially constrained firms that have either had loan applications at least partially rejected or did not apply for loans despite needing them (see Appendix D for details). A natural interpretation is that p must at least amount to 37 percent among small firms. To produce a share of firms at binding borrowing constraints of even around 30 percent, in the model p would have to be 1, emphasizing that the Compustat-based estimates likely fall short of the real figure.

As a simple back-of-the-envelope calculation, one can combine the two benchmarks using employment shares. Publicly traded Compustat firms account for about 23 percent of employment (Schlingemann and Stulz, 2022), while we calculate firms small enough to be covered by the Small Business Credit Survey (2026) to account for roughly 50 percent of employment based on US Bureau of Labor Statistics (2025) data. Using conservative values for both groups, namely 0.069 for Compustat firms and 0.37 for small firms, gives an employment-weighted benchmark of about 27.51 percent among the covered workers.[15]

Finally, we consider two liquidity-based measures. Pursell and Hussey (2025) report that roughly 41 percent of firms hold less than one month of liquid reserves, while Farrell and Wheat (2016) find that 50 percent of firms hold fewer than 27 days of a cash buffer. These firms may be unable to absorb sufficiently adverse productivity shocks, making the two

[14]The value 0.33 likely is an overly high estimate for p in this sample, as binding credit constraints need not map one-to-one into binding payroll constraints; 0.069 likely is a very low estimate, as firms already facing binding credit constraints can be expected to also face binding payroll constraints when hit by a negative productivity shock.

[15]This calculation is only illustrative: the two empirical measures are not directly comparable, and the remaining workers who work in neither publicly traded nor small firms are excluded.

estimates natural direct benchmarks for p.

Taken together, we interpret these findings as suggesting that values of $p > 0.275$ are empirically plausible, in which case short-time work may be the superior policy instrument. All of the estimates point to a value of $p > 0.063$, implying that the optimal combination of short-time work and layoff taxes leads to welfare improvements over implementing only a layoff tax. We stress, however, that further empirical validation is needed to more directly discipline the value of p.

7 Conclusion

We set out by asking the simple question, which of the two policy instruments that governments in many countries already employ - layoff taxes or short-time work - is better at counteracting the fiscal externalities of unemployment insurance. While the existing literature emphasizes the desirable properties of layoff taxes, we show that their effectiveness is highly sensitive to the assumption that firms are unconstrained in their ability to smooth shocks.

By introducing firm-level financial constraints into a rich yet tractable Diamond-Mortensen-Pissaridis (DMP) search-and-matching model featuring endogenous separations, risk-averse workers, and flexible working hours, we show that layoff taxes struggle to counteract these externalities when firms are constrained. Short-time work schemes, on the other hand, can act on constrained firms but have the disadvantage of introducing new inefficiencies into the economy through distorting working hours.

Our theoretical analysis, grounded in a Ramsey policy approach, delivers closed-form expressions for optimal layoff taxes and short-time work subsidies as a function of the share of financially constrained firms. We show that layoff taxes are the better policy instrument without or with only a few financially constrained firms, but that short-time work benefits are better if sufficiently many firms are financially constrained. Quantitatively, we find that when 27.5 percent or more of firms are financially constrained, short-time work dominates layoff taxes in welfare terms. If the level of unemployment benefits is optimized simultaneously, this threshold falls to 24 percent. We proceed to show that combining both short-time work and layoff taxes can be superior to using only one of the two policy instruments. We find that combining the two instruments yields non-negligible welfare gains relative to using each instrument in isolation when sufficiently many firms of both types are present in the economy. These gains are largest when financially constrained firms make up around 30 percent of the economy.

Which of the two policy instruments should be recommended must therefore depend on the prevalence of financial constraints in a given economy. Anecdotally, it seems highly plausible that in the US, where a layoff tax is implemented through experience-rated unemployment insurance, financial constraints play a smaller role than in countries like Germany or France which have less developed financial sectors and smaller firms where short-time work is widely used. When relating existing empirical evidence of how prevalent financial constraints are in the US it is empirically plausible that even in the US, short-time work may be the better policy tool, particularly for small businesses. Still, our findings highlight that empirically determining the extent of financial constraints in the US and other countries, and understanding their implications for optimal policy, is key to informing policy recommendations and warrants further investigation.

Another interesting way forward is to explore how our results change over the business cycle. It is well documented that financial constraints tighten during downturns. Hence, there could be a case for relying more on short-time work during downturns and more on layoff taxes during good times.

We believe this offers a promising direction for future research.

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A Table of Model Parameters, Variables and Functions

Table 3: Important Variables and Functions

Parameter	Description
Notation	
(u / c)	Financially unconstrained/ constrained
p	Fraction of financially constrained firms
Policy	
D	Minimum hours reduction threshold (Eligibility condition)
s_{stw}	STW benefits
b	UI benefits
τ	Tax on firms
F	Layoff tax
Productivity Thresholds	
ϵ_{stw}	Eligibility threshold
$\epsilon_s^u, \epsilon_s^c$	Separation thresholds (u/ c, under STW)
ξ_s^u, ξ_s^c	Separation thresholds (u/ c, no STW)
ϵ^p	Productivity where financial constraint binds
Production	
ϵ	Idiosyncratic productivity
$g(\epsilon), G(\epsilon)$	Pdf and cdf of idiosyncratic productivity shock
$h(\epsilon), h_{stw}(\epsilon)$	Working hours on and off STW
$y(\epsilon, h)$	Production function
$\phi(h)$	Disutility of work
$z(\epsilon, h), z$	Realized and expected output net of disutility
$z(\epsilon), z_{stw}(\epsilon)$	Shorthand for $z(\epsilon, h)$ on and outside STW
Firms and Firm Owner	
$J^u(\epsilon), J^c(\epsilon)$	Realized firm value (u/ c, no STW)
$J^u_{\text{stw}}(\epsilon), J^c_{\text{stw}}(\epsilon)$	Realized firm value (u/ c, under STW)
$\bar{J}$	Expected firm value
Π	Dividends
$w^u(\epsilon), w^c(\epsilon)$	(Monthly) Wage functions (u/ c, no STW)
$w^u_{\text{stw}}(\epsilon), w^c_{\text{stw}}(\epsilon)$	(Monthly) Wage function (u/ c) under STW
ν^f	Number of firm owners
c^f	Consumption firm owners

Table 4: Important Variables and Functions

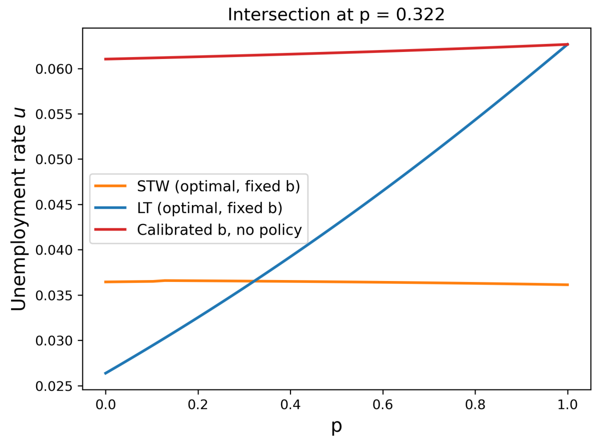
Parameter	Description
Worker	
$V^u(\epsilon), V^c(\epsilon)$	Worker value (u/ c, no STW)
$V^u_{\text{stw}}(\epsilon), V^c_{\text{stw}}(\epsilon)$	Worker value (u/ u, with STW)
$\bar{V}$	Expected worker value
U	Value of unemployment
c^w	Constant consumption-equivalent
$c^w(\epsilon)$	Constrained consumption-equivalent function
$c^w_{\text{stw}}(\epsilon)$	Constrained consumption-equivalent function STW
Labor Market	
n	Mass of employed workers
n^s	Mass of matches receiving new shock
n^u, n^c	Mass of workers at u/c firms
$u = 1 - n$	Mass of unemployed workers
v	Mass of posted vacancies
$\theta = \frac{v}{1-n}$	Labor market tightness
f	Job-finding rate
q	Vacancy-filling rate
ρ	Separation rate
ρ^u, ρ^c	Separation rates (u/ c)
From Propositions	
$\Omega(\epsilon)$	Welfare costs STW in match at productivity ϵ
Ω^u	Average welfare loss STW in (u/ c) firms
Ω	Average welfare loss net of disutility of all firms
T_{UI}	Expected time a separated worker spends on UI
T_{STW}	Expected time a worker spends on STW
BE_i	Bargaining effect
L_v	Social value of a new hire
L^u_s, L^c_s	Social value of the marginal worker in (u/ c) firms
$L^c_{s,\xi}$	Social value of the marginal worker in c firm without STW

Table 5: Important Variables and Functions

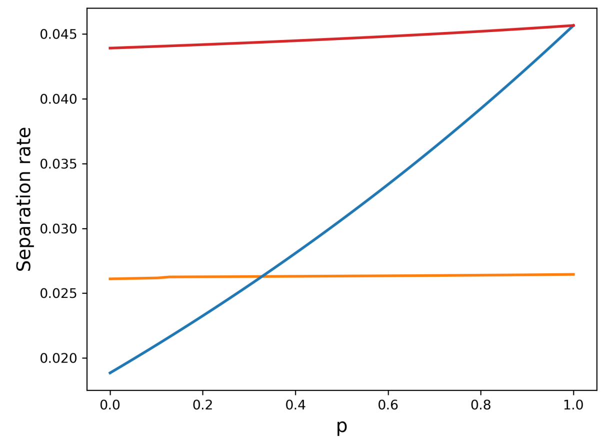
Parameter	Description
$MMS_{s_{stw}}$	Marginal number of matches preserved with STW benefits
$MMS^u_{s_{stw}}, MMS^c_{s_{stw}}$	Marginal number of matches preserved with STW benefits in (u/ c) firms
$MWL_{s_{stw}}$	Marginal welfare loss caused by STW benefits
$MWL^u_{s_{stw}}, MWL^c_{s_{stw}}$	Marginal welfare loss in (u/ c) firms caused by STW benefits
$MMS_{\epsilon_{stw}}$	Marginal number of matches added to STW with looser eligibility threshold
$MMS^u_{\epsilon_{stw}}$, $MMS^c_{\epsilon_{stw}}$	Marginal number of matches added to STW with a looser eligibility threshold in (u/ c) firms
$MWL_{\epsilon_{stw}}$	Marginal welfare loss caused by looser eligibility threshold.
$MWL^u_{\epsilon_{stw}}, MWL^c_{\epsilon_{stw}}$	Marginal welfare loss in (u/ c) firms caused by looser eligibility threshold.
MIE^c_b	Marginal insurance loss in c firms caused by UI benefits
$MIE^c_{s_{stw}}, MIE^c_{\epsilon_{stw}}$	Marginal insurance gain in c firms caused by higher STW benefits, looser eligibility threshold

B Aggregate Outcomes

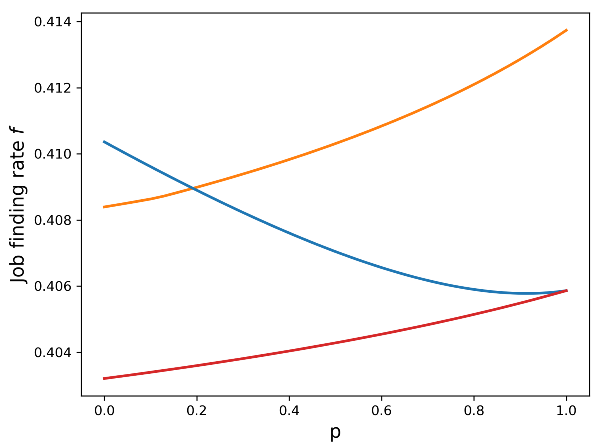
Figure 11: Aggregate Outcomes



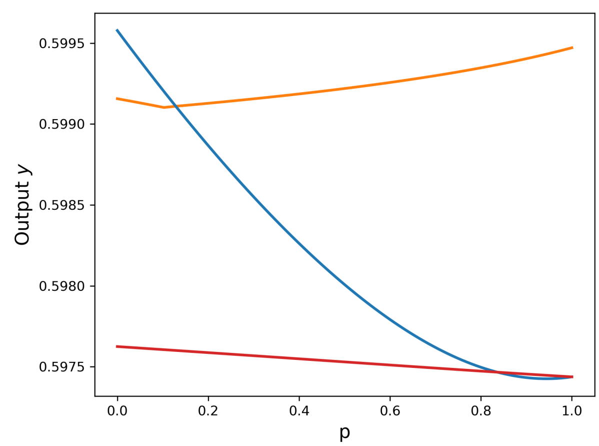
(a) Unemployment Rate



(b) Separation Rate



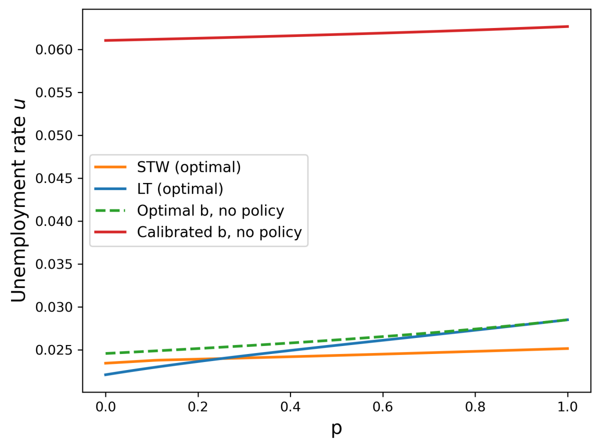
(c) Job Finding Rate



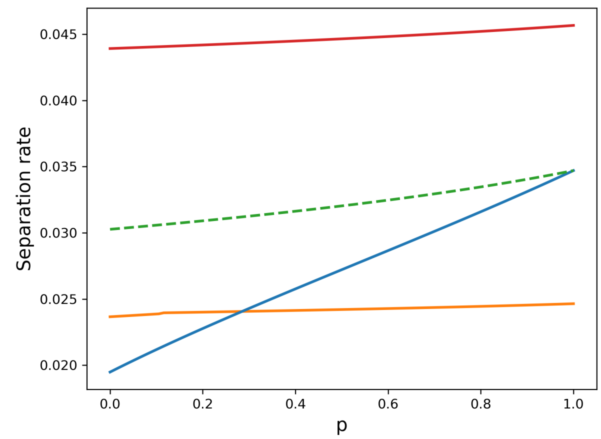
(d) Output

Notes: Aggregate outcomes for the different policy regimes taking the calibrated unemployment benefit level of $b = 0.45$ as given against the share of constrained firms, p. The red lines are associated with a regime with no layoff tax and no short-time work, the blue lines with a regime with optimal UI benefits, and the orange lines with optimal short-time work. **(a)** Unemployment rate, **(b)** separation rate, **(c)** job finding rate and **(d)** output.

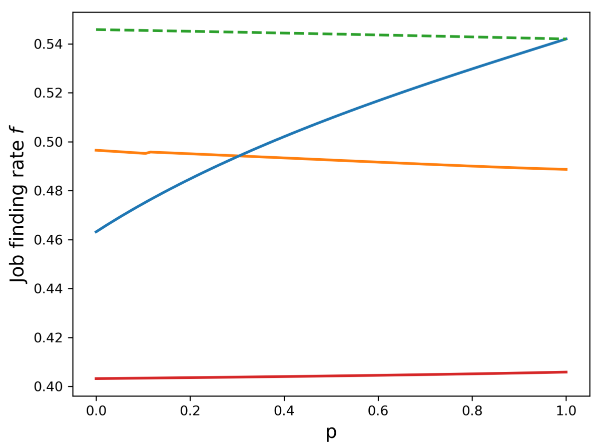
Figure 12: Aggregate Outcomes - Full Optimization



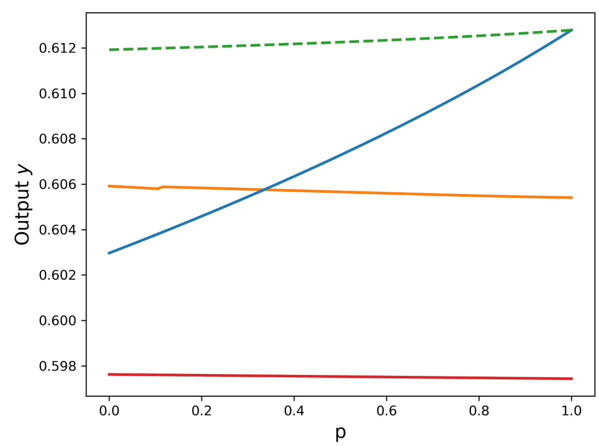
(a) Unemployment Rate



(b) Separation Rate



(c) Job Finding Rate



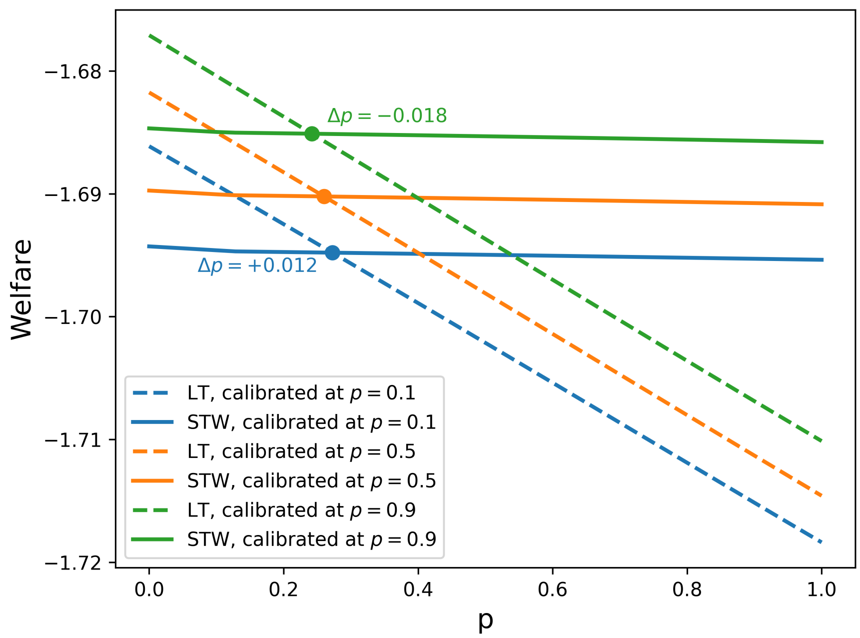
(d) Output

Notes: Aggregate outcomes for the different policy regimes (calibrated UI, optimal UI, optimal UI with optimal LT, and optimal UI with optimal STW) as a function of the share of constrained firms, p. **(a)** Unemployment rate, **(b)** separation rate, **(c)** job finding rate and **(d)** output.

C Calibration Robustness

In our calibration, we set $p = 0.5$. We do this because, as discussed in Section 6, there is no universally accepted estimate for p. However, as Figure 13 shows, our main result, namely for which values of p short-time work is superior to layoff taxes, hardly changes when using $p = 0.1$ or $p = 0.9$ during the calibration instead, showing that our results do not depend on our choice of p during the calibration.

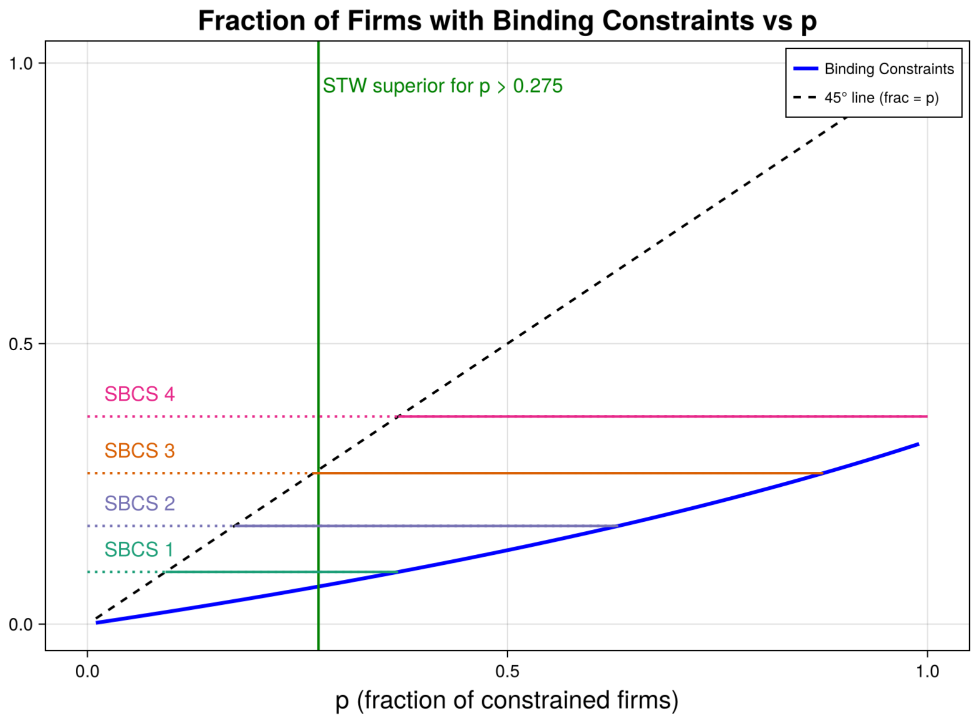
Figure 13: Impact of Chaning p



Notes: Solid lines show welfare under short-time work; dashed lines show welfare under the layoff tax. Each color corresponds to a different calibration value of (p). Dots mark the (p) at which the two welfare curves intersect for that calibration. Labels report the difference between each intersection and the middle-calibration intersection; the middle point is left unlabeled.

D Empirical Value of p

Figure 14: Empirical Value of p

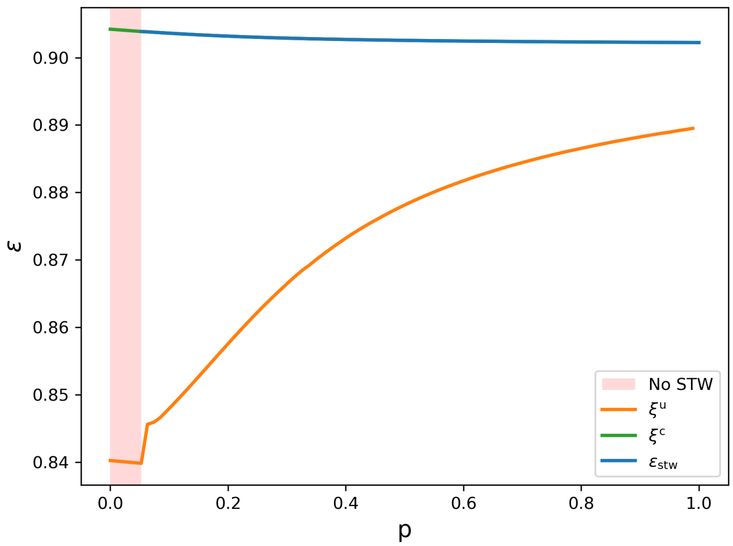


Notes: The figure determines the fraction of firms that are financially constrained. The x-axis describes the number of firms p that have a financial constraint. The blue line shows the fraction of firms in the economy whose financial constraint is binding. The dashed line marks the 45-degree line. The green line marks the p-value after which STW is superior to layoff taxes. The vertical lines labeled SBCS 1 - 4 show different degrees of financial constraints.

The figure illustrates how the measure of financial constraints drawn from the Small Business Credit Survey (2026) is constructed. SBCS 1 captures the fraction of firms whose loan applications were denied outright. SBCS 2 adds firms that did not apply for financing despite needing it, for instance, because they expected rejection or were averse to taking on debt. The Small Business Credit Survey (2026) refers to these firms collectively as having unmet financing needs. SBCS 3 further adds firms that had between 1 and 49 percent of their requested financing approved, and SBCS 4 adds firms that had between 50 and 99 percent approved. The Small Business Credit Survey (2026) refers to firms in SBCS 1, SBCS 3, and SBCS 4, those that received no or only partial loan approval, as firms with financing shortfalls.

E Optimal Eligiblity Condition Joint Optimal Design

Figure 15: Optimal Eligiblity Condition, Joint Optimal Design



Notes: The optimal eligibility threshold ϵ_{stw} set by the Ramsey planner across $p \in [0, 1]$. ξ^u_s is the separation threshold of an unconstrained firm without access to short-time work. ξ^c_s is the separation threshold of a constrained firm without access to short-time work. The calibrated UI parameter of $b = 0.45$ is taken as given.

Numerically, we can show that in our calibration, the Ramsey planner can fully screen out financially constrained firms. The reason for this is that unconstrained firms are deterred from separation via the layoff tax, while financially constrained firms are not. When it comes to the eligibility condition, the most important decision the Ramsey planner can take is whether to screen out workers or not. If he decides to screen out constrained workers, Corollary 4 argues that the planner sets the STW benefits to zero, eliminating the STW system.

The planner decides to implement the STW system when the social value of the preserved matches outweighs the social costs of distortion in working hours distortion imposed on unconstrained firms, respectively, the social costs of additional windfall effects. For $\epsilon_{stw} \in [\epsilon^c_s, \xi^c_s]$ we get:

$$\underbrace{MMS^c_{\epsilon_{\text{stw}}} \cdot L^c_{s,\epsilon}(\epsilon_{\text{stw}})}_{\text{Social value of preserved matches}} > \underbrace{MWL^u_{\epsilon_{\text{stw}}}}_{\substack{\text{Social cost of hours distortion} \\ \text{in unconstrained firms}}} + BE_{\text{stw},2} \quad \forall \epsilon \in I,$$

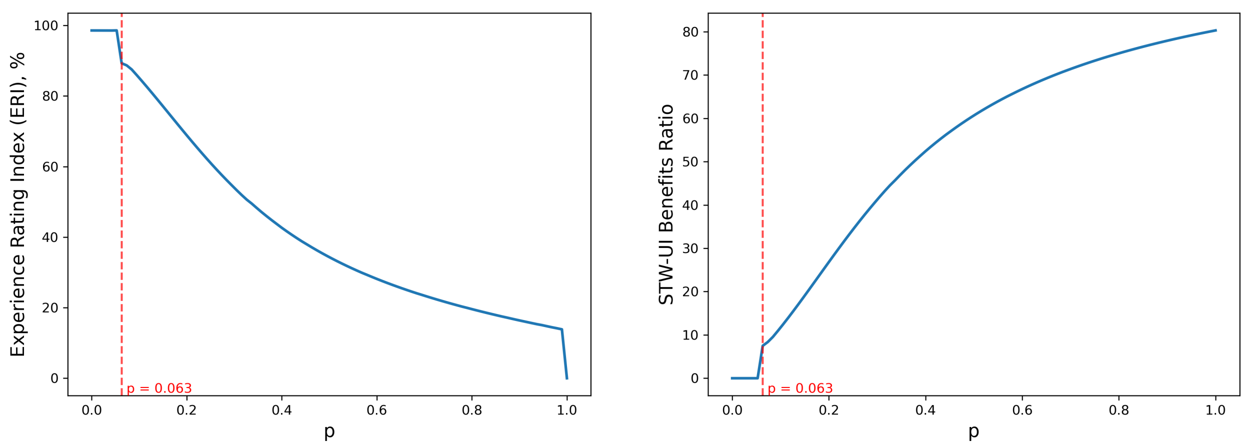
From the formula, we see again that the decision depends on the number of financially constrained firms, respectively, the number of constrained matches that can be saved with

STW, $MMS^c_{\epsilon_{\text{stw}}}$. If this number is small, the costs of distortion in working hours distortion might outweigh the welfare gain from rescuing additional workers.

Figure 15 illustrates the results numerically for the optimal eligibility condition under the joint optimal design. For $p < 6.3$ percent, the figure shows a red region, corresponding to the Ramsey planner's decision to forgo STW entirely. Beyond this threshold, the optimal eligibility condition is set equal to the separation threshold of constrained firms, which remains relatively stable as the fraction of constrained firms increases. Note that the separation threshold of unconstrained firms without access to STW rises as the layoff tax falls.

F Experience Rating and STW in Comparable Units

Figure 16: Welfare Comparison



(a) Optimal Layoff Taxes *(b) Optimal STW policy*

Notes: The left figure shows the Experience Rating Index (ERI) when the layoff tax is optimally implemented, measuring how much of the UI benefits caused by an employer are paid for through its layoff tax. At 100 percent, UI creates no incentive to lay off a worker in an unconstrained firm. The right figure shows the STW-UI benefits ratio, measuring total STW benefits divided by total UI benefits. At 100 percent, UI creates no layoff incentive under STW. Layoff taxes and STW are set jointly optimally.

To gauge how much layoff taxes and short-time work each contribute to discouraging separations induced by UI, it helps to express both instruments in comparable units. Each panel of Figure 16 reports an index measuring the share of the UI-induced separation incentive that the respective system neutralizes.

The left panel shows the Experience Rating Index (ERI), which measures the degree to which employers pay for the unemployment benefits their layoffs generate. An ERI of 50 percent means firms pay 50 cents for every dollar their laid-off workers collect in UI benefits. An ERI of 100 percent means full experience rating: employers bear the entire UI cost of their layoffs, eliminating any incentive that UI provides to separation for financially unconstrained firms. When no firms are financially constrained, layoff taxes eliminate almost all inefficient separations caused by UI. As the share of financially constrained firms rises, that fraction falls.

The right panel shows the STW-UI benefits ratio, which compares the total benefits a worker expects to receive under each program over the entire spell. A ratio of 50 percent means that for every dollar a worker would collect during a full unemployment spell under UI, they receive 50 cents under STW. A ratio of 100 percent would mean STW fully replaces UI benefits, removing any financial advantage of separation over retention.

The two panels differ in one important respect. The optimal ERI lies close to 100 percent: layoff taxes deter separations without introducing new distortions, so the planner lets firms bear nearly the full UI cost of their layoffs. The optimal benefits ratio, by contrast, remains below 100 percent. STW deters separations by subsidizing reductions in working hours, so greater generosity comes at the cost of inefficiently low hours in retained matches. The planner therefore stops short of full replacement, trading off discouraging separations against distortion in working hours. At the other end of the scale, a ratio of 0 percent means that STW provides no benefits and separation incentives are addressed through layoff taxes alone. Taken together, the two panels illustrate how the burden of discouraging inefficient separations shifts from layoff taxes toward STW as the share of financially constrained firms grows.

G Equilibrium Layoff Tax

G.1 Bargaining

Nash bargaining problem:

$$\max_{w^u(\epsilon),w^c(\epsilon),h^u(\epsilon),h^c(\epsilon),\epsilon_s^u,\epsilon_s^c} \quad \bar{J}^{(1-\eta)}\left(\bar{V}-U\right)^{\eta}$$

subject to

1. Financial constraint of financially constrained firms:

$$y(\epsilon,h(\epsilon))-w^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\geq 0$$

2. Commitment problem (Worker):

$$V^c(\epsilon) \geq U, \quad V^u(\epsilon) \geq 0$$

First of all, note that we can rewrite the financial constraint as

$$z(\epsilon) - c^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] \geq 0$$

Second, we can integrate the commitment problem of the worker into the value functions for the firm and worker surplus. For $i \in \{c, u\}$, we can rearrange the condition as:

$$u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \geq 0$$

Let $\epsilon_s^{i,\text{cp}}$ denote the threshold at which the worker wants to separate unitarily:

$$u(c^i(\epsilon_s^{i,\text{cp}})) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) = 0$$

We can rewrite the value functions of the firm and the surplus of the worker as:

$$\begin{aligned}
\bar{J} &= -\tau \\
&\quad + (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{z(\epsilon) - c^u(\epsilon) + \lambda \cdot \beta \cdot \bar{J}}{1 - \beta \cdot (1-\lambda)} dG(\epsilon) \\
&\quad + (1-p) \cdot G(\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}) \cdot \beta \cdot F \\
&\quad + p \cdot \int_{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}}^{\infty} \frac{z(\epsilon) - c^c(\epsilon) + \lambda \cdot \beta \cdot \bar{J}}{1 - \beta \cdot (1-\lambda)} dG(\epsilon) \\
&\quad + p \cdot G(\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}) \cdot \beta \cdot F \\
(\bar{V} - U) &= (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{u(c^u(\epsilon)) - u(b) + \lambda \cdot \beta \cdot (\bar{V} - U)}{1 - \beta \cdot (1-\lambda)} dG(\epsilon) \\
&\quad + p \cdot \int_{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}}^{\infty} \frac{u(c^c(\epsilon)) - u(b) + \lambda \cdot \beta \cdot (\bar{V} - U)}{1 - \beta \cdot (1-\lambda)} dG(\epsilon)
\end{aligned}$$

Note that $\epsilon_s^{i,B}$ denotes the separation threshold in the bargaining outcome. Separations occur either when the productivity falls below the bargained separation threshold or when the commitment constraint of the worker is violated $\epsilon < \max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}$. Finally, note that it is equivalent to maximize over $w^i(\epsilon)$ and $c^i(\epsilon)$ as

$$c^i(z) = w^i(\epsilon) - \phi(h^i(\epsilon)).$$

Set up the Kuhn-Tucker problem:

$$\max_{c^c(\epsilon),c^u(\epsilon),h^c(\epsilon),h^u(\epsilon),\epsilon_s^{c,B},\epsilon_s^{u,B}} \mathcal{B}$$

where

$$\begin{aligned}\mathcal{B} = &\left(\bar{J}\right)^{\eta} \cdot \left(\bar{V} - U\right)^{1-\eta} \\ &- \int_0^{\infty} \mathcal{M}(\epsilon) \left[z(\epsilon) - c^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (\lambda - f) \cdot J^c(\epsilon)\right]\right] d\epsilon\end{aligned}$$

With Kuhn-Tucker conditions for a maximum:

$$\begin{aligned}(I) \quad & \mathcal{M}(\epsilon) \leq 0 \\ (II) \quad & \mathcal{M}(\epsilon) \cdot \left[z(\epsilon) - c^c(\epsilon) + \beta \cdot \left[\lambda \cdot \bar{J} + (1 - \lambda) \cdot J^c(\epsilon)\right]\right] = 0\end{aligned}$$

FOC for the consumption equivalent of unconstrained firms:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^u(\epsilon)} &= -(1-\eta) \cdot (1-p) \cdot g(\epsilon) \cdot \left(\frac{\bar{V} - U}{\bar{J}}\right)^{\eta} \\ &\quad + \eta \cdot (1-p) \cdot g(\epsilon) \cdot u'\left(c^u(\epsilon)\right) \cdot \left(\frac{\bar{J}}{\bar{V} - U}\right)^{1-\eta} \\ &= 0\end{aligned}$$

$$\Longleftrightarrow \frac{1-\eta}{\eta} \cdot \frac{\bar{V} - U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \quad \Rightarrow \quad u'\left(c^u(\epsilon)\right) = u'(c^w)$$

Note that the firm wants to perfectly insure workers against idiosyncratic productivity shocks, provided they are unconstrained.

Define the joint surplus as

$$\bar{S} = \bar{J} + \frac{\bar{V} - U}{u'(c^w)}$$

The resulting surplus splitting rule gives:

$$J = (1-\eta) \cdot \bar{S}, \quad \frac{\bar{V} - U}{u'(c^u)} = \eta \cdot \bar{S}$$

FOC for the consumption equivalent of constrained firms:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial c^c(\epsilon)} = &-(1-p)\cdot g(\epsilon)\cdot(1-\eta)\cdot\left(\frac{\bar{V}-U}{\bar{S}}\right)^{\eta} \\
&+(1-p)\cdot g(\epsilon)\cdot u'\left(c^u(\epsilon)\right)\cdot U\cdot\left(\frac{J}{\bar{V}-U}\right)^{1-\eta} \\
&+\mathcal{M}(\epsilon)\overset{!}{=}0
\end{aligned}
$$

Insert the surplus splitting rule:

$$
\begin{aligned}
\Rightarrow \quad &-(1-p)\cdot g(\epsilon)\cdot\left(\frac{\eta}{1-\eta}\cdot u'(c^w)\right)^{\eta} \\
&+(1-p)\cdot g(\epsilon)\cdot u'\left(c^c(\epsilon)\right)\cdot\eta \\
&+\mathcal{M}(\epsilon)\cdot\left(u'(c^w)\cdot\frac{1-\eta}{\eta}\right)^{1-\eta}=0
\end{aligned}
$$

$$
\Leftrightarrow (1-p)\cdot g(\epsilon)\cdot\eta\cdot\left(u'(c^c(\epsilon))-u'(c^w)\right)=-\mathcal{M}(\epsilon)\cdot\left(u'(c^u)\cdot\frac{\eta}{1-\eta}\right)^{1-\eta}
$$

Note that if $c^c(\epsilon)<c^w$, then the RHS of the equation is positive due to risk aversion. Thus, firms and workers gain joint surplus until $c^c(\epsilon)=c^w$.

If

$$
z(\epsilon)-c^w+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\geq 0
$$

then the constraint is non-binding and $\mathcal{M}(\epsilon)=0$. ✓

If

$$
z(\epsilon)-c^w+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]< 0
$$

then the constraint is binding and $\mathcal{M}(\epsilon)<0$. ✓

Thus, it is optimal for the firm to pay as much as it can:

$$
J^c(\epsilon)=z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]=0
$$

$$
\Leftrightarrow \quad c^c(\epsilon)=z(\epsilon)+\lambda\cdot\beta\cdot\bar{J}
$$

Using the surplus splitting rule, the Nash bargaining problem can be simplified to

$$\max_{w^u(\epsilon),w^c(\epsilon),h^u(\epsilon),h^c(\epsilon),\epsilon_s^u,\epsilon_s^c} (1-\eta)^{(1-\eta)} \cdot (u'(c^w)\eta)^\eta \cdot \bar{S}$$

That is, we just have to maximize the joint surplus with the remaining contracts:

$$\begin{aligned}
\bar{S} = & -\tau \\
& + (1-p) \cdot \int_{\max\{\epsilon_s^{u,B},\epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (1-\eta \cdot f) \cdot \beta \cdot \bar{S}}{1-\beta \cdot (1-\lambda)} dG(\epsilon) \\
& - (1-p) \cdot G(\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}) \cdot \beta \cdot F \\
& + p \cdot \int_{\epsilon^p}^{\infty} \frac{z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (1-\eta \cdot f) \cdot \beta \cdot \bar{S}}{1-\beta \cdot (1-\lambda)} dG(\epsilon) \\
& + p \cdot \int_{\max\{\epsilon_s^{c,B},\epsilon_s^{c,\text{cp}}\}}^{\epsilon_p} \frac{z(\epsilon) + \frac{u(c^c(\epsilon))-u(b)}{u'(c^w)} - c^c(\epsilon) + (1-\eta \cdot f) \cdot \beta \cdot \bar{S}}{1-\beta \cdot (1-\lambda)} dG(\epsilon) \\
& - (1-p) \cdot G(\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}) \cdot \beta \cdot F
\end{aligned}$$

The FOC for working hours in unconstrained firms is:

$$\frac{\partial \mathcal{B}}{\partial h^u(\epsilon)} = (1-p) \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} - \phi'(h^u(\epsilon)) \right) = 0$$

$$\iff \quad \frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} = \phi'(h^u(\epsilon))$$

The FOC for working hours in constrained firms is:

$$\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} = p \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) \right) \cdot \frac{u'(c^c(\epsilon))}{u'(c^w)} = 0$$

$$\iff \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} = \phi'(h^c(\epsilon))$$

Suppose that $\epsilon_s^{u,B} > \epsilon_s^{u,cp}$

$$\begin{aligned}
\frac{\partial \mathcal{B}}{\partial \epsilon_s^{u,b}} = -(1-p) \cdot g(\epsilon_s^{u,b}) \cdot \Bigg[& \frac{1}{1-\beta \cdot (1-\lambda)} \cdot \Big(z(\epsilon_s^{u,b}) + \frac{u(c^w)-u(b)}{u'(c^w)} \\
& - c^w + (\lambda - \eta f) \cdot \beta \cdot \bar{S} \Big) + \beta \cdot F \Bigg] = 0
\end{aligned}$$

$$\iff z(\epsilon_s^{u,b}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry condition:

$$z(\epsilon_s^{u,b}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + \frac{\lambda-\eta f}{1-\eta}\cdot\left(\frac{k_v}{q}\right) = 0$$

Note that workers do not have a commitment problem $\epsilon_s^{u,B} > \epsilon_s^{u,cp}$, as workers would never separate under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f)\cdot\beta\cdot(\bar{V} - U) > 0$$

Workers in unconstrained firms always have a positive surplus!

Suppose that $\epsilon_s^{c,B} \geq \epsilon_s^{c,cp}$

$$\begin{aligned}\frac{\partial\mathcal{B}}{\partial\epsilon_s^{c,B}} = -(1-p)\cdot g(\epsilon_s^{c,B})\cdot\Bigg[&\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Bigg(z(\epsilon_s^{c,B}) + \frac{u\left(c^c(\epsilon_s^{c,B})\right)-u(b)}{u'(c^w)} \\ &- c^c(\epsilon_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Bigg) + F\Bigg] = 0\end{aligned}$$

$$\iff z(\epsilon_s^{c,B}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^c(\epsilon_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry condition:

$$z(\epsilon_s^{c,B}) + (1-\beta\cdot(1-\lambda))\cdot\beta\cdot F + \frac{u(c^c(\epsilon_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\epsilon_s^{cp}) + \frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q} = 0$$

The participation constraint of workers can be written with the free-entry condition as:

$$z(\epsilon_s^{c,cp}) + \frac{u(c^c(\epsilon_s^{c,cp}))-u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,cp}) + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

Note that to avoid paying future layoff taxes, firms would like workers to stay within the firm for negative surplus values. Thus, the worker decides to leave the firm unilaterally $\epsilon_s^{c,B} < \epsilon_s^{c,cp}$.

G.2 Job Creation and Wage Equation

The notation of this section follows Appendix I for layoff taxes.

(I) Value of an unconstrained firm after the idiosyncratic productivity shock has been realized:

$$
\begin{aligned}
& J^u(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda\, \bar{J} + (1-\lambda) \cdot J^u(\epsilon)\right] \\
\Leftrightarrow \quad & J^u(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left[z(\epsilon) - c^w + \lambda \cdot \beta \cdot \bar{J}\right]
\end{aligned}
$$

(II) Value of a constrained firm without binding constraints after the idiosyncratic productivity shock has been realized:

$$
\begin{aligned}
& J^c(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda\, \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] \\
\Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(z(\epsilon) - c^w + \lambda \cdot \beta \cdot \bar{J}\right)
\end{aligned}
$$

(III) Value of a constrained firm with binding constraints after the idiosyncratic productivity shock has been realized:

$$
\begin{aligned}
& J^c(\epsilon) = z(\epsilon) - c^w(\epsilon) + \beta \cdot \left[\lambda\, \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] \\
\Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(z(\epsilon) - c^w(\epsilon) + \lambda \cdot \beta \cdot \bar{J}\right)
\end{aligned}
$$

(IV) Value of a firm before the idiosyncratic productivity shock has been realized:

$$
\bar{J} = -\tau + (1-p) \cdot \int_{\epsilon_s^u}^{\infty} J^u(\epsilon)\, dG(\epsilon) + p \cdot \int_{\epsilon_s^c}^{\infty} J^c(\epsilon)\, dG(\epsilon) - \rho \cdot \beta \cdot F
$$

Inserting (I)-(III) into (IV) gives:

$$
\begin{aligned}
\Leftrightarrow \quad \bar{J} = -\tau + \frac{1}{1-\beta \cdot (1-\lambda)} \Big[& (1-\rho) \cdot z - (1-p) \cdot (1-\rho^u) \cdot c^w - p \cdot (1-\rho^c) \cdot e^c \\
& + (1-p) \cdot \lambda \cdot \beta \cdot \bar{J} \Big] - \rho \cdot \beta \cdot F
\end{aligned}
$$

Here z denotes the average production (without distortions) and e^c denotes the average cost of a constrained worker for a firm. Both are defined in Section I of the appendix.

(V) Value of an unconstrained worker after the idiosyncratic productivity shock has been realized:

$$
\begin{aligned}
& V^u(\epsilon) = u(c^w) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) \cdot V^u(\epsilon)\right] \\
\Leftrightarrow \quad & V^u(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w) + \lambda \cdot \beta \cdot \bar{V}\right)
\end{aligned}
$$

(VI) Value of a constrained worker without binding constraints after the idiosyncratic productivity shock has been realized:

$$
\begin{aligned}
& V^c(\epsilon) = u(c^w) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) \cdot V^c(\epsilon)\right] \\
\Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w) + \lambda \cdot \beta \cdot \bar{V}\right)
\end{aligned}
$$

(VII) Value of a constrained worker with binding constraints after the idiosyncratic productivity shock has been realized:

$$
\begin{aligned}
& V^c(\epsilon) = u(c^w(\epsilon)) + \beta \cdot \left[\lambda \bar{V} + (1-\lambda) \cdot V^c(\epsilon)\right] \\
\Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w(\epsilon)) + \lambda \cdot \beta \cdot \bar{V}\right)
\end{aligned}
$$

(VIII) Value of a worker before the idiosyncratic productivity shock has been realized:

$$
\bar{V} = (1-p) \cdot \int_{\epsilon_s^u}^{\infty} V^u(\epsilon)\, dG(\epsilon) + p \cdot \int_{\xi_s^c}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \rho \cdot U
$$

Inserting (V)-(VII) into (VIII) gives:

$$
\begin{aligned}
\Leftrightarrow \quad \bar{V} = & \frac{1}{1-\beta \cdot (1-\lambda)} \Big[(1-p)(1-\rho^u) \cdot u(c^w) + p \cdot (1-\rho^c) \cdot u^c \\
& + \beta \cdot \left[(1-\rho) \cdot \lambda \cdot \beta \cdot \bar{V} + \rho \cdot U\right] \Big]
\end{aligned}
$$

u^c is defined as the average utility of a worker in a constrained firm. Its definition can be found in Appendix I.

(IX) Unemployment:

$$
U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1-f) \cdot U\right]
$$

Next, we turn to calculating the joint surplus.

A) First, let us calculate the surplus of the worker after the idiosyncratic productivity shock has been realized:

$$\begin{aligned}
& V^i(\epsilon) - U = u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot V^i(\epsilon) + (\lambda - f) \cdot \beta \cdot \bar{V} - (\lambda - f) \cdot \beta \cdot U \\
\Leftrightarrow \quad & V^i(\epsilon) - U = u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (V^i(\epsilon) - U) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \\
\Leftrightarrow \quad & V^i(\epsilon) - U = \frac{1}{1 - \beta \cdot (1 - \lambda)} \big(u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \big)
\end{aligned}$$

B) Next, we calculate the expected surplus of the worker, before the shocks have been realized:

$$\begin{aligned}
\bar{V} - U &= (1 - \rho) \cdot \int_{\epsilon_s^u}^{\infty} (V^u(\epsilon) - U)\ dG(\epsilon) + p \cdot \int_{\epsilon_s^c}^{\infty} (V^c(\epsilon) - U)\ dG(\epsilon) \\
&= \frac{1}{1 - \beta \cdot (1 - \lambda)} \Bigg[(1 - \rho) \cdot (1 - \rho^u) \cdot (u(c^u) - u(b)) \\
&\quad + p \cdot (1 - \rho^c) \cdot (u(c^c) - u(b)) \\
&\quad + (1 - \rho) \cdot (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \Bigg]
\end{aligned}$$

C) Next, we can calculate the joint surplus from the expected value of a worker for a firm and the expected surplus of the worker:

$$\begin{aligned}
S &= \bar{J} + \frac{\bar{V} - U}{u'(c^u)} \\
&= -\tau + \frac{1}{1 - \beta \cdot (1 - \lambda)} \Bigg[(1 - \rho) \cdot z \\
&\quad + (1 - \rho) \cdot (1 - \rho^u) \cdot \left(\frac{u(c^u) - u(b)}{u'(c^u)} - c^u \right) \\
&\quad + p \cdot (1 - \rho^c) \cdot \left(\frac{u(c^c) - u(b)}{u'(c^u)} - e^c \right) \\
&\quad + (1 - \rho) \cdot \lambda \cdot \beta \cdot \bar{J} + (1 - \rho) \cdot (\lambda - f) \cdot \beta \cdot \frac{\bar{V} - U}{u'(c^u)} \Bigg] - \rho \cdot \beta \cdot F
\end{aligned}$$

Using the surplus splitting rule, we can rewrite the equation above as:

$$
\begin{aligned}
S = -\tau + \frac{1}{1-\beta\cdot(1-\lambda)} \Bigg[& (1-\rho)\cdot z \\
& + (1-\rho)\cdot(1-\rho^u)\cdot\left(\frac{u(c^u)-u(b)}{u'(c^u)} - c^u\right) \\
& + p\cdot(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^u)} - e^c\right) \\
& + (1-\rho)\cdot(\lambda-\eta f)\cdot\beta\cdot S \Bigg] - \rho\cdot\beta\cdot F
\end{aligned}
$$

Next, we want to replace the tax in the surplus equation. We can do this by using the budget constraint of the government:

$$
\begin{aligned}
n^s\cdot\tau &= -n^s\cdot\rho\cdot F + (1-n)\cdot b \\
\Leftrightarrow \quad \tau &= -\rho\cdot F + \frac{1-n}{n^s}\cdot b \\
\text{Insert } n^s &= \frac{\lambda}{1-\rho}\cdot n \\
\Rightarrow \quad \tau &= \rho\cdot F + \frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b
\end{aligned}
$$

Inserting it into the equation for the joint surplus gives:

$$
\begin{aligned}
S = \frac{1}{1-\beta\cdot(1-\lambda)} \Bigg[& (1-\rho)\cdot\left(z - \frac{1-n}{n}\cdot b\right) + (1-p)\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) \\
& + p\cdot(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)} - e^c\right) + (1-\rho)\cdot(\lambda-\eta f)\cdot\beta\cdot S \Bigg] \\
& -\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b + \rho\cdot(1-\beta)\cdot F
\end{aligned}
$$

Next, insert the free-entry condition: $\frac{k_v}{q} = \beta\cdot\bar{J}$

$$
\begin{aligned}
\frac{1}{1-\eta}\cdot\frac{k_v}{q} = \frac{\beta}{1-\beta\cdot(1-\lambda)}\cdot\Bigg[&(1-\rho)\cdot z\\
&+(1-p)\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)\\
&+p\cdot(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}-e^c\right)\\
&+(1-\rho)\cdot\left(\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}\right)\Bigg]\\
&-\frac{\beta}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b+\rho\cdot\beta\cdot(1-\beta)\cdot F
\end{aligned}
$$

The subsequent derivation of the wage equation is completely analogous to the derivation under the STW regime described in Section H.2. The wage equation thus reads:

$$
\begin{aligned}
(1-p)\cdot(1-\rho^u)\cdot(1-\eta)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}+\eta\cdot c^w\right) = \\
&\eta\cdot[(1-\rho)\cdot z+(1-\rho)\cdot\theta\cdot k_v]\\
&-\frac{1-\beta\cdot(1-\lambda)}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b\\
&+\frac{1-\beta\cdot(1-\lambda)}{\lambda}\cdot\rho\cdot\beta\cdot(1-\beta)\cdot F\\
&-p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}+\eta\cdot e^c\right)
\end{aligned}
$$

H Equilibrium STW

H.1 Bargaining

Nash bargaining problem:

$$
\max_{\substack{w^u(\epsilon),w^c(\epsilon),w^u_{\text{stw}}(\epsilon),w^c_{\text{stw}}(\epsilon),\\ h^u(\epsilon),h^c(\epsilon),h^u_{\text{stw}}(\epsilon),h^c_{\text{stw}}(\epsilon),\\ \epsilon^u_s,\epsilon^c_s,\xi^u_s,\xi^c_s}} \bar{J}^{1-\eta}\cdot(\bar{V}-U)^\eta
$$

Subject to:

1. Financial constraints of financially constrained firms:

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right]$$
$$y(\epsilon, h_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) \geq \beta \cdot \left[\lambda \cdot \bar{J} + (1-\lambda) \cdot J^c_{\text{stw}}(\epsilon)\right]$$

2. Workers have a commitment problem:

$$U \leq V^u(\epsilon), \quad U \leq V^u_{\text{stw}}(\epsilon)$$
$$U \leq V^c(\epsilon), \quad U \leq V^c_{\text{stw}}(\epsilon)$$

Second, we can integrate the commitment problem of the worker into the value functions for the firm and worker surplus. For $i \in \{\text{stw}, \text{no stw}\}$ and $j \in \{u, c\}$, we can reformulate the condition as:

$$u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \geq 0$$

Let $\epsilon_s^{j,\text{cp}}$ denote the threshold at which the worker wants to separate from a firm with STW support:

$$u(c_i^j(\epsilon_s^{j,\text{stw},\text{cp}})) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) = 0$$

Likewise, we can determine the threshold at which workers leave firms without STW support, $\xi_s^{j,\text{cp}}$, as:

$$u(c_i^j(\xi_s^{j,\text{cp}})) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) = 0$$

We can rewrite the value functions of the firm and the surplus of the worker as:

$$\begin{aligned}
\bar{J} = -\tau + (1-p) \cdot \Bigg[& \int_{\max\{\xi_s^{u,B}, \xi_s^{u,\text{cp}}, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^u(\epsilon) + \lambda\beta\bar{J}\right) dG(\epsilon) \\
& + \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}, \epsilon_{\text{stw}}\}} \frac{1}{1-\beta\cdot(1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^u_{\text{stw}}(\epsilon) + \lambda\beta\bar{J}\right) dG(\epsilon) \Bigg] \\
+ p \cdot \Bigg[& \int_{\max\{\xi_s^{c,B}, \xi_s^{c,\text{cp}}, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^c(\epsilon) + \lambda\beta\bar{J}\right) dG(\epsilon) \\
& + \int_{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,\text{cp}}\}}^{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,cp}, \epsilon_{\text{stw}}\}} \frac{1}{1-\beta\cdot(1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^c_{\text{stw}}(\epsilon) + \lambda\beta\bar{J}\right) dG(\epsilon) \Bigg]
\end{aligned}$$

$$
\begin{aligned}
&(\bar{V}-U)\\
&=(1-p)\cdot\Bigg[\int_{\max\{\xi_s^{u,B},\,\xi_s^{u,\text{cp}},\,\epsilon_{\text{stw}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^u(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\\
&\quad+\int_{\max\{\epsilon_s^{u,B},\,\epsilon_s^{u,\text{cp}}\}}^{\max\{\epsilon_s^{u,B},\epsilon_s^{u,cp},\epsilon_{\text{stw}}\}}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^u_{\text{stw}}(\epsilon))+s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\Bigg]\\
&\quad+p\cdot\Bigg[\int_{\max\{\xi_s^{c,B},\,\xi_s^{c,\text{cp}},\,\epsilon_{\text{stw}}\}}^{\infty}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^c(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\\
&\quad+\int_{\max\{\epsilon_s^{c,B},\,\epsilon_s^{c,\text{cp}}\}}^{\max\{\epsilon_s^{c,B},\epsilon_s^{c,cp},\epsilon_{\text{stw}}\}}\frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^c_{\text{stw}}(\epsilon))+s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))-u(b)+\lambda\beta(\bar{V}-U)\right)dG(\epsilon)\Bigg]
\end{aligned}
$$

Note that $\xi_s^{u,B}, \xi_s^{c,\text{B}}, \epsilon_s^{u,B}$ and $\epsilon_s^{c,\text{B}}$ denote the separation thresholds at bargaining outcome. Note that we can rewrite the problem to optimize over consumption equivalents rather than salaries. $c^i_{\text{stw}}(\epsilon)$ denotes the consumption equivalent paid by the firm to the worker on STW. $c^i(\epsilon)$ is the consumption equivalent consumed by the worker.

$$
\max_{\substack{c^u(\epsilon),\,c^c(\epsilon),\,c^u_{\text{stw}}(\epsilon),\,c^c_{\text{stw}}(\epsilon),\\ h^u(\epsilon),\,h^c(\epsilon),\,h^u_{\text{stw}}(\epsilon),\,h^c_{\text{stw}}(\epsilon),\\ \xi_s^{u,B},\,\xi_s^{c,B},\,\epsilon_s^{u,B},\,\epsilon_s^{c,B}}} \left(\bar{J}\right)^{1-\eta}\cdot\left(\bar{V}-U\right)^{\eta}
$$

Financial constraints of financially constrained firms

$$
\begin{aligned}
y(\epsilon,h(\epsilon))-w^c(\epsilon) &\geq \beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\\
y(\epsilon,h_{\text{stw}}(\epsilon))-w^c_{\text{stw}}(\epsilon) &\geq \beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\text{stw}}(\epsilon)\right]
\end{aligned}
$$

Set up Kuhn-Tucker conditions:

$$
\begin{aligned}
&\left(\bar{J}\right)^{\eta}\cdot\left(\bar{V}-U\right)^{1-\eta}-\int_0^{\infty}\mathcal{M}(\epsilon)\left[z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\right]dG(\epsilon)\\
&\quad-\int_0^{\infty}\mathcal{M}_{\text{stw}}(\epsilon)\left[z_{\text{stw}}(\epsilon)-c^c_{\text{stw}}(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\text{stw}}(\epsilon)\right]\right]dG(\epsilon)
\end{aligned}
$$

With Kuhn-Tucker conditions for a maximum:

(A) (i) $\mathcal{M}(\epsilon)\leq 0$

(ii) $\mathcal{M}(\epsilon)\left[z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\right]=0$

(B) (i) $\mathcal{M}_{\text{stw}}(\epsilon)\leq 0$

(ii) $\mathcal{M}_{\text{stw}}(\epsilon)\left[z_{\text{stw}}(\epsilon)-c^c_{\text{stw}}(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\text{stw}}(\epsilon)\right]\right]=0$

FOC for $c^u(\epsilon)$:

$$\frac{\partial\mathcal{B}}{\partial c^u(\epsilon)}=-(1-\eta)\cdot(1-p)\cdot g(\epsilon)\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta}$$
$$+\eta\cdot(1-p)\cdot g(\epsilon)\cdot u'(c^u(\epsilon))\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta}=0$$

Unconstrained firms want to insure workers against idiosyncratic productivity shocks, outside STW:

$$\iff\quad\frac{1-\eta}{\eta}\cdot\frac{\bar{V}-U}{\bar{J}}=u'\left(c^u(\epsilon)\right)\quad\Rightarrow\quad u'(c^u(\epsilon))=u'(c^u(\epsilon'))=u'(c^u)$$

FOC for $c^u_{\text{stw}}(\epsilon)$:

$$\frac{\partial\mathcal{B}}{\partial c^u_{\text{stw}}(\epsilon)}=-(1-\eta)(1-p)\cdot g(\epsilon)\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta}$$
$$+\eta\cdot(1-p)\cdot g(\epsilon)\cdot u'\left(c^u_{\text{stw}}(\epsilon)\right)\cdot\eta\cdot\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta}=0$$

Unconstrained firms want to insure workers against idiosyncratic productivity shocks, also on STW:

$$\iff\quad\frac{1-\eta}{\eta}\cdot\frac{\bar{V}-U}{\bar{J}}=u'\left(c^u(\epsilon)\right)$$
$$\Rightarrow\quad u'\left(c^u_{\text{stw}}(\epsilon)\right)=u'\left(c^u_{\text{stw}}(\epsilon')\right)=u'(c^w)$$

FOC for $c^c(\epsilon)$:

$$\frac{\partial\mathcal{B}}{\partial c^c(\epsilon)}=-(1-p)\cdot g(\epsilon)\cdot(1-\eta)\cdot\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta}$$
$$+(1-p)\cdot g(\epsilon)\cdot u'(c^c(\epsilon))\cdot\eta\cdot\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta}+\mathcal{M}(\epsilon)$$
$$\stackrel{!}{=}0$$

Insert the surplus splitting rule:

$$
\begin{aligned}
&-(1-p)\cdot g(\epsilon)\cdot\left(\frac{\eta}{1-\eta}\cdot u'(c^w)\right)^{\eta} \\
&+(1-p)\cdot g(\epsilon)\cdot u'(c^c(\epsilon))\cdot\eta\cdot\left(\frac{1-\eta}{\eta}\cdot\frac{1}{u'(c^w)}\right)+\mathcal{M}(\epsilon)\overset{!}{=}0
\end{aligned}
$$

$$
\iff\quad (1-p)\cdot g(\epsilon)\cdot\eta\cdot(u'(c^c(\epsilon))-u'(c^w))=-\mathcal{M}(\epsilon)\cdot\left(u'(c^w)\cdot\frac{\eta}{1-\eta}\right)^{1-\eta}
$$

Note: If $c^c(\epsilon)<u'(c^w)$, then the RHS of the equation is positive.
Thus, firms and workers gain joint surplus until $c^c(\epsilon)=c^w$.

If

$$z(\epsilon)-c^w+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]\geq 0,$$

then the constraint is non-binding and $\mathcal{M}(\epsilon)=0$. ✓

If

$$z(\epsilon)-c^w+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]<0,$$

then the constraint is binding and $\mathcal{M}(\epsilon)<0$. ✓

Thus, it is optimal for the firm to pay as much as it can:

$$J^c(\epsilon)=z(\epsilon)-c^c(\epsilon)+\beta\cdot\left[\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]=0$$

$$\iff\quad c^c(\epsilon)=z(\epsilon)+\lambda\cdot\beta\cdot\bar{J}$$

Using the same arguments, we get that under STW.

$$c_{\text{stw}}(\epsilon)=c^w\quad\text{if the financial constraint is non-binding,}$$

and

$$c_{\text{stw}}(\epsilon)=\epsilon_{\text{stw}}(\epsilon)+s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon))+\lambda\cdot\beta\cdot\bar{J}\quad\text{when it is binding.}$$

Using the surplus-splitting rule, we can simplify the Nash bargaining problem to:

$$\max_{h^u(\epsilon),h^c(\epsilon),h^u_{\text{stw}}(\epsilon),h^c_{\text{stw}}(\epsilon),\, \xi^u_s,\xi^c_s,\xi^{u^c}_s,\xi^{c,c}_s} (1-\eta)^{1-\eta} \cdot (u'(c^w)\cdot\eta)^{\eta} \cdot \bar{S}$$

That is, we just have to maximize the joint surplus:

$$\begin{aligned}
\bar{S} = & -\tau \\
& + (1-p)\cdot \int_{\max\{\xi_s^{u,B},\xi_s^{u,cp},\epsilon_{\text{stw}}\}}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
& + (1-p)\cdot \int_{\max\{\epsilon_s^{u,B},\epsilon_s^{u,cp}\}}^{\max\{\epsilon_s^{u,B},\epsilon_s^{u,cp},\epsilon_{\text{stw}}\}} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w \\
& \qquad + s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon)) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
& + p\cdot \int_{\epsilon^p}^{\infty} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
& + p\cdot \int_{\max\{\xi_s^{c},\xi_s^{c,cp},\epsilon_{\text{stw}}\}}^{\epsilon^p} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z(\epsilon) + \frac{u(c^c(\epsilon))-u(b)}{u'(c^w)} - c^c(\epsilon) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon) \\
& + p\cdot \int_{\max\{\epsilon_s^{c},\epsilon_s^{c,cp}\}}^{\max\{\epsilon_s^{c},\epsilon_s^{c,cp},\epsilon_{\text{stw}}\}} \frac{1}{1-\beta\cdot(1-\lambda)} \Big(z_{\text{stw}}(\epsilon) + \frac{u(c^c_{\text{stw}}(\epsilon))-u(b)}{u'(c^w)} \\
& \qquad - c^c(\epsilon) + (1-\eta\cdot f)\cdot\beta\cdot\bar{S} \Big) dG(\epsilon)
\end{aligned}$$

Note that without financial constraints, the firm smooths the consumption equivalent consumed by the worker:

$$c^w = c^u_{\text{stw}}(\epsilon) = c^u_{\text{stw}}(\epsilon) + s_{stw}\cdot(\bar{h} - h_{\text{stw}}(\epsilon))$$

$$\Leftrightarrow \quad c^u_{\text{stw}} = c^w - s_{stw}\cdot(\bar{h} - h_{\text{stw}}(\epsilon))$$

STW makes it less expensive to smooth consumption equivalents consumed by the worker. The FOC for working hours in unconstrained firms outside STW is:

$$\begin{aligned}
\frac{\partial\mathcal{B}}{\partial h^u(\epsilon)} &= (1-p)\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon,h^u(\epsilon))}{\partial h^u(\epsilon)} - \phi'(h^u(\epsilon))\right) = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon,h^u(\epsilon))}{\partial h^u(\epsilon)} &= \phi'(h^u(\epsilon))
\end{aligned}$$

The FOC for working hours in constrained firms *outside* STW is:

$$\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} = p \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) \right) \cdot \frac{u'(c^c(\epsilon))}{u'(c^w)} = 0$$

$$\Leftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} = \phi'(h^c(\epsilon))$$

The FOC for working hours in unconstrained firms *on* STW is:

$$\frac{\partial \mathcal{B}}{\partial h^u_{\text{stw}}(\epsilon)} = (1-p) \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^u_{\text{stw}}(\epsilon))}{\partial h^u_{\text{stw}}(\epsilon)} - \phi'(h^u_{\text{stw}}(\epsilon)) - s_{stw} \right) = 0$$

$$\Leftrightarrow \quad \frac{\partial y(\epsilon, h^u_{\text{stw}}(\epsilon))}{\partial h^u_{\text{stw}}(\epsilon)} = \phi'(h^u_{\text{stw}}(\epsilon)) + s_{stw}$$

The FOC for working hours in constrained firms *on* STW is:

$$\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} = p \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) - s_{stw} \right) \cdot \frac{u'(c^c(\epsilon))}{u'(c^w)} = 0$$

$$\Leftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} = \phi'(h^c(\epsilon)) + s_{stw}$$

Suppose that

$$\max\{\xi^{u,B}_s,\, \xi^{u,\text{cp}}_s,\, \epsilon_{\text{stw}}\} = \xi^{u,B}_s$$

Then the FOC for the separation threshold of the unconstrained firm without STW support is:

$$\frac{\partial \mathcal{B}}{\partial \xi^{u,B}_s} = -(1-p) \cdot g(\xi^{u,B}_s) \cdot \frac{1}{1-\beta \cdot (1-\lambda)} \left(z(\xi^{u,B}_s) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (\lambda - \eta f) \cdot \beta \cdot \bar{S} \right) = 0$$

$$\Longleftrightarrow \; z(\xi^{u,B}_s) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (\lambda - \eta f) \cdot \beta \cdot \bar{S} = 0$$

Insert free-entry condition:

$$z(\xi^{u,B}_s) + z(\xi^{u,B}_s) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{(\lambda - \eta f)}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that $\xi^{u,B}_s > \xi^{u,\text{cp}}_s$ must hold, as workers would never separate under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) > 0$$

Workers always gets a positive surplus!

Suppose that

$$\max\{\xi_s^{c,B},\ \xi_s^{c,\mathrm{cp}},\ \epsilon_{\mathrm{stw}}\} = \xi_s^{c,B}$$

Then the FOC for the separation threshold of the unconstrained firm without STW support is:

$$\frac{\partial \mathcal{B}}{\partial \xi_s^{c,B}} = -(1-p)\cdot g(\xi_s^{c,B})\cdot\Bigg[\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Bigg(z(\xi_s^{c,B}) + \frac{u\left(c^c(\xi_s^{c,B})\right)-u(b)}{u'(c^w)}$$
$$- c^c(\xi_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Bigg)\Bigg] = 0$$

$$\iff z(\xi_s^{c,B}) + \frac{u(c^c(\xi_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry condition:

$$z(\xi_s^{c,B}) + \frac{u(c^c(\xi_s^{c,B}))-u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

The participation constraint of workers can be written with the free-entry condition as:

$$z(\xi_s^{c,\mathrm{cp}}) + \frac{u(c^c(\xi_s^{c,\mathrm{cp}}))-u(b)}{u'(c^w)} - c^c(\xi_s^{c,\mathrm{cp}}) + \frac{(\lambda-\eta f)}{1-\eta}\cdot\frac{k_v}{q} = 0$$

Note that these are the same conditions. This implies that given the inability to insure workers against idiosyncratic productivity shocks, separations for a constrained firm without access to STW are efficient.

Suppose that

$$\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\mathrm{cp}}, \epsilon_{\mathrm{stw}}\} = \epsilon_s^{u,B}$$

Then the FOC for the separation threshold of the unconstrained firm with STW support is:

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{u,B}} = -(1-p)\cdot g(\epsilon_s^{u,B})\cdot\Bigg[\frac{1}{1-\beta\cdot(1-\lambda)}\cdot\Bigg(z_{\mathrm{stw}}(\epsilon_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w$$
$$+ s_{stw}\cdot(\bar{h} - h_{\mathrm{stw}}(\epsilon_s^{u,B})) + (\lambda-\eta f)\cdot\beta\cdot\bar{S}\Bigg)\Bigg] = 0$$

$$\iff z_{\mathrm{stw}}(\epsilon_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + s_{stw}(\bar{h} - h_{\mathrm{stw}}(\epsilon_s^{u,B})) + (\lambda-\eta f)\cdot\beta\cdot\bar{S} = 0$$

Insert free-entry condition:

$$z_{\text{stw}}(\epsilon_s^{u,B}) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that $\epsilon_s^{u,B} > \epsilon_s^{u,\text{cp}}$ must hold, as workers would never separate under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) > 0$$

Workers always gets a positive surplus!

Suppose that

$$\max\{\epsilon_s^{c,B},\, \epsilon_s^{c,\text{cp}},\, \epsilon_{\text{stw}}\} = \epsilon_s^{c,B}$$

Then the FOC for the separation threshold of the constrained firm with STW support is:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial \epsilon_s^{c,B}} = -(1-p) \cdot g(\epsilon_s^{c,B}) \cdot \Bigg[& \frac{1}{1 - \beta \cdot (1-\lambda)} \cdot \Bigg(z_{\text{stw}}(\epsilon_s^{c,B}) + \frac{u\left(c^c_{\text{stw}}(\epsilon_s^{c,B})\right) - u(b)}{u'(c^w)} \\ & - c^c_{\text{stw}}(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \beta \cdot \bar{S} \Bigg) \Bigg] = 0\end{aligned}$$

$$\iff z(\epsilon_s^{c,B}) + \frac{u(c^c_{\text{stw}}(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c^c_{\text{stw}}(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \beta \cdot \bar{S} = 0$$

Insert free-entry condition:

$$z_{\text{stw}}(\epsilon_s^{c,B}) + \frac{u(c^c_{\text{stw}}(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c^c_{\text{stw}}(\epsilon_s^{c,B}) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

The participation constraint of workers can be written with the free-entry condition as:

$$z_{\text{stw}}(\epsilon_s^{c,\text{cp}}) + \frac{u(c^c_{\text{stw}}(\epsilon_s^{c,\text{cp}})) - u(b)}{u'(c^w)} - c^c_{\text{stw}}(\epsilon_s^{c,\text{cp}}) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that these are the same conditions. This implies that, given the inability to insure workers against idiosyncratic productivity shocks, separations for a constrained firm with access to STW are efficient.

H.2 Job Creation and Wage Equation

This section derives the job creation and the equation for STW. It follows the notation of Appendix I.

(I) Value of an unconstrained firm outside STW after the idiosyncratic productivity shock has been realized:

$$\begin{aligned} & J^u(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda\, \bar{J} + (1-\lambda)\, J^u(\epsilon)\right] \\ \Leftrightarrow \quad & J^u(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w + \lambda\beta\bar{J}\right) \end{aligned}$$

(II) Value of an unconstrained firm on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^u_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) + s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon)) - c^w + \beta\left[\lambda\, \bar{J} + (1-\lambda)\, J^u_{\text{stw}}(\epsilon)\right] \\ \Leftrightarrow \quad & J^u_{\text{stw}}(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z_{\text{stw}}(\epsilon) - c^w + \lambda\beta\bar{J}\right) \end{aligned}$$

(III) Value of a constrained firm outside STW with non-binding constraints after the idiosyncratic productivity shock has been realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w + \beta \cdot \left[\lambda\, \bar{J} + (1-\lambda)\, J^c(\epsilon)\right] \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w + \lambda\beta\bar{J}\right) \end{aligned}$$

(IV) Value of a constrained firm outside STW with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w(\epsilon) + \beta\left[\lambda\, \bar{J} + (1-\lambda)\, J^c(\epsilon)\right] \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} \left(z(\epsilon) - c^w(\epsilon) + \lambda\beta\bar{J}\right) \end{aligned}$$

(V) Value of a constrained firm on STW after the idiosyncratic productivity shock has been realized:

$$\begin{aligned} & J^c_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) - c^w_{\text{stw}}(\epsilon) + \beta \cdot [\lambda J + (1-\lambda) J^c_{\text{stw}}(\epsilon)] \\ \Leftrightarrow \quad & J^c_{\text{stw}}(\epsilon) = \frac{1}{1-\beta\cdot(1-\lambda)} (z_{\text{stw}}(\epsilon) - c^w_{\text{stw}}(\epsilon) + \lambda\beta J) \end{aligned}$$

(VI) Value of a firm before the idiosyncratic productivity shock has been realized:

$$\bar{J} = -\tau + (1-p)\cdot\left(\int_{\epsilon_{\text{stw}}}^{\infty} J^u(\epsilon)\,dG(\epsilon) + \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} J^u_{\text{stw}}(\epsilon)\,dG(\epsilon)\right)$$
$$+ p\cdot\left(\int_{\max\{\epsilon_{\text{stw}},\xi_s^c\}}^{\infty} J^c(\epsilon)\,dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} J^c_{\text{stw}}(\epsilon)\,dG(\epsilon)\right)$$

Inserting (I) - (IV) into (V) gives:

$$\begin{aligned}
\bar{J} &= -\tau \\
&+ \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[(1-\rho)\cdot(z-\Omega) \\
&- (1-p)\cdot(1-\rho^u)\cdot\left(c^w - \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right) \\
&- p\cdot(1-\rho^c)\cdot\left(e^c - \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right) \\
&+ (1-\rho)\cdot\lambda\cdot\beta\cdot\bar{J}\Bigg]
\end{aligned}$$

Here, z denotes the average production (without distortions) and e^c denotes the average cost of a constrained worker for a firm. Both are defined in section D of the appendix.

(VII) Value of an unconstrained worker outside STW after the idiosyncratic productivity shock has been realized:

$$\begin{aligned}
V^u(\epsilon) &= u(c^w) + \beta\cdot\left[\lambda\,\bar{V} + (1-\lambda)\,V^u(\epsilon)\right] \\
\Leftrightarrow\quad V^u(\epsilon) &= \frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^w) + \lambda\beta\bar{V}\right)
\end{aligned}$$

(VIII) Value of an unconstrained worker on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned}
V^u_{\text{stw}}(\epsilon) &= u(c^w) + \beta\cdot\left[\lambda\,\bar{V} + (1-\lambda)\,V^u_{\text{stw}}(\epsilon)\right] \\
\Leftrightarrow\quad V^u_{\text{stw}}(\epsilon) &= \frac{1}{1-\beta\cdot(1-\lambda)}\left(u(c^w) + \lambda\beta\bar{V}\right)
\end{aligned}$$

(IX) Value of a constrained worker outside STW with non-binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^c(\epsilon) &= u(c^w) + \beta \cdot \left[\lambda \, \bar{V} + (1-\lambda) \, V^c(\epsilon)\right] \\ \Leftrightarrow \quad V^c(\epsilon) &= \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w) + \lambda\beta\bar{V}\right) \end{aligned}$$

(X) Value of a constrained worker outside STW with binding constraints after the idiosyncratic productivity shock has been realized:

$$\begin{aligned} V^c(\epsilon) &= u(c^w(\epsilon)) + \beta \cdot \left[\lambda \, \bar{V} + (1-\lambda) \, V^c(\epsilon)\right] \\ \Leftrightarrow \quad V^c(\epsilon) &= \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c^w(\epsilon)) + \lambda\beta\bar{V}\right) \end{aligned}$$

(XI) Value of a constrained worker on STW after the idiosyncratic productivity shock has been realized:

$$\begin{aligned} V^c_{\text{stw}}(\epsilon) &= u(c_{\text{stw}}(\epsilon)) + \beta \cdot \left[\lambda \, \bar{V} + (1-\lambda) \, V^c_{\text{stw}}(\epsilon)\right] \\ \Leftrightarrow \quad V^c_{\text{stw}}(\epsilon) &= \frac{1}{1-\beta \cdot (1-\lambda)} \left(u(c_{\text{stw}}(\epsilon)) + \lambda\beta\bar{V}\right) \end{aligned}$$

(XII) Value of a worker before the idiosyncratic productivity shock has been realized:

$$\begin{aligned} \bar{V} = (1-p) \cdot \Bigg(& \int_{\epsilon_{\text{stw}}}^{\infty} V^u(\epsilon) \, dG(\epsilon) \\ & + \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} V^u_{\text{stw}}(\epsilon) \, dG(\epsilon) \Bigg) \\ & + p \cdot \Bigg(\int_{\epsilon_p}^{\infty} V^c(\epsilon) \, dG(\epsilon) \\ & + \int_{\max\{\xi_s^c, \epsilon_{\text{stw}}\}}^{\epsilon_p} V^c(\epsilon) \, dG(\epsilon) \\ & + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} V^c_{\text{stw}}(\epsilon) \, dG(\epsilon) \Bigg) \\ & + \rho \cdot U \end{aligned}$$

Inserting (VII) - (XI) into (XII) gives:

$$\bar{V} = \frac{1}{1 - \beta \cdot (1 - \lambda)} \Big[(1-p)(1-\rho^u) \cdot u(c^w) + p \cdot (1 - \rho^c) \cdot u^c \\ + (1 - \rho) \cdot \lambda \cdot \beta \cdot \bar{V} + \rho \cdot U \Big]$$

Here, u^c denotes the average utility of a constrained worker for a firm. Both are defined in Appendix I.

(XIII) Unemployment:

$$U = u(b) + \beta \cdot \left[f \cdot \bar{V} + (1 - f) \cdot U \right]$$

Next, we want to calculate the joint surplus of firms and workers.

(A) The surplus of the worker after the idiosyncratic productivity shock has been realized is:

$$\begin{aligned}
& V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot V_i^j(\epsilon) + (\lambda - f) \cdot \beta \cdot \bar{V} - (\lambda - f) \cdot \beta \cdot U \\
\Leftrightarrow \quad & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (V_i^j(\epsilon) - U) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \\
\Leftrightarrow \quad & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot (V_i^j(\epsilon) - U) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \\
\Leftrightarrow \quad & V_i^j(\epsilon) - U = \frac{1}{1 - \beta \cdot (1 - \lambda)} \left(u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot \beta \cdot (\bar{V} - U) \right)
\end{aligned}$$

for $i \in \{\text{stw}, \text{no stw}\}$ and $j \in \{u, c\}$.

(B) The surplus of the worker before shock realizations are known:

$$\begin{aligned}
\bar{V} - U = (1-p) \cdot & \left(\int_{\epsilon_{\text{stw}}}^{\infty} V^u(\epsilon) - U \, dG(\epsilon) + \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} V_{\text{stw}}^u(\epsilon) - U \, dG(\epsilon) \right) \\
& + p \cdot \left(\int_{\epsilon_p}^{\infty} V^c(\epsilon) - U \, dG(\epsilon) + \int_{\max\{\xi_s^c, \epsilon_{\text{stw}}\}}^{\epsilon_p} V^c(\epsilon) - U \, dG(\epsilon) \right. \\
& \left. + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} V_{\text{stw}}^c(\epsilon) - U \, dG(\epsilon) \right)
\end{aligned}$$

Inserting A into B gives:

$$
\begin{aligned}
\bar{V} - U = \frac{1}{1-\beta\cdot(1-\lambda)} \Big[&(1-p)(1-\rho^u)\cdot(u(c^w)-u(b)) \\
&+ p\cdot(1-\rho^c)\cdot(u^c - u(b)) \\
&+ (\lambda - f)\cdot\beta\cdot(\bar{V}-U) \Big]
\end{aligned}
$$

(C) Finally, we can calculate the joint surplus before shock realizations are known:

$$
\begin{aligned}
S &= J + \frac{\bar{V}-U}{u'(c^w)} \\
&= -\tau + \frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[(1-\rho)(1-\rho^u)\cdot\Bigg(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w \\
&\quad + \frac{1}{1-\rho^u}\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{stw}\cdot(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon)\Bigg) \\
&\quad + p(1-\rho^c)\cdot\Bigg(\frac{u^c-u(b)}{u'(c^w)} - e^c \\
&\quad + \frac{1}{1-\rho^c}\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon)\Bigg) \\
&\quad + (1-\rho)\lambda\beta J + (1-\rho)(\lambda - f)\cdot\beta\cdot\frac{\bar{V}-U}{u'(c^w)}\Bigg]
\end{aligned}
$$

Next, we start deriving the job-creation condition. Inserting the surplus splitting rule

$$
J = \eta\cdot S, \quad \frac{\bar{V}-U}{u'(c^w)} = (1-\eta)\cdot S
$$

into the equation for the joint surplus gives:

$$
\begin{aligned}
S = J + \frac{\bar{V} - U}{u'(c^w)} &= \eta S + (1-\eta) S \\
&= -\tau + \frac{1}{1-\beta\cdot(1-\lambda)} \Bigg[(1-\rho)\left(z - \Omega - \frac{1-n}{n} b\right) \\
&\quad + (1-p)(1-\rho^u)\left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w\right) \\
&\quad + (1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{stw}\cdot(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
&\quad + p(1-\rho^c)\left(\frac{u^c - u(b)}{u'(c^w)} - e^c\right) \\
&\quad + p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg] \\
\Leftrightarrow \quad S &= -\tau + \frac{1}{1-\beta\cdot(1-\lambda)} \Bigg[(1-\rho)\left(z - \Omega - \frac{1-n}{n} b\right) \\
&\quad + (1-p)(1-\rho^u)\left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w\right) \\
&\quad + (1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
&\quad + p(1-\rho^c)\left(\frac{u^c - u(b)}{u'(c^w)} - e^c\right) \\
&\quad + p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
&\quad + (1-\rho)(\lambda - \eta\cdot f)\cdot\beta\cdot S \Bigg]
\end{aligned}
$$

Next, we want to replace the tax. To do this, we need to know the budget constraint of the government:

$$
\begin{aligned}
n^s \cdot \tau = \frac{n}{1-\rho} \cdot \Bigg(& (1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + (1-n)\cdot b
\end{aligned}
$$

Inserting the employment equation

$$n = (1-\rho) \cdot \frac{n^s}{\lambda}$$

gives:

$$\begin{aligned}
\tau = \frac{1}{\lambda} \cdot \Bigg(& (1-p) \int_{\epsilon_s^u}^{\infty} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + \frac{(1-\rho)}{n^s} \cdot b
\end{aligned}$$

Inserting the expression for the number of firms that receive a shock

$$n^s = \frac{n}{1-\rho} \cdot \lambda$$

gives:

$$\begin{aligned}
\tau = \frac{1}{\lambda} \cdot \Bigg(& (1-p) \int_{\epsilon_s^u}^{\infty} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + \frac{1}{\lambda} \cdot (1-\rho) \cdot \left(\frac{1-n}{n} \cdot b \right)
\end{aligned}$$

Inserting the tax into the surplus equation gives:

$$
\begin{aligned}
S = \frac{1}{1-\beta\cdot(1-\lambda)} \Bigg[& (1-\rho)\,(z-\Omega) \\
& + (1-p)(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) \\
& + p(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)} - e^c\right) \\
& + (1-\rho)(\lambda-f)\cdot\beta\cdot S \Bigg] \\
& - \left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} (\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
& - \left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
& - \frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b
\end{aligned}
$$

Inserting the free-entry condition

$$\frac{k_v}{q} = \beta\cdot J$$

gives:

$$
\begin{aligned}
\frac{1}{1-\eta}\cdot\frac{k_v}{q} = \frac{\beta}{1-\beta\cdot(1-\lambda)} \Bigg[& (1-\rho)\left(z-\Omega-\frac{1-n}{n}\cdot b\right) \\
& + (1-p)(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) \\
& + p(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)} - e^c\right) \\
& + (1-\rho)\cdot\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q} \Bigg]
\end{aligned}
$$

This is the job-creation condition used in the Ramsey problem for STW in Appendix I.

Next, we want to calculate the wage-equation. Remember the surplus splitting rule from Nash bargaining:

$$\eta\cdot\bar{J} = (1-\eta)\cdot\frac{\bar{V}-U}{u'(c^w)}$$

Inserting the tax into the value of a worker for a firm gives:

$$\begin{aligned}
\bar{J} = &\frac{1}{1-\beta\cdot(1-\lambda)}\Big[(1-\rho)(z-\Omega) - p(1-\rho^u)c^w - (1-p)(1-\rho^c)e^c + (1-\rho)\lambda\beta\bar{J}\Big] \\
&-\left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
&-\left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
&-\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b
\end{aligned}$$

Likewise, we can calculate the surplus of a worker:

$$\begin{aligned}
\bar{V} - U = &\frac{1}{1-\beta\cdot(1-\lambda)}\Bigg[(1-p)(1-\rho^u)\left(u(c^w) - u(b)\right) + p(1-\rho^c)\left(u^c - u(b)\right) \\
&+ (1-\rho)(\lambda - f)\beta(\bar{V} - U)\Bigg]
\end{aligned}$$

Next, we can insert the value of a worker for a firm and the surplus of a worker into the surplus splitting rule:

$$\begin{aligned}
\frac{\eta}{1-\beta\cdot(1-\lambda)}\cdot\Big[&(1-\rho)(z-\Omega) - p(1-\rho^u)\cdot c^w - (1-p)(1-\rho^c)\cdot e^c \\
&+ (1-\rho)\cdot\lambda\cdot\beta\cdot\bar{J}\Big] \\
&-\left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
&-\left(\frac{1}{\lambda} - \frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
&-\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
= &\frac{1-\eta}{1-\beta\cdot(1-\lambda)}\cdot\frac{1}{u'(c^w)}\cdot\Big[(1-p)(1-\rho^u)\cdot(u(c^w) - u(b)) \\
&+ p(1-\rho^c)\cdot(u^c - u(b)) \\
&+ (1-\rho)(\lambda - f)\beta(\bar{V} - U)\Big]
\end{aligned}$$

Insert the surplus splitting rule again: $\eta\cdot\bar{J} = (1-\eta)\cdot\frac{\bar{V}-U}{u'(c^w)}$

$$
\begin{aligned}
\frac{\eta}{1-\beta\cdot(1-\lambda)}\cdot\Big[\quad & (1-\rho)(z-\Omega)-p(1-\rho^u)\cdot c^w-(1-p)(1-\rho^c)\cdot e^c \\
& +(1-\rho)\cdot\lambda\cdot\beta\cdot\bar{J}\Big] \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\mathrm{stw}}} s_{stw}(\bar{h}-h_{\mathrm{stw}}(\epsilon))\,dG(\epsilon) \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\mathrm{stw}},\epsilon_s^c\}} s_{stw}(\bar{h}-h_{\mathrm{stw}}(\epsilon))\,dG(\epsilon) \\
& -\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
= {} & \frac{1-\eta}{1-\beta\cdot(1-\lambda)}\cdot\frac{1}{u'(c^w)}\cdot\Big[(1-p)(1-\rho^u)\cdot(u(c^w)-u(b)) \\
& +p(1-\rho^c)\cdot(u^c-u(b)) \\
& +(1-\rho)(\lambda-f)\cdot u'(c^w)\cdot\frac{\eta}{1-\eta}\cdot\beta\cdot\bar{J}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad \frac{\eta}{1-\beta\cdot(1-\lambda)}\Big[& (1-\rho)(z-\Omega)-p(1-\rho^u)\cdot c^w-(1-p)(1-\rho^c)\cdot e^c \\
& +(1-\rho)\cdot\lambda\cdot f\cdot\beta\cdot\bar{J}\Big] \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\mathrm{stw}}} s_{stw}(\bar{h}-h_{\mathrm{stw}}(\epsilon))\,dG(\epsilon) \\
& -\left(\frac{1}{\lambda}-\frac{1}{1-\beta\cdot(1-\lambda)}\right)\cdot p\int_{\epsilon_s^c}^{\max\{\epsilon_{\mathrm{stw}},\epsilon_s^c\}} s_{stw}(\bar{h}-h_{\mathrm{stw}}(\epsilon))\,dG(\epsilon) \\
& -\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
= {} & \frac{1-\eta}{1-\beta\cdot(1-\lambda)}\cdot\frac{1}{u'(c^w)}\Big[(1-p)(1-\rho^u)(u(c^w)-u(b)) \\
& +p(1-\rho^c)(u^c-u(b))\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow \quad & \eta \cdot \Big[(1-\rho)(z-\Omega) + (1-\rho)\cdot f \cdot \beta \cdot \bar{J}\Big] \\
& \quad - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\mathrm{stw}}} s_{stw}(\bar{h} - h_{\mathrm{stw}}(\epsilon))\, dG(\epsilon) \\
& \quad - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right)\cdot p \int_{\epsilon_s^c}^{\max\{\epsilon_{\mathrm{stw}},\epsilon_s^c\}} s_{stw}(\bar{h} - h_{\mathrm{stw}}(\epsilon))\, dG(\epsilon) \\
& \quad - \frac{1-\beta\cdot(1-\lambda)}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
& = (1-p)(1-\rho^u)(1-\eta)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)} + \eta\cdot c^w\right) \\
& \quad + p(1-\rho^c)(1-\eta)\cdot\left(\frac{u^c - u(b)}{u'(c^w)} + \eta\cdot e^c\right)
\end{aligned}
$$

Inserting the free-entry condition again, we get the wage equation from Appendix I for STW.

$$
\begin{aligned}
& (1-p)(1-\rho^u)(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)} + \eta\cdot c^w \\
& \quad - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right)\cdot(1-p)\int_{\epsilon_s^u}^{\epsilon_{\mathrm{stw}}} s_{stw}(\bar{h} - h_{\mathrm{stw}}(\epsilon))\, dG(\epsilon) \\
& \quad - \left(\frac{1-\beta\cdot(1-\lambda)}{\lambda} - 1\right)\cdot p \int_{\epsilon_s^c}^{\max\{\epsilon_{\mathrm{stw}},\epsilon_s^c\}} s_{stw}(\bar{h} - h_{\mathrm{stw}}(\epsilon))\, dG(\epsilon) \\
& \quad - \frac{1-\beta\cdot(1-\lambda)}{\lambda}\cdot(1-\rho)\cdot\frac{1-n}{n}\cdot b \\
& = \eta\cdot\Big[(1-\rho)(z-\Omega) - (1-\rho)\cdot\frac{1-n}{n}\cdot b + (1-\rho)\cdot\theta\cdot k_v\Big] \\
& \quad - p(1-\rho^c)(1-\eta)\cdot\left(\frac{u^c - u(b)}{u'(c^w)} + \eta\cdot e^c\right)
\end{aligned}
$$

I The Full Ramsey Problems

I.1 Layoff Tax

For $\beta \to 1$, the Ramsey planner's problem reads:

$$
\begin{aligned}
\max_{b, s_{stw}, \epsilon_{\mathrm{stw}}} \; & n^u \cdot u(c^w) + n^c \cdot u^c + (1-n)\cdot u(b) \\
& + v^f \cdot u((n\cdot z - n^u\cdot c^w - n^c\cdot e^c - \tau(b) - \theta\cdot(1-n)\cdot k_v)/v^f)
\end{aligned}
$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$n^u = \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s$$

(II) Separation rate, unconstrained workers:

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1-\rho^c) \cdot n^s$$

(IV) Separation rate, constrained workers:

$$\rho^c = G(\epsilon_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1-n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of constrained worker:

$$u^c = \frac{1}{1-\rho^c} \left((1 - G(\epsilon^p)) \cdot u(c^w) + \int_{\epsilon_s^c}^{\epsilon^p} u(c(\epsilon))\, dG(\epsilon) \right)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1-\rho^c} \left[(1 - G(\epsilon^p)) \cdot c^w + \int_{\epsilon_s^c}^{\epsilon^p} c^w(\epsilon)\, dG(\epsilon) \right]$$

(X) Average production (without distortions):

$$z = \frac{1}{1-\rho}\left((1-p)\int_{\epsilon_s^u}^{\infty} z(\epsilon)\,dG(\epsilon) + p\int_{\epsilon_s^c}^{\infty} z(\epsilon)\,dG(\epsilon)\right)$$

(XI) Job-creation condition:

$$\begin{aligned}\frac{1}{1-\eta}\cdot\frac{k_v}{q} = \frac{1}{\lambda}\Bigg[&(1-\rho)\cdot\left(z-\frac{1-n}{n}\cdot b\right)\\ &+(1-p)(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)\\ &+(1-p)(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right)\\ &+(1-\rho)\cdot\frac{\lambda-f\cdot\eta}{1-\eta}\cdot\frac{k_v}{q}\Bigg]\end{aligned}$$

(XII) Wage:

$$\begin{aligned}WE =&(1-p)\cdot(1-\rho^u)\cdot\left(\eta\cdot c^w+(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right)\\ &-\eta\cdot(1-\rho)\cdot\left(z-\frac{1-n}{n}\cdot b+\theta\cdot\frac{k_v}{q}\right)\\ &+p\cdot(1-\rho^c)\cdot\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}+\eta\cdot e^c\right]=0\end{aligned}$$

(XIII) Separation condition unconstrained firm:

$$z(\epsilon_s^u)+\lambda\cdot F+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w+\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XIV) Threshold at which the constraint is binding:

$$z(\epsilon^p)+\lambda\cdot\frac{k_v}{q}=c^w$$

(XV) Separation condition constrained firm:

$$\frac{u(c^w(\epsilon_s^c))-u(b)}{u'(c^w)}+(\lambda-f)\cdot\frac{\eta}{1-\eta}\cdot\frac{k_v}{q}=0$$

(XVI) Consumption of a constrained worker:

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \frac{k_v}{q}$$

I.2 Short-Time Work

For $\beta \to 1$, the Ramsey planner's problem reads:

$$\begin{aligned}\max_{b, s_{stw}, \epsilon_{\text{stw}}} \; & n^u \cdot u(c^w) + n^c \cdot u^c + (1-n) \cdot u(b) \\ & + v^f \cdot u((n \cdot (z - \Omega) - n^u \cdot c^w - n^c \cdot e^c - \tau(b) - \theta \cdot (1-n) \cdot k_v)/v^f)\end{aligned}$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$n^u = \frac{1-p}{\lambda} \cdot (1 - \rho^u) \cdot n^s$$

(II) Separation rate (unconstrained workers):

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

(IV) Separation rate (constrained workers):

$$\rho^c = G\left(\max\{\xi_s^c, \epsilon_{\text{stw}}\}\right) - G\left(\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}\right) + G(\xi_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1-n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of a constrained worker:

$$u^c = \frac{1}{1-\rho^c} \Bigg((1 - G(\epsilon^p)) \cdot u(c^w) + \int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon_p} u(c^w(\epsilon)) \, dG(\epsilon)$$
$$+ \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} u(c^w_{\text{stw}}(\epsilon)) dG(\epsilon) \Bigg)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1-\rho^c} \Bigg[(1 - G(\epsilon^p)) \cdot c^w + \int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon_p} c^w(\epsilon) \, dG(\epsilon)$$
$$+ \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \big(c^w_{\text{stw}}(\epsilon) - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon)) \big) \, dG(\epsilon) \Bigg]$$

(X) Average production (without distortions):

$$z = \frac{1}{1-\rho} \Bigg((1-p) \int_{\xi_s^u}^{\infty} z(\epsilon) \, dG(\epsilon) + p \int_{\max\{\xi_s^c, \epsilon_{\text{stw}}\}}^{\infty} z(\epsilon) \, dG(\epsilon)$$
$$+ p \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} z(\epsilon) \, dG(\epsilon) \Bigg)$$

(XI) Average distortion of working hours:

$$\Omega = \frac{1}{1-\rho} \left((1-p) \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} \Omega(\epsilon) \, dG(\epsilon) + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \Omega(\epsilon) \, dG(\epsilon) \right)$$
$$\text{with} \quad \Omega(\epsilon) = \bar{z}(\epsilon) - z_{\text{stw}}(\epsilon)$$

(XII) Job-creation condition:

$$\frac{1}{1-\eta} \cdot \frac{k_v}{q} = \frac{1}{\lambda} \Bigg[(1-\rho)(z - \Omega - \frac{1-n}{n} b) \\ + p(1-\rho^u) \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\ + (1-p)(1-\rho^c) \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\ + (1-\rho) \cdot \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} \Bigg]$$

(XIII) Wage:

$$WE = (1-p) \cdot (1-\rho^u) \left(\eta \cdot c^w + (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \right) \\ - \eta(1-\rho) \left(z - \Omega - \frac{1-n}{n} b + \theta \cdot k_v \right) \\ + p \cdot (1-\rho^c) \left[(1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} + \eta \cdot e^c \right] = 0$$

(XIV) Separation condition for an unconstrained firm without STW:

$$z(\xi_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XV) Separation condition for an unconstrained firm with STW:

$$z_{\text{stw}}(\epsilon_s^u) + s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XVI) Threshold at which the constraint is binding:

$$z(\epsilon_p) + \lambda \cdot \frac{k_v}{q} = c^w$$

(XVII) Separation condition, constrained firm without STW:

$$\frac{u(c^w(\xi_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XVIII) Separation condition, constrained firm with STW:

$$\frac{u(c^w_{\text{stw}}(\epsilon^c_s)))}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XIX) Consumption, constrained worker outside STW

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \frac{k_v}{q}$$

(XX) Consumption, constrained worker on STW

$$c^w_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) + s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \frac{k_v}{q}$$

J Ramsey FOCs with Layoff Tax

In the following, multipliers from the Lagrangian, implied by the maximization problem from the previous section, are denoted by λ_{idx}, the index depending on the constraint. Here, λ_n denotes the Lagrange multiplier for the total employment equation, λ_{n^u} for the number of unconstrained firms, λ_{n^c} for the number of constrained firms, λ_{n^s} for the number of firms that received a shock, λ_θ for the job-creation condition, λ_c for the wage equation, $\lambda_{\epsilon^u_s}$ for the separation condition of unconstrained firms and $\lambda_{\epsilon^c_s}$ for the separation condition of constrained firms. Every other equation listed in the Ramsey problem for the layoff tax is assumed to be plugged in.

J.1 Employment (Layoff Tax)

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n^s} &= -\lambda_{n^s} + \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot (u(c^w) - u'(c^w) \cdot c^u) \\ &\quad + \frac{p}{\lambda} \cdot (1-\rho^c) \cdot (u(c^w) - u'(c^w) \cdot e^c) \\ &\quad + \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \lambda_{n^u} + \frac{p}{\lambda} \cdot (1-\rho^c) \cdot \lambda_{n^c} = 0\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial n^u} = -\lambda_{n^u} + \lambda_n = 0$$

$$\frac{\partial \mathcal{L}}{\partial n^c} = -\lambda_{n^c} + \lambda_n = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n} &= -\lambda_n + \lambda_{n^s} \cdot (\lambda - q(\theta) \cdot \theta) + u'(c^w) \cdot [z + b + \theta \cdot c] - u(b) \\ &\quad + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \lambda_\theta + \eta(1-\rho) \cdot \frac{b}{n^2} \cdot \lambda_c = 0\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad \frac{\lambda_n}{u'(c^w)} &= (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + [z + b + \theta \cdot c] - \frac{u(b)}{u'(c^u)} \\ &\quad + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \frac{\lambda_\theta}{u'(c^w)} + \frac{\lambda \cdot b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)}\end{aligned}$$

J.2 Optimal Job-Creation Condition (Layoff Tax)

Before we begin, let us define the insurance effect as:

$$\begin{aligned}\tilde{IE}_\theta &= \frac{p}{\lambda} \left(\int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right) \cdot k_v \\ IE_\theta &= n^s \cdot u'(c^w) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \tilde{IE}_\theta\end{aligned}$$

The FOC for labor market tightness denotes:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \theta} &= -k_v(1-n)u'(c^w) + IE_\theta \cdot k_v + (1-\gamma)q(\theta)(1-n)\lambda_{n^s} \\ &\quad - \frac{\gamma}{1-\eta}k_v\lambda_\theta + \frac{\lambda\gamma - f\eta}{(1-\eta)f}\left(\frac{1-\rho}{\lambda}\lambda_\theta - \lambda_{\epsilon_s^u}\right) \\ &\quad - \left(\lambda \cdot \frac{\gamma}{f}u'(c(\epsilon_s^u)) + \frac{\lambda\gamma - f}{(1-\eta)f}\eta\right)\lambda_{\epsilon_s^c} \\ &\quad - \lambda_c \cdot \frac{\partial WE}{\partial \theta}\end{aligned}$$

$$\Leftrightarrow \qquad \frac{\lambda_{n^s}}{u'(c^w)} = \frac{1+\chi}{1-\eta} \cdot \frac{k_v}{q}$$

with

$$\begin{aligned}\chi = \frac{1}{u \cdot u'(c^w)} \cdot &\left[\frac{1}{f} \cdot \frac{1}{1-\eta} \cdot \left(\gamma\lambda_\theta - (\lambda\gamma - f\eta)\left(\frac{1-\rho}{\lambda}\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right)\right. \\ &+ \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w(\epsilon_s^u)) + \frac{\lambda\gamma - f}{(1-\eta)f} \cdot \eta\right)\lambda_{\epsilon_s^c} \\ &\left.+ \frac{1}{k_v} \cdot \left(\lambda_c \cdot \frac{\partial WE}{\partial \theta} - IE_\theta\right)\right]\end{aligned}$$

Using the FOCs for employment and labor market tightness gives:

$$\begin{aligned}u'(c^w) \cdot \frac{1+\chi}{1-\gamma} \cdot \frac{k_v}{q} &= \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot (u(c^w) - u(b) - u'(c^w) \cdot c^w) \\ &\quad + \frac{p}{\lambda} \cdot (1-\rho^c) \cdot (u^c - u(b) - u'(c^w) \cdot e^c) \\ &\quad + u'(c^w) \cdot \frac{1-\rho}{\lambda} \cdot \left(z + b + \frac{\lambda - \gamma f + \chi(\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q}\right) \\ &\quad + \frac{1-\rho}{\lambda} \cdot \frac{\lambda_\theta}{n^s} \cdot \frac{b}{n} + \frac{1-\rho}{n^s} \cdot \frac{b}{n} \cdot \lambda_c\end{aligned}$$

Rearranging gives the *optimal job-creation condition:*

$$\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q}=\frac{1-\rho}{\lambda}\cdot(z+b)+\frac{1-\rho}{\lambda}\cdot\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\cdot\frac{b}{n}+\frac{1-\rho}{\lambda}\cdot\frac{\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\cdot\frac{b}{n}$$
$$+\frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)$$
$$+\frac{p}{\lambda}\cdot(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right)$$
$$+\frac{1-\rho}{\lambda}\cdot\frac{\lambda-\gamma f+\chi\cdot(\lambda-f)}{1-\gamma}\cdot\frac{k_v}{q}$$

Subtracting the decentralized job-creation condition from the optimal gives:

$$\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}=\left(1+\frac{\lambda_\theta+\eta+\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{1-\rho}{\lambda}\cdot\frac{b}{n}$$
$$+\frac{(1-\rho)\cdot(\lambda-f)}{\lambda}\cdot\left(\chi-\frac{\eta-\gamma}{1-\gamma}\right)\cdot\frac{k_v}{q}$$

Rearranging gives:

$$\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}=\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{\frac{1-\rho}{\lambda}}{\rho+(1-\rho)\cdot\frac{f}{\lambda}}\cdot\frac{b}{n}$$
$$=\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}$$

J.3 Optimal Separation Condition (Unconstrained Firms)

FOC for the separation threshold of unconstrained firms:

$$-\frac{\partial\mathcal{L}}{\partial\epsilon_s^u}=\frac{n}{1-\rho}\cdot(1-p)\cdot g(\epsilon_s^u)\cdot u'(c^w)\cdot\left(z(\epsilon_s^u)+\frac{u(c^w)}{u'(c^w)}-c^w-z\right)$$
$$+\lambda_\theta\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left[z(\epsilon_s^u)+\frac{u(c^w)-u(b)}{u'(c^w)}-c^w-\frac{1-n}{n}\cdot b+\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}\right]$$
$$+\lambda_c\cdot\frac{\partial WE}{\partial\epsilon_s^u}+\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot\lambda_{n^u}+\lambda_{\epsilon_s^u}\cdot\frac{\partial S_{lt}^u(\epsilon_s^u)}{\partial\epsilon_s^u}=0$$

Insert for λ_{n^u} and subtract decentralized separation condition of unconstrained firms:

$$\begin{aligned}
g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \cdot \left[z(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w + \frac{\lambda_n}{u'(c^w)} \right] \\
- \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[\lambda F + \frac{1-n}{n} \cdot b \right] \\
+ \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S_{\mathrm{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}$$

Insert for λ_n:

$$\begin{aligned}
& g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \left[z(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w - z \right] \\
& + g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \left[(\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + (z + b + \theta \cdot c) - \frac{u(b)}{u'(c^w)} \right. \\
& \qquad \left. + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \frac{\lambda_\theta}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} \right] \\
& + \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \left[\lambda F + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S_{\mathrm{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}$$

$$\begin{aligned}
\iff \quad & z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(\lambda F + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\mathrm{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}$$

Inserting λ_n^s gives the *optimal Separation condition*:

$$\begin{aligned}
& z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1 - \gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot (\lambda \cdot F - b) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) = 0
\end{aligned}$$

Subtracting the decentralized separation condition and rearranging gives:

$$\begin{aligned}
\lambda \cdot F = b &+ (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\gamma}\right) \cdot \frac{k_v}{q} \\
&+ \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u} \\
&- \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot (\lambda F - b) \\
&+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)
\end{aligned}$$

$$\begin{aligned}
\iff 0 = &\left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s}\right) \cdot \left(b + (\lambda - f) \cdot \frac{\frac{1-\rho}{\lambda}}{\rho + (1-\rho) \cdot \frac{f}{\lambda}} \cdot \frac{b}{n} - \lambda \cdot F\right) \\
&+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right) \\
&+ \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u}
\end{aligned}$$

$$\begin{aligned}
\iff \quad 0 = &\left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s}\right) \cdot \left(b + (\lambda - f) \cdot \frac{b}{f} - \lambda F\right) \\
&+ \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u} \\
&+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)
\end{aligned}$$

$$\begin{aligned}
\iff \quad 0 = &\left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda F\right) \\
&+ \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u} \\
&+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)
\end{aligned}$$

Rearranging gives the Lagrange multiplier for unconstrained firms:

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{1-p}{\lambda} \cdot \frac{g(\epsilon_s^u)}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda F\right) \\
+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right) \Bigg)
\end{aligned}
$$

J.4 Optimal Separation Condition (Constrained Firms)

The FOC for the separation threshold in the constrained firm is:

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = & \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \cdot u'(c^w) \left(z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c)) - u(c^w)}{u'(c^w)} - c^w(\epsilon_s^c) - z \right) \\
& + \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left[z(\epsilon_s^c) + \frac{c^w(\epsilon_s^c) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
& + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \lambda_{n^c} \\
& + \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^c} \\
& + \lambda_{\epsilon_s^c} \cdot \frac{\partial S^c_{\text{lt}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Note that we can reformulate the decentralized separation threshold of a worker in a constrained firm as:

$$
\begin{aligned}
& \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0 \\
\iff \quad & \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = c^w(\epsilon_s^c) - z(\epsilon_s^c) - \lambda \cdot \frac{k_v}{q} \\
\iff \quad & z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0
\end{aligned}
$$

This simplifies the equation to:

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = & \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \cdot \left(z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c))}{u'(c^w)} - c^w(\epsilon_s^c) - z \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \frac{\lambda_{n^c}}{u'(c^w)} \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{1-n}{n} \cdot b \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} + \frac{\lambda_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c(\epsilon_s^c)}{\partial \epsilon_s^c} \overset{!}{=} 0
\end{aligned}
$$

Inserting λ_n^s gives the *optimal separation condition*

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c))}{u'(c^w)} - c^w(\epsilon_s^c) + b + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
+ \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot b \\
+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \\
+ \frac{\lambda}{p} \cdot \frac{1}{n^s \cdot g(\epsilon_s^c)} \cdot \frac{\lambda_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Subtracting the decentralized separation condition gives:

$$
\begin{aligned}
0 = b + (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\eta} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \\
+ \frac{\lambda \cdot \epsilon_s^c}{n^s \cdot u'(c^w)} \cdot \frac{1}{\rho} \cdot \frac{1}{\lambda} \cdot \frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} \cdot \frac{1}{g(\epsilon_s^c)} \\
+ \frac{\lambda \cdot \theta}{n^s \cdot u'(c^w)} \cdot b + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right)
\end{aligned}
$$

Rearranging gives back the Lagrange multiplier:

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \frac{\lambda}{f} \cdot b \\
+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg)
\end{aligned}
$$

J.5 The Optimal Layoff Tax

We start from

$$
\frac{\partial \mathcal{L}}{\partial F} = -\lambda_{\epsilon_s^u} = 0
$$

Note that the optimal layoff tax sets the Lagrange multiplier for the separation condition of unconstrained firms equal to zero. This implies that the layoff tax implements the optimal separation threshold for unconstrained firms. Then

$$-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^u)}{\frac{\partial S_{\mathrm{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F \right) \right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \right)$$

This leads to:

$$F = \frac{1}{f} \cdot b + BE_{lt}$$
$$BE_{lt} = \frac{1}{\lambda} \frac{1}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} \cdot \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)$$

Since the optimal layoff tax sets the Lagrange multiplier of the separation condition for unconstrained firms equal to zero, we can infer that the layoff tax implements the optimal separation rate in the economy when the number of financially constrained firms is equal to zero, $p = 0$.

J.6 Optimal UI given Layoff Tax

From the FOC of UI, we can derive the optimality condition for unemployment insurance. With an optimal layoff tax, the Lagrange multiplier for the separation condition of the unconstrained firm is equal to zero.

$$(1-\eta) \cdot (u'(b) - u'(c^w)) = \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)} \right)$$
$$+ \left(\lambda_{\epsilon_s^c} + \lambda_{\xi_s^c} \right) \cdot \left(-\frac{u'(b)}{u'(c^w)} \right)$$
$$+ \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} - (1-\eta) \cdot \frac{u'(b)}{u'(c^w)} \right)$$

To get better insight, we need to determine the Lagrange multipliers for λ_θ, λ_c. We already know the Lagrange multipliers for the separation conditions of constrained firms:

$$-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right)$$
$$+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg]$$

$$-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{1}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b$$
$$+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg]$$

First, let us find an expression for λ_c:

$$\frac{\partial \mathcal{L}}{\partial c^w} = -\Bigg((1-p) \cdot (1-\rho^u) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)}$$
$$+ p \cdot (1-\rho^c) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)}\Bigg) \cdot \lambda_\theta$$
$$- (1-\eta) \cdot \Bigg((1-p) \cdot (1-\rho^u) \cdot \left(1 - (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)}\right)$$
$$- p \cdot (1-\rho^c) \cdot (1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)}\Bigg) \cdot \lambda_c$$
$$+ \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_{\epsilon_s^u}$$
$$+ \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_{\epsilon_s^c} = 0$$

Next, we can solve for the Lagrange multiplier of the wage equation:

$$\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right)$$
$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b$$
$$+ \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta\Bigg]$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right)$$
$$+ \lambda_c \cdot \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg)$$

$$\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b$$
$$+ \lambda_c \cdot \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg)$$

Inserting λ_c gives:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right)$$
$$+ \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right)$$
$$\cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right)$$
$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg]\Bigg)$$

$$\lambda_{\epsilon_s^c} = \frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b$$
$$+ \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right)$$
$$\cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right)$$
$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg]\Bigg)$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(- n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + \lambda_c \cdot \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg)$$

$$\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b + \lambda_c \cdot \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg)$$

Inserting λ_c gives:

$$\begin{aligned}\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(& n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(\lambda + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ & + \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right) \\ & \cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ & + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta\Bigg]\Bigg)\end{aligned}$$

$$\begin{aligned}\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(& n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \\ & + \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right) \\ & \cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ & + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta\Bigg]\Bigg)\end{aligned}$$

Now we have everything to calculate λ_θ from the two equations:

$$(1)\quad \left(\chi - \frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}$$

$$\begin{aligned}(2)\quad \chi = &\frac{1}{1-\eta}\cdot\frac{1}{u\cdot f}\cdot\frac{1}{u'(c^w)}\cdot\left[\gamma-(\lambda\gamma-f\eta)\cdot\left(\frac{1}{\lambda}(1-\rho)\lambda_\theta-\lambda_{\epsilon_s^u}\right)\right]\\ &+\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot\lambda_{\epsilon_s^c}\\ &+\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\lambda_c\cdot\frac{\partial WE}{\partial\theta}\cdot\frac{1}{k_v}\\ &-\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot IE_\theta\end{aligned}$$

We can rearrange (1) to:

$$\chi\cdot k_v = (1-\gamma)\cdot q\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)$$

Note that

$$(1-\gamma)\cdot q = \frac{\partial f}{\partial\theta}$$

This follows from the fact that:

$$f'(\theta) = q(\theta)+\theta\cdot q'(\theta) = q(\theta)\cdot\left(1+\frac{q'(\theta)\cdot\theta}{q(\theta)}\right) = q(\theta)\cdot(1-\gamma)$$

So we can write:

$$\chi\cdot k_v = \left(\frac{\partial f}{\partial\theta}\right)\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)$$

Inserting χ gives:

$$\begin{aligned}&\left[\gamma-(\lambda\gamma-f\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta\\ &= u'(c^w)\cdot\left(\frac{\partial f}{\partial\theta}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)\\ &\quad+(\lambda\gamma-f\eta)\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot(-\lambda_{\epsilon_s^u})\\ &\quad+\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot k_v\cdot(-\lambda_{\epsilon_s^c})\end{aligned}$$

$$+\frac{\partial WE}{\partial \theta}\cdot(-\lambda_c) \quad + IE_\theta$$

Inserting $\lambda_{\epsilon_s^u}, \lambda_{\epsilon_s^c}, IE_\theta$ gives

$$\begin{aligned}
&\left[\gamma-(\lambda\gamma-f\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta\\
&=u'(c^w)\cdot\frac{\partial f}{\partial\theta}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{b}{f}\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^u)\cdot\frac{1-p}{\lambda}\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda F\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c)\cdot\frac{p}{\lambda}\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\\
&\quad+\lambda_\theta\cdot\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total},2}\cdot\left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}}\\
&\quad+u'(c^w)\cdot n^s\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\tilde{IE}_\theta
\end{aligned}$$

with

$$\begin{aligned}
\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}}&=\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)+\left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^u}{\partial c^w}\right)\\
&\quad+\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)\cdot\left[\left(-\frac{\partial n^u}{\partial\epsilon_s^u}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial\epsilon_s^u}\right)^{\text{total}}\right]\cdot\left(\frac{\partial\epsilon_s^u}{\partial c^w}\right)\\
&\quad+f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^u}{\partial c^w}
\end{aligned}$$

$$\begin{aligned}
\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}&=\Bigg[\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)+\left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^c}{\partial c^w}\right)\\
&\quad+\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)\cdot\left[\left(-\frac{\partial n^c}{\partial\epsilon_s^c}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial\epsilon_s^c}\right)^{\text{total}}\right]\cdot\frac{\partial\epsilon_s^c}{\partial c^w}\\
&\quad+f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^c}{\partial c^w}\Bigg]
\end{aligned}$$

$$
\begin{aligned}
\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} = & \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}} \right) \\
& + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}} \right) \\
& + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}
\end{aligned}
$$

This allows us to calculate the Lagrange multiplier for the job-creation condition

$$
\begin{aligned}
M \cdot \lambda_\theta = & u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f} \right) \\
& + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - \lambda F\right) \\
& + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\
& + u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta
\end{aligned}
$$

with

$$
\begin{aligned}
M = & \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \\
& + \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f}\right) \\
& + \left(\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot \left(\frac{\lambda}{f} b - \lambda \cdot F\right) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\
& + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
& + \tilde{IE}_\theta
\end{aligned}
$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase in the joint surplus on θ:

$$
\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}
$$

Further, note that:

$$\frac{\partial n^u}{\partial \epsilon_s^u} = -g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s$$
$$\frac{\partial n^c}{\partial \epsilon_s^c} = -g(\epsilon_s^c) \cdot \frac{1-p}{\lambda} \cdot n^s$$

Rearranging for λ_θ gives:

$$\begin{aligned}\lambda_\theta = u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right) \\ + u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(\frac{\lambda}{f} b - \lambda F\right) \\ + u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \frac{\lambda}{f} b \\ + u'(c^w) \cdot n^c \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta\end{aligned}$$

Note that

$$\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$

$$\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{total}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}$$

Further define:

$$\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)$$

Finally, we can calculate the Lagrange multiplier for λ_c by inserting into λ_θ:

$$\begin{aligned}
\lambda_c = \frac{-1}{\left(\frac{\partial S}{\partial c^w}\right)^{\text{total}}} \cdot \Bigg[\\
&\left(\frac{\partial \epsilon_s^u}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} b - \lambda F\right) \\
&+ \left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b \\
&+ \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&- n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{ge}} \cdot \left(\frac{\partial \theta}{\partial S}\right) \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

Simplifying:

$$\begin{aligned}
\lambda_c = -\frac{u'(c^w)}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[&\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \frac{\lambda}{f} b \\
&+ \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + n^c \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

Following the same arguments, we can express the Lagrange multiplier for the separation conditions as:

$$\begin{aligned}
\lambda_{\epsilon_s^u} = -\frac{u'(c^w)}{\left(\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}\right)^{\text{total}}} \Bigg[&\left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \frac{\lambda}{f} b \\
&+ \frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
&+ n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}$$

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}} \Bigg[\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c} \cdot \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u} \right) \right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \lambda F \right)$$

$$+ \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c} \right) \right)^{\text{ge}} \cdot \frac{\lambda}{f} b + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial f}{\partial c^w} \right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f} \right)$$

$$+ n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial \theta}{\partial c^w} \right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

For convenience, the optimal UI benefits are replicated here:

$$\begin{aligned}(1-n) \cdot (u'(b) - u'(c^w)) = \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)} \right) \cdot \left(-\frac{u'(b)}{u'(c^w)} \right) \\ + (\lambda_{\epsilon_s^u} + \lambda_{\epsilon_s^c}) \cdot \left(-\frac{u'(b)}{u'(c^w)} \right) \\ + \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} - (1-\eta) \cdot \frac{u'(b)}{u'(c^w)} \right)\end{aligned}$$

Inserting the Lagrange multiplier gives:

$$\begin{aligned}(1-n)\cdot &(u'(b) - u'(c^w)) \\ &= u'(c^w) \cdot \left[\frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial f}{\partial c^w} \right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f} \right) \right] \\ &+ u'(c^w) \cdot \left[\left(\frac{\partial \epsilon_s^u}{\partial b} \right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u} \right) \cdot \left(\frac{\lambda}{f} b - \lambda F \right) \right] \\ &+ u'(c^w) \cdot \left[\left(\frac{\partial \epsilon_s^c}{\partial b} \right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c} \right) \cdot \frac{\lambda}{f} b \right] \\ &+ u'(c^w) \cdot \left[n^c \cdot \left(-\frac{\partial \theta}{\partial b} \right)^{\text{ge}} \cdot \hat{IE}_\theta \right].\end{aligned}$$

With:

$$\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) = \underbrace{\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \left(\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b} \cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}}\right] \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)}_{\text{indirect effect}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b} \cdot \left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b} \cdot \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}}\right] \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)}_{\text{indirect effect}}$$

$$\left(\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\left(\frac{\partial S}{\partial b} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}\right)}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b} \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

J.7 Optimal UI under Optimal Layoff Tax

From the FOC of UI, we can derive the optimality condition for unemployment insurance. In contrast to the section where we take layoff taxes as given, the government implements the optimal layoff tax. This implies that the Lagrange multiplier of the separation condition in unconstrained firms is equal to zero. This is also fundamental for the optimality condition for the UI benefits:

$$(1-\eta)\cdot(u'(b)-u'(c^w)) = \lambda_\theta\cdot(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)+\lambda_{\epsilon_s^u}\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)$$
$$+\lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)$$

The Lagrange multiplier for the separation condition of unconstrained firms is equal to

$$-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} = \frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial\epsilon_s^c}}\cdot\Bigg[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b$$
$$+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\Bigg]$$

First, let us find an expression for λ_c:

$$\frac{\partial\mathcal{L}}{\partial c^w} = -\Bigg((1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}$$
$$+p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta$$
$$-(1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)$$
$$-p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_c$$
$$+\frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c}=0$$

Insert Lagrange multipliers for the separation condition:

$$
\begin{aligned}
&-\Bigg((1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\\
&\quad+p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta\\
&-(1-\eta)\cdot(1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\\
&\quad\cdot p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_c\\
&+\frac{\partial\epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\Bigg((n^s\cdot u'(c^w)+\lambda_\theta)\cdot\left(\frac{\lambda}{f}\cdot b\right)\\
&\quad+\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\Bigg)=0
\end{aligned}
$$

Rearranging gives:

$$
\begin{aligned}
0=\lambda_\theta\cdot&\frac{\partial S^{\text{total}}}{\partial c^w}\\
&+\lambda_c\cdot\frac{\partial WE^{\text{total}}}{\partial c^w}\\
&+\frac{\partial\epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}b
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial S^{\text{total}}}{\partial c^w}=\Bigg[&p(1-\rho^u)\frac{u(c^w)-u(b)}{u'(c^w)}\frac{u''(c^w)}{u'(c^w)}\\
&-(1-p)(1-\rho^c)\frac{u^c-u(b)}{u'(c^w)}\frac{u''(c^w)}{u'(c^w)}\\
&+\frac{\partial\epsilon_s^c}{\partial\theta}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{\lambda}{f}b\Bigg]
\end{aligned}
$$

and

$$\frac{\partial WE^{\text{total}}}{\partial c^w} = \Bigg[-(1-\eta)(1-p\rho^u)\left(1-(1-\eta)\frac{u(c^w)-u(b)}{u'(c^w)}\frac{u''(c^w)}{u'(c^w)}\right)$$
$$+ (1-(1-p)\rho^c)(1-\eta)\frac{u^c-u(b)}{u'(c^w)}\frac{u''(c^w)}{u'(c^w)}$$
$$+ \frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\lambda_c\left(\eta\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg]$$

Next, we can solve for the Lagrange multiplier of the wage equation:

$$\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}\Bigg[\frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}\cdot b$$
$$+\left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right)\cdot\lambda_\theta\Bigg]$$

Now we have everything to calculate λ_θ from the two equations:

$$(1)\quad \left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}$$

$$(2)\quad \chi = \frac{1}{1-\eta}\cdot\frac{1}{u\cdot f}\cdot\frac{1}{u'(c^w)}\cdot\left[\gamma-(\lambda\gamma-f\eta)\cdot\left(\frac{1}{\lambda}(1-\rho)\lambda_\theta\right)\right]$$
$$+\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot\lambda_{\epsilon_s^c}$$
$$+\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\lambda_c\cdot\frac{\partial WE}{\partial\theta}\cdot\frac{1}{k_v}$$
$$-\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot IE_\theta$$

We can rearrange (1) to:

$$\chi\cdot k_v = (1-\gamma)\cdot q\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)$$

Note that

$$(1-\gamma)\cdot q = \frac{\partial f}{\partial\theta}$$

This follows from the fact that:

$$f'(\theta) = q(\theta) + \theta \cdot q'(\theta) = q(\theta) \cdot \left(1 + \frac{q'(\theta) \cdot \theta}{q(\theta)}\right) = q(\theta) \cdot (1 - \gamma)$$

So we can write:

$$\chi \cdot k_v = \left(\frac{\partial f}{\partial \theta}\right) \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right)$$

Inserting χ gives:

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \left(\frac{\partial f}{\partial \theta}\right) \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\epsilon_s^c}) \\
&\quad + \frac{\partial WE}{\partial \theta} \cdot (-\lambda_c) \quad + IE_\theta
\end{aligned}$$

Inserting $\lambda_{\epsilon_s^c}, IE_\theta$ gives

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \\
&\quad + \lambda_\theta \cdot \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + u'(c^w) \cdot n^s \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \tilde{IE}_\theta
\end{aligned}$$

with

$$\begin{aligned}
\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} &= \left[\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^c}{\partial c^w}\right)\right. \\
&\quad \left. + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right)\right] \cdot \frac{\partial \epsilon_s^c}{\partial c^w} \\
&\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^c}{\partial c^w}
\end{aligned}$$

$$
\begin{aligned}
\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total},2} &= \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \\
&\quad + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right) \\
&\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}
\end{aligned}
$$

This allows us to calculate the Lagrange multiplier for the job-creation condition

$$
\begin{aligned}
M \cdot \lambda_\theta = u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
+ u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\
+ u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta
\end{aligned}
$$

with

$$
\begin{aligned}
M = \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \\
+ \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f}\right) \\
+ \left(\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\
+ \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
+ \tilde{IE}_\theta
\end{aligned}
$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase in the joint surplus on θ:

$$
\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}
$$

Rearranging for λ_θ gives:

$$\lambda_\theta = u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right)$$
$$+ u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \frac{\lambda}{f} b$$
$$+ u'(c^w) \cdot n^c \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta$$

Note that

$$\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$
$$\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}$$

Further define:

$$\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)$$

Finally, we can calculate the Lagrange multiplier for λ_c by inserting into λ_θ:

$$\lambda_c = \frac{-1}{\left(\frac{\partial S}{\partial c^w}\right)^{\text{total}}} \Bigg[\left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b$$
$$+ \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right)$$
$$+ n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right) \cdot \hat{IE}_\theta \Bigg]$$

Simplifying:

$$\lambda_c = -\frac{u'(c^w)}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \frac{\lambda}{f} b$$
$$+ \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + n^c \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

Following the same arguments, we can express the Lagrange multiplier for the separation condition as:

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}}\left[\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c} \cdot \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f}b - \lambda F\right) + \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)\right)^{\text{ge}} \cdot \frac{\lambda}{f}b\right.$$

$$\left. + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta\right]$$

For convenience, the optimal UI benefits are replicated here:

$$\begin{aligned}(1-n)\cdot(u'(b) - u'(c^w)) = {} & \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)}\right) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right) \\ & + \lambda_{\epsilon_s^c} \cdot \left(-\frac{u'(b)}{u'(c^w)}\right) \\ & + \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} + (1-\eta) \cdot \frac{u'(b)}{u'(c^w)}\right)\end{aligned}$$

Inserting the Lagrange multiplier gives

$$\begin{aligned}(1-n)\cdot(u'(b) - u'(c^w)) = u'(c^w) \cdot \Bigg[& \left(-\frac{\partial f}{\partial b}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\ & + \left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \frac{\lambda}{f} b \\ & + n^c \cdot \left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]\end{aligned}$$

with

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)}_{\text{direct effects}} + \underbrace{\left[\frac{\partial S}{\partial b} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b} \cdot \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}}\right] \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)}_{\text{indirect effect}}$$

$$\left(\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\left(\frac{\partial S}{\partial b} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}\right)}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b} \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}} + \underbrace{\left[\frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

Inserting the Lagrange multiplier gives

$$(1-n)\cdot\frac{(u'(b)-u'(c^w))}{u'(c^w)} = \left(-\frac{\partial f}{\partial b}\right)^{\text{ge}}\cdot u\cdot L_v + \left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)\cdot L_s^c + n^c\cdot\left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}}\cdot \hat{IE}_\theta$$

Note that when we set the layoff tax optimally, then UI will not impose any distortions on the separation condition of unconstrained firms. In the context of Proposition 1, this means that $MLS^u = 0$, bolstering the fact that optimal layoff taxes can implement the optimal number of separations.

K Ramsey FOCs with STW

In the following, multipliers from the Lagrangian, implied by the maximization problem from the previous section, are denoted by λ_{idx}, the index depending on the constraint. Here, λ_n denotes the Lagrange multiplier for the total employment equation, λ_{n^u} for the number of unconstrained firms, λ_{n^c} for the number of constrained firms, λ_{n^s} for the number of firms that received a shock, λ_θ for the job-creation condition, λ_c for the wage equation, $\lambda_{\epsilon_s^u}$ for the separation condition of unconstrained firms with STW support, $\lambda_{\xi_s^c}$ for the separation condition of constrained firms without STW support, and $\lambda_{\epsilon_s^c}$ for the separation condition of constrained firms with STW support. Every other equation listed in the Ramsey problem for STW is assumed to be plugged in.

K.1 Employment

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n^s} &= -\lambda_{n^s} + \frac{1-p}{\lambda}(1-\rho^u)\cdot(u(c^w) - u'(c^w)\cdot c^w)\\ &+ \frac{p}{\lambda}(1-\rho^c)\cdot(u^c - u'(c^w)\cdot e^c)\\ &+ \frac{1-p}{\lambda}\cdot(1-\rho^c)\cdot\lambda_{n^u} + \frac{p}{\lambda}(1-\rho^c)\cdot\lambda_{n^c} = 0\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial n^u} = -\lambda_{n^u} + \lambda_n = 0$$

$$\frac{\partial \mathcal{L}}{\partial n^c} = -\lambda_{n^c} + \lambda_n = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n} &= -\lambda_n + \lambda_{n^s}\cdot(\lambda - q(\theta)\cdot\theta)\\ &+ u'(c^w)\cdot[(z-\Omega) + b + \theta\cdot k_v] - u(b)\\ &+ \frac{(1-\rho)}{\lambda}\cdot\frac{b}{n^2}\cdot\lambda_\theta + \eta(1-\rho)\cdot\frac{b}{n^2}\cdot\lambda_c\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad \frac{\lambda_n}{u'(c^w)} &= (\lambda - q(\theta)\cdot\theta) + [z - \Omega + b + \theta\cdot k_v] - \frac{u(b)}{u'(c^w)}\\ &+ \frac{1-\rho}{\lambda}\cdot\frac{b}{n^2}\cdot\frac{\lambda_\theta}{u'(c^w)} + \eta\cdot(1-\rho)\cdot\frac{b}{n^2}\cdot\frac{\lambda_c}{u'(c^w)}\end{aligned}$$

K.2 Job Creation

Before we start, let us define the insurance effect:

$$IE_\theta = n^s\cdot u'(c^w)\cdot\left(1 + \frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\tilde{IE}_\theta$$

$$\begin{aligned}\tilde{IE}_\theta = \frac{p}{\lambda}\Bigg(&\int_{\max\{\xi_s^c,\epsilon_{\text{stw}}\}}^{\epsilon_p}\frac{\lambda}{f}\cdot\gamma\cdot\frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)}\,dG(\epsilon)\\ &+\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}\frac{\lambda}{f}\cdot\gamma\cdot\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\,dG(\epsilon)\Bigg)\cdot k_v\end{aligned}$$

The FOC for labor market tightness can be written as:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \theta} = & -k_v(1-n)\cdot u'(c^w) + IE_\theta + (1-\gamma)\cdot q(\theta)\cdot(1-n)\cdot\lambda_{n^s} \\
& - \frac{1}{1-\eta}\cdot\gamma\cdot k_v + \frac{\lambda_\theta}{f} + \frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f\eta}{f}\cdot\left(\frac{1}{\lambda}\cdot(1-\rho)\cdot\lambda_\theta - \lambda_{\epsilon_s^u}\right) \\
& - \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w_{\mathrm{stw}}(\epsilon_s^c)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\epsilon_s^c} \\
& - \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w(\xi_s^c)) + \frac{1}{\lambda}\cdot\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\xi_s^c} \\
& - \lambda_c\cdot\frac{\partial WE}{\partial \theta}
\end{aligned}
$$

$$
\iff \qquad \frac{\lambda_{n^s}}{u'(c^w)} = \frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q}
$$

$$
\begin{aligned}
\text{with} \quad \chi = & \frac{1}{(1-n)\cdot u'(c^w)}\cdot\frac{1}{f} \\
\cdot \quad & \frac{1}{1-\eta}\cdot\left(\gamma\cdot\lambda_\theta - (\lambda\cdot\gamma - f\cdot\eta)\cdot\left(\frac{1}{\lambda}\cdot(1-\rho)\cdot\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right) \\
& + \frac{1}{(1-n)\cdot u'(c^w)}\left(\lambda\cdot\frac{\gamma}{f}u'(c^w_{\mathrm{stw}}(\epsilon_s^c)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\epsilon_s^c} \\
& + \frac{1}{(1-n)\cdot u'(c^w)}\left(\lambda\cdot\frac{\gamma}{f}u'(c^w(\xi_s^c)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\xi_s^c} \\
& + \frac{\lambda_c}{(1-n)\cdot u'(c^w)}\cdot\frac{\partial WE}{\partial \theta}/k_v \\
& - \frac{1}{(1-n)\cdot u'(c^w)}\cdot IE_\theta/k_v
\end{aligned}
$$

Collecting terms from FOCs for employment and job-creation conditions gives:

$$
\begin{aligned}
u'(c^w)\cdot\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} = & \frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot(u(c^w) - u(b) - u'(c^w)\cdot c^w) \\
& + \frac{p}{\lambda}\cdot(1-\rho^c)\cdot(u^c - u(b) - u'(c^w)\cdot e^c) \\
& + \left(\frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_\theta}{n^s}\right) + \left(\frac{1-\rho}{\lambda}\cdot\frac{\lambda_c}{n^s}\cdot\eta\cdot\lambda_c\cdot\frac{b}{n}\right) \\
& + u'(c^w)\cdot\frac{1-\rho}{\lambda}\cdot\left(z - \Omega + b + \frac{\lambda - \gamma f + \chi(1-f)}{1-\gamma}\cdot\frac{k_v}{q}\right)
\end{aligned}
$$

Optimal job-creation condition:

$$
\begin{aligned}
\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} = & \frac{1-\rho}{\lambda}\cdot(z-\Omega+b) \\
& + \frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_\theta}{n^s\cdot u'(c^w)} \\
& + \frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_c\cdot\lambda}{n^s\cdot u'(c^w)} \\
& + \frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right) \\
& + \frac{p}{\lambda}\cdot(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right) \\
& + \frac{1-\rho}{\lambda}\cdot\frac{\lambda-\gamma f+\chi(\lambda-f)}{1-\gamma}\cdot\frac{k_v}{q}
\end{aligned}
$$

Subtracting the decentralized job-creation condition from the optimal gives:

$$
\begin{aligned}
\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = & \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{1-\rho}{\lambda}\cdot\frac{b}{n} \\
& + \frac{(1-\rho)(\lambda-f)}{\lambda}\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}
\end{aligned}
$$

Rearranging gives:

$$
\begin{aligned}
\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} & = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{\frac{1-\rho}{\lambda}}{\rho+(1-\rho)\cdot\frac{f}{\lambda}}\cdot\frac{b}{n} \\
& = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}
\end{aligned}
$$

K.3 Lagrange Multipliers for the Separation Condition

Optimal separation condition unconstrained firms

$$
\begin{aligned}
& -\frac{\partial \mathcal{L}}{\partial \epsilon_s^u} \\
& = \frac{n}{1-\rho} \cdot (1-p) \cdot g(\epsilon_s^u) \cdot u'(c^w) \cdot \left(z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w - (z + \Omega) \right) \\
& \quad + \lambda_\theta \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right. \\
& \quad \left. - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
& \quad + \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot \lambda_{n^u} + \lambda_{\epsilon_s^u} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \overset{!}{=} 0
\end{aligned}
$$

Insert for λ_{n^u}:

$$
\begin{aligned}
\iff \quad & g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \\
& \cdot \left[z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w - (z + \Omega) + \frac{\lambda_{n^u}}{u'(c^w)} \right] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \overset{!}{=} 0
\end{aligned}
$$

$$
\begin{aligned}
\iff \quad & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \left(s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \lambda_c \cdot \frac{1}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s} \right) \\
& + \lambda_{\epsilon_s^u} \cdot \frac{1}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

Insert for λ_{n^s} :

$$\begin{aligned}\Longleftrightarrow \quad & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b \\ & + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\ & - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\ & + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\ & + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{1}{n^s} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0\end{aligned}$$

Insert λ_{n^s}

$$\begin{aligned}\Longleftrightarrow \; & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b \\ & + \frac{\lambda - \gamma f + \chi(\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\ & - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) - b \right) \\ & + \lambda \cdot \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\ & + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0\end{aligned}$$

Subtract the decentralized separation condition:

$$\begin{aligned}s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) = b + (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\eta} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} & \\ + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} & \\ - \frac{\lambda_\theta \cdot \theta}{n^s \cdot u'(c^w)} \cdot \left(s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) - b \right) & \\ + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) &\end{aligned}$$

Hence

$$\begin{aligned}
0 = &\left(1+\frac{\lambda_\theta}{u'(c^w)\cdot n^s}\right)\\
&\cdot\left(b+(\lambda-f)\cdot\frac{(1-\rho/\lambda)}{(\rho+(1-\rho)\cdot f/\lambda)}\cdot\frac{b}{n}-s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)}\cdot\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}\\
&+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\\
\iff\quad 0 = &\left(1+\frac{\lambda_\theta}{u'(c^w)\cdot n^s}\right)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)}\cdot\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}\\
&+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)
\end{aligned}$$

And finally:

$$\begin{aligned}
-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} = &\frac{(1-p)\cdot g(\epsilon_s^u)}{\lambda}\cdot\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}}\cdot\Bigg(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\\
&\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^u)\right)\right)\\
&+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\Bigg)
\end{aligned}$$

The optimal separation condition constrained firms

$$\begin{aligned}
-\frac{\partial\mathcal{L}}{\partial\epsilon_s^c} = &\Bigg[\frac{n}{1-\rho}\cdot p\cdot g(\epsilon_s^c)\cdot u'(c^w)\left(z(\epsilon_s^c)-\Omega(\epsilon_s^c)\right.\\
&\left.+\frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)}-c_{\text{stw}}(\epsilon_s^c)-(z+\Omega)\right)\\
&+\lambda_\theta\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\Bigg[z(\epsilon_s^c)-\Omega(\epsilon_s^c)+\frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}-c^w(\epsilon_s^c)\\
&-\frac{1-n}{n}\cdot b+\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}\Bigg]\\
&-\lambda_c\cdot\frac{\partial WE}{\partial\epsilon_s^c}+\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot\lambda_{n^c}\Bigg]\cdot\mathbb{1}(\epsilon_{\text{stw}}\geq\epsilon_s^c)+\lambda_{\epsilon_s^c}^c\cdot\frac{\partial S_{\text{stw}}^u(\epsilon_s^c)}{\partial\epsilon_s^c}=0
\end{aligned}$$

Subtract decentralized separation condition (constrained firms):

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \left(z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(c^w)}{u'(c^w)} - (z + \Omega) \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \frac{\lambda_{n^c}}{u'(c^w)} \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left[s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \frac{\lambda_{\epsilon_s^c}^c}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Insert λ_{n^c} and λ_n

$$
\begin{aligned}
\iff -\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& \frac{p}{\lambda} \cdot n^s g(\epsilon_s^c) \left(z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - u(c^w) - (z + \Omega) \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \Bigg((\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} \\
& + [z - \Omega + b + \theta \cdot c] - \frac{u(b)}{u'(c^w)} \\
& \quad + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \lambda_\theta \cdot \frac{1}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} \Bigg) \Bigg] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left[s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \frac{\lambda_{\epsilon_s^c}^c}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Insert λ_{n^s}

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) + b \\
& + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left[s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) - b \right] \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \xi_s^c) \\
& + \frac{\lambda}{p} \cdot \frac{1}{n^s \cdot g(\epsilon_s^c)} \cdot \frac{\lambda_{\epsilon_s^c}^c}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

And finally:

$$-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial s^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$

Analogously, it can be derived that

$$-\frac{\lambda_{\xi_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\xi_s^c)}{\frac{\partial s^c_{\text{stw}}(\xi_s^c)}{\partial \xi^c}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b\right) \right.$$
$$\left. + \frac{\lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \xi^c}\right) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$

K.4 Optimal STW Benefits

$$\frac{\partial \mathcal{L}}{\partial s_{stw}} = \frac{p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} (u'(c_{\text{stw}}(\epsilon)) - u'(c^w)) \left(\bar{h} - h_{\text{stw}}(\epsilon)\right) dG(\epsilon)$$
$$+ \lambda_\theta \cdot \frac{p}{\lambda} \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\right) \left(\bar{h} - h_{\text{stw}}(\epsilon)\right) dG(\epsilon)$$
$$- \frac{n^s \cdot (1-\rho)}{\lambda} \cdot u'(c^w) \cdot \frac{\partial \Omega}{\partial s_{stw}} - \lambda_\theta \cdot \frac{(1-\rho)}{\lambda} \cdot \frac{\partial \Omega}{\partial s_{stw}} - \lambda_c \cdot \frac{\partial WE}{\partial s_{stw}}$$
$$- \lambda_{\epsilon_s^u} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial s_{stw}} - \lambda_{\epsilon_s^c} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial s_{stw}} = 0$$

Insert Lagrange multiplier separation condition:

$$\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{p}{\lambda} \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \right.$$
$$\left. -(1-\rho) \cdot \frac{\partial \Omega}{\partial s_{stw}}\right) - \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \frac{\partial WE}{\partial s_{stw}}$$
$$+ \frac{(1-p)}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{\partial \epsilon_s^u}{\partial s_{stw}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\right)$$
$$+ \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\partial \epsilon_s^c}{\partial s_{stw}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)\right) = 0$$

Rearranging gives:

$$
\begin{aligned}
\bar{\tau}_{\text{stw}} = \quad & \underbrace{\frac{\lambda}{f} b}_{\text{Fiscal Ext.}} \\
& + \underbrace{\frac{\mathbb{1}\{\epsilon_{\text{stw}} \geq \epsilon_s^c\}}{MMS_{s_{stw}}} \cdot n^s \cdot \frac{p}{\lambda} \int_{\epsilon_s^c}^{\epsilon_{\text{stw}}} \left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \right) (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon)}_{\text{Insurance}} \\
& - \underbrace{\frac{n^s}{MMS_{s_{stw}}} \cdot \frac{1}{\lambda} \cdot (1-\rho) \frac{\partial\, \Omega}{\partial\, s_{stw}}}_{\text{Distortion}} + \underbrace{BE_{\text{stw},3}}_{\text{Bargaining Effect}}
\end{aligned}
$$

$$
\begin{aligned}
\bar{\tau}_{\text{stw}} = & \frac{MMS^u_{s_{stw}}}{MMS_{s_{stw}}} s_{stw} (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \\
& + \frac{MMS^c_{s_{stw}}}{MMS_{s_{stw}}} s_{stw} (\bar{h} - h_{\text{stw}}(\epsilon_s^c))
\end{aligned}
$$

where

$$
MMS^u_{s_{stw}} = \frac{(1-p)}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{\partial \epsilon_s^u}{\partial s_{stw}} \cdot n^s, \quad MMS^c_{s_{stw}} = \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\partial \epsilon_s^c}{\partial s_{stw}} \cdot n^s,
$$

$$
MMS_{s_{stw}} = MMS^c_{s_{stw}} + MMS^u_{s_{stw}}
$$

and

$$
\begin{aligned}
BE_{\text{stw},3} = & \frac{1}{\left(1 + \frac{\lambda_\theta}{n^s\, u'(c^w)}\right)} \\
& \times \Bigg[- \frac{\lambda_c}{\varphi\, n^s\, u'(c^w)} \frac{\partial WE}{\partial s_{stw}} + \frac{MMS^u_{s_{stw}}}{MMS_{s_{stw}}} \frac{\lambda_c}{n^s\, u'(c^w)} \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \frac{\lambda}{1-p} \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& \quad + \frac{MMS^c_{s_{stw}}}{MMS_{s_{stw}}} \frac{\lambda_c}{n^s\, u'(c^w)} \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \frac{\lambda}{p} \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg]
\end{aligned}
$$

K.5 Optimal Eligibility Condition

Assumption throughout: $\epsilon^p \geq \epsilon_{\text{stw}} \geq \epsilon_s^u$

Further assume that:

$$
n^s \cdot u'(c^w) + \lambda_\theta - \eta \cdot \lambda_c > 0, \qquad n^s \cdot u'(c^w) + \lambda_\theta > 0
$$

These conditions exclude the case where the Planner would want to use STW's distortion in working hours to destroy production and thus reduce vacancy posting. The planner might want to do that when the Hosios condition is not fulfilled. For reasonable parameter values, however, the condition should hold anyway. Recall that by Lemma 1 $\epsilon_s^c > \epsilon_s^u$ and $\xi_s^c > \xi_s^u$.

Candidate I:
FOC for eligibility threshold:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} = g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Big(& u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}})) \\
& - u'(c^w) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})] \Big) \\
& - n \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \\
& + \lambda_\theta \cdot \frac{p}{\lambda} \cdot \Big(u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}})) \\
& - u'(c^w) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})] \Big) \\
& - \lambda_\theta \cdot \frac{1-\rho}{\lambda} \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} \overset{!}{\geq} 0
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \quad \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{1-\rho}{\lambda} \cdot n^s \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} + BE\right) \\
\overset{!}{\geq} \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \\
\cdot \left(\frac{\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}}))}{c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})} - u'(c^w)}{u'(c^w)} \right) \\
\cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})]
\end{aligned}$$

$$\Leftrightarrow \quad \left(\frac{1-\rho}{\lambda} \cdot n^s \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} + BE\right) \overset{!}{\geq} \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \cdot \left(\frac{\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}}))}{c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})} - u'(c^w)}{u'(c^w)} \right) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})]$$

If the equation holds with strict inequality, the cost of a distortion in working hours exceeds the benefit of providing additional insurance to workers in constrained firms. Consequently, the Ramsey planner chooses not to allow firms and workers to enter short-time work when they could survive without reaching the threshold $\epsilon_{\text{stw}} = \xi_s^c$. Conversely, if the equation holds with exact equality, the participation threshold is determined by balancing the additional cost of distortion in working hours against the benefit of providing extra insurance.

Candidate II:

FOC for the eligibility threshold:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} &= -n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega^u}{\partial \epsilon_{\text{stw}}} u'(c^w) \\ &\quad - (\lambda_\theta - \eta \cdot \lambda_c) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega^u}{\partial \epsilon_{\text{stw}}} \\ &= -\left(n^s \cdot u'(c^w) + \lambda_\theta + \eta \cdot \lambda_c\right) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega^u}{\partial \epsilon_{\text{stw}}} \end{aligned}$$

Under reasonable parameter values (assuming that the Ramsey planner never wants to use short-time work to impede vacancy posting by destroying the efficiency of the match, captured by the Lagrange multipliers λ_θ and λ_c), we get:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} < 0 \quad \text{thus it is optimal to set} \quad \epsilon_{\text{stw}} = \epsilon_s^u.$$

Candidate III:

FOC for the eligibility threshold:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{\mathrm{stw}}} = & -n^u \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\mathrm{stw}}} \cdot u'(c^w) \\
& + \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_{\mathrm{stw}}) \cdot u'(c^w) \\
& \cdot \left(z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) + \frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}))}{u'(c^w)} - c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) - (z+\Omega) \right) \\
& + \lambda_\theta \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\mathrm{stw}}} \\
& + \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\mathrm{stw}}) \\
& \cdot \left[z(\epsilon_{\mathrm{stw}}) + \frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})) - u(b)}{u'(c^w)} \right. \\
& \quad \left. - c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\mathrm{stw}}} + \frac{p}{\lambda} \cdot g(\epsilon_{\mathrm{stw}}) \cdot n^s \cdot \lambda_{n^c} \overset{!}{>} 0.
\end{aligned}
$$

Insert n^u

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{\mathrm{stw}}} = & -n^s \cdot (1-\rho^u) \cdot \left(\frac{1-p}{\lambda} \right) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\mathrm{stw}}} \cdot u'(c^w) \\
& + n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\mathrm{stw}}) \cdot u'(c^w) \cdot \left(z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) + \frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}))}{u'(c^w)} \right) \\
& - n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\mathrm{stw}}) \cdot u'(c^w) \cdot (c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) + z + \Omega) \\
& + \lambda_\theta \cdot \frac{g(\epsilon_{\mathrm{stw}})}{\lambda} \cdot (1-p)(1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\mathrm{stw}}} \\
& + \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\mathrm{stw}}) \cdot \left(z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) + \frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})) - u(b)}{u'(c^w)} - c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}) - \frac{1-n}{n} \cdot b \right. \\
& \qquad \left. + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \left(\frac{k_v}{q} \right) \right) \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\mathrm{stw}}} + \frac{p}{\lambda} \cdot g(\epsilon_{\mathrm{stw}}) \cdot n^s \cdot \lambda_{n^c} \overset{!}{>} 0
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\ & -\left(n^s \cdot u'(c^w) + \lambda_\theta\right) \cdot (1-\rho^u) \cdot \left(\frac{1-p}{\lambda}\right) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \\
& + n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \\
& \cdot \left(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) - (z+\Omega) + \frac{\lambda_{n^c}}{u'(c^w)} \right) \\
& + \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) \\
& \qquad - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \left(\frac{k_v}{q}\right) \Bigg] \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} \overset{!}{>} 0
\end{aligned}
$$

Insert λ_{n^c} and subtract the decentralized separation condition for constrained firms:

$$
\begin{aligned}
\Leftrightarrow \quad & -\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \\
& + \frac{p}{\lambda} \cdot \Bigg[b + z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) \\
& + \frac{\lambda - \eta \cdot f + \chi \cdot (\lambda - f)}{1-\eta} \cdot \left(\frac{k_v}{q}\right) \Bigg] \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \frac{p}{\lambda} \cdot \left[s_{stw} \cdot (h - h_{\text{stw}}(\epsilon_{\text{stw}})) - b \right] \\
& - \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot \eta \cdot \frac{b}{n} \right) \overset{!}{>} 0
\end{aligned}
$$

Subtract the decentralized separation condition:

$$
\begin{aligned}
\Leftrightarrow \quad & -\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{(1-p)}{\lambda}\cdot(1-\rho^u)\cdot\frac{\partial\Omega_u}{\partial\epsilon_{\mathrm{stw}}}\cdot\frac{1}{g(\epsilon_{\mathrm{stw}})}\\
& +\frac{p}{\lambda}\cdot\Bigg[z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})-z_{\mathrm{stw}}(\epsilon_s)+\frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}))-u(c_{\mathrm{stw}}(\epsilon_s^c))}{u'(c^w)}\\
& \quad -s_{stw}\cdot(h-h_{\mathrm{stw}}(\epsilon_s^c))+b+(1-\rho)\cdot(\lambda-f)\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}\Bigg]\\
& +\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\cdot\frac{p}{\lambda}\cdot\Bigg[z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})-z_{\mathrm{stw}}(\epsilon_s^c)+\frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}))-u(c_{\mathrm{stw}}(\epsilon_s^c))}{u'(c^w)}\\
& -s_{stw}\cdot(h-h_{\mathrm{stw}}(\epsilon_s^c))+b]\\
& -\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\frac{\partial WE}{\partial\epsilon_{\mathrm{stw}}}\cdot\frac{1}{g(\epsilon_{\mathrm{stw}})}\cdot\frac{\partial S_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})}{\partial\epsilon_{\mathrm{stw}}}+\frac{p}{\lambda}\cdot\eta\cdot\frac{b}{n}\right)\overset{!}{>}0
\end{aligned}
$$

Inserting for $(\chi-\frac{\eta-\gamma}{1-\eta})\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}$ gives us the final condition. We can distinguish between three cases:

(A)

$$
\begin{aligned}
& g(\epsilon_{\mathrm{stw}})\cdot\frac{p}{\lambda}\cdot n^s\cdot\Bigg[z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})-z_{\mathrm{stw}}(\epsilon_s^c)\\
& \quad +\frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}))-u(c_{\mathrm{stw}}(\epsilon_s^c))}{u'(c^w)}+\frac{\lambda}{f}\cdot b-s_{stw}\cdot\big(h-h_{\mathrm{stw}}(\epsilon_s^c)\big)\Bigg]\\
& > n^s\cdot\frac{1-p}{\lambda}\cdot(1-\rho)\cdot\frac{\partial\Omega_u}{\partial\epsilon_{\mathrm{stw}}}+BE\\
& \forall\,\epsilon_{\mathrm{stw}}\text{ in case 3}\quad\Rightarrow\quad\epsilon_{\mathrm{stw}}=\xi_s^c
\end{aligned}
$$

(B)

$$
\begin{aligned}
& g(\epsilon_{\mathrm{stw}})\cdot\frac{p}{\lambda}\cdot n^s\cdot\Bigg[z_{\mathrm{stw}}(\epsilon_{\mathrm{stw}})-z_{\mathrm{stw}}(\epsilon_s^c)\\
& \quad +\frac{u(c_{\mathrm{stw}}(\epsilon_{\mathrm{stw}}))-u(c_{\mathrm{stw}}(\epsilon_s^c))}{u'(c^w)}+\frac{\lambda}{f}\cdot b-s_{stw}\cdot\big(h-h_{\mathrm{stw}}(\epsilon_s^c)\big)\Bigg]\\
& < n^s\cdot\frac{1-p}{\lambda}\cdot(1-\rho)\cdot\frac{\partial\Omega_u}{\partial\epsilon_{\mathrm{stw}}}+BE\\
& \forall\,\epsilon_{\mathrm{stw}}\text{ in case 3}\quad\Rightarrow\quad\epsilon_{\mathrm{stw}}=\max\{\xi_s^c,\xi_s^u\}
\end{aligned}
$$

(C)

$$
\begin{aligned}
&g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s^c) \\
&\quad + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} + \frac{\lambda}{f} \cdot b - s_{stw} \cdot \big(h - h_{\text{stw}}(\epsilon_s^c)\big) \Bigg] \\
&= n^s \cdot \frac{1-p}{\lambda} \cdot (1-\rho) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} + BE
\end{aligned}
$$

BE is defined as:

$$
\begin{aligned}
BE = g(\epsilon_{\text{stw}}) \cdot \frac{\lambda_c}{u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \eta \cdot \frac{b}{n} \right) \\
\Bigg/ \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right)
\end{aligned}
$$

This concludes the proof of Proposition 4.

K.6 Optimal Unemployment Insurance with STW

From the FOC of UI, we can derive the optimality condition of unemployment insurance:

$$
\begin{aligned}
(1-n) \cdot (u'(b) - u'(c^w)) = \lambda_\theta (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)} \right) \\
+ \left(\lambda_{\epsilon_s^u} + \lambda_{\epsilon_s^c} + \lambda_{\xi_s^c} \right) \cdot \left(- \frac{u'(b)}{u'(c^w)} \right) \\
+ \lambda_c (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} - (1-\eta) \cdot \frac{u'(b)}{u'(c^w)} \right)
\end{aligned}
$$

To get better insight, we need to determine the Lagrange multipliers for λ_θ, λ_c. We already know the Lagrange multipliers for the separation conditions:

$$-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{1-p}{\lambda} \cdot \frac{g(\epsilon_s^u)}{\frac{\partial S^u_{\mathrm{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot \left(\bar{h} - h_{\mathrm{stw}}(\epsilon_s^u)\right)\right) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\right)$$

$$-\frac{\lambda_{\xi_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\xi_s^c)}{\frac{\partial S^c_{\mathrm{stw}}(\xi_s^c)}{\partial \xi_s^c}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \xi_s^c}\right)\right) \cdot \mathbb{1}(\epsilon_{\mathrm{stw}} \leq \xi_s^c)$$

$$-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial S^c_{\mathrm{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot \left(\bar{h} - h_{\mathrm{stw}}(\epsilon_s^c)\right)\right) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)\right) \cdot \mathbb{1}(\epsilon_{\mathrm{stw}} \geq \epsilon_s^c)$$

First, let us find an expression for λ_c:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c^w} = -\Bigg[&(1-p) \cdot (1-\rho^u) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
&+ p \cdot (1-\rho^c) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)}\Bigg] \cdot \lambda_\theta \\
&- (1-\eta) \cdot (1-p) \cdot (1-\rho^u) \cdot \left(1 - (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)}\right) \\
&- (1-p) \cdot (1-\rho^c) \cdot (1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_c \\
&+ \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_{\epsilon_s^u} \\
&+ \frac{u(c(\xi_s^c)) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_{\xi_s^c} \\
&+ \frac{u(c_{\mathrm{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \cdot \lambda_{\epsilon_s^c} = 0
\end{aligned}$$

Insert Lagrange multipliers for separation conditions

$$
\begin{aligned}
&-\left(p(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}+(1-p)(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\lambda_\theta\\
&-(1-\eta)\cdot\left((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\right.\\
&\left.-p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\lambda_c\\
&+\frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\left(n^s\cdot u'(c^w)+\lambda_\theta\right)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\\
&+\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\left(n^s\cdot u'(c^w)+\lambda_\theta\right)\cdot\left(\frac{\lambda}{f}\cdot b+\lambda_c\cdot\left(\eta\cdot\frac{b}{n}-\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\right)\\
&\cdot\mathbb{1}(\epsilon_{\text{stw}}\leq\xi_s^c)\\
&+\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\left(n^s\cdot u'(c^w)+\lambda_\theta\right)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\xi_s^c))\right)\\
&+\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\geq\epsilon_s^c)=0
\end{aligned}
$$

Rearranging yields

$$
\begin{aligned}
0=\lambda_\theta\cdot\frac{\partial S^{\text{total}}}{\partial c^w}&+\lambda_c\cdot\frac{\partial WE^{\text{total}}}{\partial c^w}\\
&+\frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot u'(c^w)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}\cdot b\cdot\mathbb{1}(\epsilon_{\text{stw}}\leq\xi_s^c)\\
&+\frac{\partial\epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\geq\epsilon_s^c)
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial S^{\text{total}}}{\partial c^w} &= (1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} \\
&\quad - p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} \\
&\quad + \frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\left(\frac{\lambda}{f}b - s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + \frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\cdot\frac{\lambda}{f}\cdot b\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c) \\
&\quad + \frac{\partial\epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\left(\frac{\lambda}{f}b - s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial WE^{\text{total}}}{\partial c^w} &= -(1-\eta)\cdot(1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) \\
&\quad + p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} \\
&\quad + \frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right) \\
&\quad + \frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\cdot\lambda_c\cdot\left(\eta\cdot\frac{b}{n}-\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c) \\
&\quad + \frac{\partial\epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\lambda_c\cdot\left(\eta\cdot\frac{b}{n}-\frac{1}{g(\epsilon_s^c)}\cdot\frac{1}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c)
\end{aligned}
$$

Collecting terms:

$$
\begin{aligned}
\lambda_c = \frac{-1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}&\Bigg[\frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot u'(c^w)\left(\frac{\lambda}{f}\cdot b - s_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^u)\right)\right) \\
&+ \frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\cdot n^s\cdot u'(c^w)\left(\frac{\lambda}{f}\cdot b\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c) \\
&+ \frac{\partial\epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\left(\frac{\lambda}{f}\cdot b - s_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^c)\right)\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c) \\
&+ \left(-\frac{\partial S_{\text{total}}}{\partial c^w}\right)\cdot\lambda_\theta\Bigg]
\end{aligned}
$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^u) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \right.$$
$$\cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right)$$
$$\left. + \lambda_c \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u} \right) \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^u} \right) \right)$$

$$\lambda_{\xi_s^c} = \frac{\mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \frac{\lambda}{f} \cdot b \right.$$
$$\left. + \lambda_c \left(\frac{\partial n}{\partial \xi_s^c} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \xi_s^c} \right) \right)$$

$$\lambda_{\epsilon_s^c} = -\frac{\mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \right.$$
$$\cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right)$$
$$\left. + \lambda_c \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c} \right) \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^c} \right) \right)$$

Inserting λ_c gives:

$$\lambda_{\epsilon_s^u} =$$
$$-\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \left[n^s u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)} \right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \right.$$
$$+ \left(\left(-\frac{\partial n^u}{\partial \epsilon_s^u} \right) \left(-\frac{\partial c^w}{\partial n} \right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u} \right)^{\text{total}} \right)$$
$$\cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \right.$$
$$+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$
$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$
$$\left. \left. + \left(-\frac{\partial S^{\text{total}}}{\partial c^w} \right) \cdot \lambda_\theta \right) \right]$$

$$
\begin{aligned}
&\lambda_{\xi_s^c} = \\
&\quad \frac{\mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)}{\frac{\partial S^c_{\text{stw}}(\xi_s^c)}{\partial \xi_s^c}} \Bigg[-n^s u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\xi_s^c))\right) \\
&\quad + \left(\left(-\frac{\partial n^c}{\partial \xi_s^c}\right) \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{total}} \right) \\
&\quad \cdot \Bigg(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&\quad + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&\lambda_{\epsilon_s^c} = \\
&\quad -\frac{\mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \Bigg[n^s u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \\
&\quad + \left(\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}} \right) \\
&\quad \cdot \Bigg(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&\quad + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg) \Bigg]
\end{aligned}
$$

Now we have everything to calculate λ_θ from two equations:

$$
\begin{aligned}
(1)\quad & \left(\chi - \frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} \\
& \quad = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}
\end{aligned}
$$

$$
\begin{aligned}
(2)\quad \chi = {} & \frac{1}{1-\eta}\cdot\frac{1}{u\cdot f}\cdot\frac{1}{u'(c^w)}\cdot\left[\gamma-(\lambda\gamma-f\eta)\cdot\left(\frac{1}{\lambda}(1-\rho)\lambda_\theta-\lambda_{\epsilon_s^u}\right)\right] \\
& + \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot\lambda_{\xi_s^c} \\
& + \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot\lambda_{\epsilon_s^c} \\
& + \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\lambda_c\cdot\frac{\partial WE}{\partial\theta}\cdot\frac{1}{k_v} \\
& - \frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot IE_\theta
\end{aligned}
$$

We can rearrange (1) to:

$$
\chi\cdot k_v = (1-\gamma)\cdot q\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)
$$

Note that

$$
(1-\gamma)\cdot q = -\frac{\partial f}{\partial\theta}
$$

with

$$
\begin{aligned}
f'(\theta) &= q(\theta)+\theta\cdot q'(\theta) \\
&= q(\theta)\cdot\left(1+\frac{q'(\theta)\cdot\theta}{q(\theta)}\right) \\
&= q(\theta)\cdot(1-\gamma)
\end{aligned}
$$

Inserting χ gives:

$$
\begin{aligned}
&[\gamma - (\lambda\gamma - f \cdot \eta)] \cdot \frac{1}{\lambda} \cdot (1-\rho) \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \left(\frac{\partial f}{\partial \theta}\right) \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + \frac{1}{f} \cdot (\lambda\gamma - f \cdot \eta) \cdot k_v \cdot (-\lambda_{\epsilon_s^u}) \\
&\quad + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w(\epsilon_s^c)) + \left(\frac{1}{1-n} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\xi_s^c}) \\
&\quad + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c(\xi_s^c)) + \left(\frac{1}{1-n} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\epsilon_s^c}) \\
&\quad + \frac{\partial WE}{\partial \theta} \cdot (-\lambda_c) \\
&\quad + IE_\theta
\end{aligned}
$$

Insert $\lambda_{\epsilon_s^u}, \lambda_{\epsilon_s^c}, \lambda_{\xi_s^c}, IE_\theta$:

$$
\begin{aligned}
&\left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \left(\frac{\partial f}{\partial \theta}\right) \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \\
&\quad \cdot \frac{1-p}{\lambda} \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \xi_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \\
&\quad \cdot \frac{p}{\lambda} \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&\quad + \lambda_\theta \cdot \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + n^s \cdot u'(c^w) \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \tilde{IE}_\theta
\end{aligned}
$$

Denote:

$$\begin{aligned}\left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} &= \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^u}{\partial c^w}\right) \\ &\quad + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left[\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right] \cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w}\right) \\ &\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^u}{\partial c^w}\end{aligned}$$

$$\begin{aligned}\left(-\frac{\partial \xi_s^c}{\partial \theta}\right)^{\text{total}} &= \Bigg[\left(-\frac{\partial \xi_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \xi_s^c}{\partial c^w}\right) \\ &\quad + \left(-\frac{\partial \xi_s^c}{\partial \theta}\right) \cdot \left[\left(\left(-\frac{\partial n^c}{\partial \xi_s^c}\right)\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{total}}\right] \cdot \left(\frac{\partial \xi_s^c}{\partial c^w}\right) \\ &\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \xi_s^c}{\partial c^w}\Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)\end{aligned}$$

$$\begin{aligned}\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} &= \Bigg[\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^c}{\partial c^w}\right) \\ &\quad + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \cdot \frac{\partial \epsilon_s^c}{\partial c^w} \\ &\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^c}{\partial c^w}\Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)\end{aligned}$$

$$\begin{aligned}\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} &= \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \\ &\quad + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left[\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right] \\ &\quad + \left(-\frac{\partial \xi_s^c}{\partial \theta}\right) \cdot \left[\left(- - \frac{\partial n^c}{\partial \xi_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{total}}\right] \\ &\quad + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \\ &\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}\end{aligned}$$

This allows us to calculate the Lagrange multiplier for the job-creation condition:

$$
\begin{aligned}
&M \cdot \lambda_\theta = \\
&u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f} \right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(\frac{\partial \epsilon_s^u}{\partial \theta} \right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(- \frac{\partial \xi_s^c}{\partial \theta} \right)^{\text{total}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(- \frac{\partial \epsilon_s^c}{\partial \theta} \right)^{\text{total}} \cdot g(\epsilon_s^c) \\
&\quad \cdot \frac{p}{\lambda} \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&\quad + u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta
\end{aligned}
$$

with

$$
\begin{aligned}
M = &\left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho) \right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \\
&+ \left(- \frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f} \right) \\
&+ \left(\frac{\partial \epsilon_s^u}{\partial \theta} \right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
&+ \left(\frac{\partial \xi_s^c}{\partial \theta} \right)^{\text{total}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&+ \left(\frac{\partial \epsilon_s^c}{\partial \theta} \right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&+ \left(\frac{\partial c^w}{\partial \theta} \right)^{\text{total}} \cdot \left(- \frac{\partial S}{\partial c^w} \right)^{\text{total}} \\
&+ \tilde{IE}_\theta
\end{aligned}
$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase in the joint surplus on θ:

$$
\left(\frac{\partial \theta}{\partial S} \right)^{\text{ge}} = \frac{1}{M}
$$

Rearranging for λ_θ gives:

$$
\begin{aligned}
&M \cdot \lambda_\theta = \\
&u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(\left(-\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda}\left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right)\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(\left(-\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)\right) \\
&\quad + u'(c^w) \cdot n^s \\
&\quad \cdot \left(\left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \tilde{IE}_\theta
\end{aligned}
$$

Define:

$$
\begin{aligned}
\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot &\left[\int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c^w(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right. \\
&\left. - \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)\right] \cdot k_v
\end{aligned}
$$

Note:

$$
\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}
$$

$$
\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{total}}
$$

$$
\left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{total}}
$$

$$
\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}
$$

Further, note that:

$$\frac{\partial n^u}{\partial \epsilon_s^u} = -g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s$$
$$\frac{\partial n^c}{\partial \epsilon_s^c} = -g(\epsilon_s^c) \cdot \frac{1-p}{\lambda} \cdot n^s$$
$$\frac{\partial n^c}{\partial \xi_s^c} = -g(\xi_s^c) \cdot \frac{1-p}{\lambda} \cdot n^s$$

This gives:

$$\begin{aligned}
\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \epsilon_s^u}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \right) \\
& \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
& + \left(\frac{\partial \xi_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} \right) \\
& \cdot \left(-\frac{\partial n^c}{\partial \xi_s^c}\right) \cdot u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \right) \\
& \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) \cdot u'(c^w) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f} \right) \\
& - n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot I\hat{E}_\theta \Bigg]
\end{aligned}$$

Following the same arguments, we can express the Lagrange multiplier for the separation conditions as:

$$
\begin{aligned}
\lambda_{\epsilon_s^u} = -\frac{u'(c^w)}{\left(\frac{\partial S^u_{\text{stw}}(\epsilon^u_s)}{\partial \epsilon^u_s}\right)^{\text{total}}} \Bigg[& \left(-\frac{\partial n^u}{\partial \epsilon^u_s}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw}\left(\bar{h} - h_{\text{stw}}(\epsilon^u_s)\right)\right) \\
& + \left(\frac{\partial \xi^c_s}{\partial \epsilon^u_s}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \xi^c_s}\right) \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s) \\
& + \left(\frac{\partial \epsilon^c_s}{\partial \epsilon^u_s}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon^c_s}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
& + \frac{\partial c^w}{\partial \epsilon^u_s} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \epsilon^u_s} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{\xi_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\xi^c_s)}{\partial \xi^c_s}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \xi^u_s}{\partial \xi^c_s}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \xi^c_s} \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\xi^u_s))\right) \\
& + \left(-\frac{\partial n^c}{\partial \xi^c_s}\right)^{\text{ge}} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi^c_s) \\
& + \left(\frac{\partial \epsilon^c_s}{\partial \xi^c_s}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon^c_s}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
& + \frac{\partial c^w}{\partial \xi^c_s} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \xi^c_s} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon^c_s)}{\partial \epsilon^c_s}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \epsilon^u_s}{\partial \epsilon^c_s}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^u}{\partial \epsilon^u_s}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\xi^u_s))\right) \\
& + \left(\frac{\partial \xi^c_s}{\partial \epsilon^c_s}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon^c_s}\right) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \epsilon^c_s) \\
& + \left(-\frac{\partial n^c}{\partial \epsilon^c_s}\right) \left(\frac{\lambda}{f} b - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon^c_s))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon^c_s) \\
& + \frac{\partial c^w}{\partial \epsilon^c_s} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \epsilon^c_s} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}
$$

For convenience, the optimal UI benefits are reproduced here:

$$\begin{aligned}(1-\eta)\cdot(u'(b)-u'(c^w)) = \lambda_\theta(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)\\
+(\lambda_{\epsilon_s^u}+\lambda_{\xi_s^c}+\lambda_{\epsilon_s^c})\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)\\
+\lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}-(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)\end{aligned}$$

Inserting the Lagrange multiplier gives:

$$\begin{aligned}&(1-n)(u'(b)-u'(c^w))\\
&=u'(c^w)\Bigg[\left(\frac{\partial c^w}{\partial b}\right)^{\text{ge}}\left(-\frac{\partial f}{\partial c^w}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right)\\
&\quad+\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&\quad+\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}}\cdot\left(-\frac{\partial n^c}{\partial \xi_s^c}\right)\cdot\frac{\lambda}{f}\cdot b\cdot\mathbb{1}(\epsilon_{\text{stw}}\leq\xi_s^c)\\
&\quad+\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)\cdot\left(\frac{\lambda}{f}\cdot b-s_{stw}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\geq\epsilon_s^c)\\
&\quad+n^c\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot\hat{IE}_\theta\Bigg]\end{aligned}$$

with

$$\begin{aligned}\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)&=\underbrace{\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)}_{\text{direct effects}}\\
&+\underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}+\frac{\partial \xi_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{ge}}+\frac{\partial \epsilon_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}}\right]\cdot\left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)}_{\text{indirect effect}}\end{aligned}$$

$$\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \xi_s^c}\right) = \underbrace{\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}} \left(-\frac{\partial n^c}{\partial \xi_s^c}\right)}_{\text{direct effects}}$$
$$+ \underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial \epsilon_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b}\left(\frac{\partial \xi_s^c}{\partial c^w}\right)\right] \cdot \left(-\frac{\partial n^c}{\partial \xi_s^c}\right)}_{\text{indirect effect}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)}_{\text{direct effects}}$$
$$+ \underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial \xi_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b}\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)\right] \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right)}_{\text{indirect effect}}$$

$$\left(-\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}}$$
$$+ \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \xi_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \xi_s^c} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\left(-\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}}$$
$$+ \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \xi_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \xi_s^c} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

L Proof of Lemma 1

Separation condition: unconstrained matches

$$z_{\text{stw}}(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + s_{stw}(\bar{h} - h(\epsilon_s^u)) + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

Separation condition: constrained matches

$$\frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} + \frac{\eta}{1 - \eta} \cdot (\lambda - f) \cdot \frac{k_v}{q}$$

Remember:

$$c_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon_s^c) + s_{stw}(\bar{h} - h(\epsilon_s^c)) + \lambda \cdot \frac{k_v}{q}$$

This allows us to rewrite the separation condition of constrained firms on STW as:

$$z_{\text{stw}}(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) + s_{stw}(\bar{h} - h(\epsilon_s^c)) + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

T.b.s.: $\epsilon_s^c > \epsilon_s^u$ **(the proof for $\xi_s^c > \xi_s^u$ is completely analogous)**

Note: $c^w > c_{\text{stw}}(\epsilon)$

$$\begin{aligned}
\epsilon_s^c > \epsilon_s^u \Leftrightarrow \quad & z_{\text{stw}}(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + s_{stw}(\bar{h} - h(\epsilon)) \\
& + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \\
& \geq z_{\text{stw}}(\epsilon) + \frac{u(c_{\text{stw}}(\epsilon)) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon) \\
& + s_{stw}(\bar{h} - h(\epsilon)) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q}
\end{aligned}$$

$$\Leftrightarrow \quad \frac{u(c^w)}{u'(c^w)} - c^w > \frac{u(c_{\text{stw}}(\epsilon))}{u'(c^w)} - c_{\text{stw}}(\epsilon)$$

$$\Leftrightarrow \quad \frac{u(c^w) - u(c_{\text{stw}}(\epsilon))}{u'(c^w)} > c^w - c_{\text{stw}}(\epsilon)$$

$$\Leftrightarrow \quad u(c^w) - u(c_{\text{stw}}(\epsilon)) > (c^w - c_{\text{stw}}(\epsilon)) \cdot u'(c^w)$$

$$\Leftrightarrow \quad \int_{c_{\text{stw}}(\epsilon)}^{c^w} u'(c)\, dc > \int_{c_{\text{stw}}(\epsilon)}^{c^w} u'(c^w)\, dc$$

$$\Leftrightarrow \quad \int_{c_{\text{stw}}(\epsilon)}^{c^w} \left[u'(c) - u'(c^w)\right]\, dc > 0 \qquad \checkmark \quad \text{(due to risk aversion)}$$

M Calibration

To calibrate the model, the following system of equations can be solved, taking targets as exogenous. The endogenous model variables used in the system are pinned down at the equilibrium level as well. Note that we use $\beta = 1$.

Exogenous:

$$f,\; q, \rho, b_{\text{rep}}, n$$

Endogenous:

$$k_f, \bar{m}, b, \epsilon_s^c, \epsilon_s^u, c^w, \omega$$

(I)

$$\rho = (1-p) \cdot G(\epsilon_s^u) + p \cdot G(\epsilon_s^c)$$

(II)

$$0 = z(\epsilon_s^u) + \lambda \cdot F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q}$$

(III)

$$0 = z(\epsilon_p) + \lambda \cdot \frac{k_v}{q} - c^w$$

(IV)

$$\frac{u(c(\epsilon_s^u)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(V)

$$
\begin{aligned}
\frac{1}{1-\eta}\cdot\frac{k_v}{q} = \frac{1}{\lambda}\Bigg[&(1-\rho)\cdot\left(z-\frac{1-\eta}{\eta}b\right)\\
&+(1-p)(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)\\
&+p(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}-e^c\right)\\
&+(1-\rho)\cdot\frac{\lambda-f\eta}{1-\eta}\cdot\frac{k_v}{q}\Bigg]
\end{aligned}
$$

(VI)

$$
\begin{aligned}
&(1-p)\cdot(1-\rho^u)\cdot\left(\eta\cdot c^w+(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right)\\
&=\eta\cdot(1-\rho)\cdot\left(z+\frac{1-n}{n}\cdot b+\theta\cdot k_v\right)\\
&\quad-p\cdot(1-\rho^c)\cdot\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}+\eta\cdot e^c\right]
\end{aligned}
$$

(VII)

$$
\begin{aligned}
\omega = \frac{1}{1-\rho}\cdot\Bigg(&(1-p)\cdot\int_{\epsilon_s^u}^{\infty}\left[c^w+\phi(h(\epsilon))\right]dG(\epsilon)\\
&+p\cdot\int_{\epsilon_p}^{\infty}\left[c^w+\phi(h(\epsilon))\right]dG(\epsilon)\\
&+p\cdot\int_{\epsilon_s^c}^{\epsilon_p}\left[c(\epsilon)+\phi(h(\epsilon))\right]dG(\epsilon)\Bigg)
\end{aligned}
$$

(VIII)

$$
b = b_{\text{rep}}\cdot\omega
$$

N Optimization

To optimize for policy parameters, we use the following algorithm: First, we use the global Covariance Matrix Adaptation Evolution Strategy (CMA-ES) with 8 random starting points. Then we select the best 4 solutions and use a local trust region optimization algorithm to further optimize these candidates. As the local optimization algorithm, we rely on BOBYQA (Bound Optimization BY Quadratic Approximation). Finally, we select the best solution among these 4 candidates as the optimal policy parameters. This procedure is conducted independently for each value of p. Note that, since the Ramsey planner must obey the equilibrium conditions, each welfare evaluation in these optimization algorithms requires solving the model for the given policy parameters. We rely on an evenly spaced grid for p of 96 points between 0 and 1.

O Joint Optimal Design of STW and Layoff Taxes

O.1 Ramsey Problem

For $\beta \to 1$, the Ramsey planner's problem reads:

$$\max_{b, s_{stw}, \epsilon_{\text{stw}}} n^u \cdot u(c^w) + n^c \cdot u^c + (1-n) \cdot u(b) \\ + v^f \cdot u((n \cdot (z - \Omega) - n^u \cdot c^w - n^c \cdot e^c - \tau(b) - \theta \cdot (1-n) \cdot k_v)/v^f)$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$n^u = \frac{1-p}{\lambda} \cdot (1 - \rho^u) \cdot n^s$$

(II) Separation rate (unconstrained workers):

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

(IV) Separation rate (constrained workers):

$$\rho^c = G\left(\max\{\xi_s^c, \epsilon_{\text{stw}}\}\right) - G\left(\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}\right) + G(\xi_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1 - n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of a constrained worker:

$$u^c = \frac{1}{1 - \rho^c} \Bigg((1 - G(\epsilon^p)) \cdot u(c^w) + \int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon_p} u(c^w(\epsilon))\, dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} u(c^w_{\text{stw}}(\epsilon)) dG(\epsilon) \Bigg)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1 - \rho^c} \Bigg[(1 - G(\epsilon^p)) \cdot c^w + \int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon_p} c^w(\epsilon)\, dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \big(c^w_{\text{stw}}(\epsilon) - s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon)) \big)\, dG(\epsilon) \Bigg]$$

(X) Average production (without distortions):

$$z = \frac{1}{1 - \rho} \Bigg((1 - p) \int_{\xi_s^u}^{\infty} z(\epsilon)\, dG(\epsilon) + p \int_{\max\{\xi_s^c, \epsilon_{\text{stw}}\}}^{\infty} z(\epsilon)\, dG(\epsilon) + p \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} z(\epsilon)\, dG(\epsilon) \Bigg)$$

(XI) Average distortion of working hours:

$$\Omega = \frac{1}{1 - \rho} \Bigg((1 - p) \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} \Omega(\epsilon)\, dG(\epsilon) + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \Omega(\epsilon)\, dG(\epsilon) \Bigg)$$

$$\text{with} \quad \Omega(\epsilon) = \bar{z}(\epsilon) - z_{\text{stw}}(\epsilon)$$

(XII) Job-creation condition:

$$\frac{1}{1-\eta} \cdot \frac{k_v}{q} = \frac{1}{\lambda} \Bigg[(1-\rho)(z - \Omega - \frac{1-n}{n} b) \\ + p(1-\rho^u) \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\ + (1-p)(1-\rho^c) \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\ + (1-\rho) \cdot \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} \Bigg]$$

(XIII) Wage:

$$WE = (1-p) \cdot (1-\rho^u) \left(\eta \cdot c^w + (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \right) \\ - \eta(1-\rho) \left(z - \Omega - \frac{1-n}{n} b + \theta \cdot k_v \right) \\ + p \cdot (1-\rho^c) \left[(1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} + \eta \cdot e^c \right] = 0$$

(XIV) Separation condition for an unconstrained firm without STW:

$$z(\xi_s^u) + \lambda \cdot F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XV) Separation condition for an unconstrained firm with STW:

$$z_{\text{stw}}(\epsilon_s^u) + s_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \lambda \cdot F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XVI) Threshold at which the constraint is binding:

$$z(\epsilon_p) + \lambda \cdot \frac{k_v}{q} = c^w$$

(XVII) Separation condition, constrained firm without STW:

$$\frac{u(c^w(\xi_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XVIII) Separation condition, constrained firm with STW:

$$\frac{u(c^w_{\text{stw}}(\epsilon^c_s)))}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XIX) Consumption, constrained worker outside STW

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \frac{k_v}{q}$$

(XX) Consumption, constrained worker on STW

$$c^w_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) + s_{stw}(\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \frac{k_v}{q}$$

Forming the FOCs for the separation thresholds with and without STW and the FOCs related to labor market tightness and employment, we can calculate the Lagrange multipliers for the separation thresholds and express them as:

$$
\begin{aligned}
-\frac{\lambda_{\xi_s^u}}{n^s \cdot u'(c^w)} &= \mathbb{1}\{\epsilon_{stw} < \xi_s^u\} \cdot \frac{1-p}{\lambda} \cdot g(\xi_s^u) \cdot \frac{1}{\frac{\partial S^u_{\text{stw}}(\xi_s^u)}{\partial \xi_s^u}} \cdot \Bigg[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\
&\quad + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \xi_s^c}\right) \Bigg] \\
-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} &= \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^u\} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \\
&\quad \cdot \Bigg[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F - s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^u)\right)\right) \\
&\quad + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right) \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} &= \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^c\} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \\
&\quad \cdot \Bigg[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^c)\right)\right) \\
&\quad + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right) \Bigg]
\end{aligned}
$$

The optimality condition for the layoff tax shows that the Lagrange multiplier of the separation thresholds for the unconstrained firms with and without STW has to add up to 0:

$$
\lambda_{\epsilon_s^u} + \lambda_{\xi_s^c} = 0
$$

The FOC for the STW benefits gives:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial s_{stw}} &= \frac{p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \left(u'(c_{\text{stw}}(\epsilon)) - u'(c^w)\right) \left(\bar{h} - h_{\text{stw}}(\epsilon)\right) \, dG(\epsilon) \\
&\quad + \lambda_\theta \cdot \frac{p}{\lambda} \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\right) \left(\bar{h} - h_{\text{stw}}(\epsilon)\right) \, dG(\epsilon) \\
&\quad - \frac{n^s \cdot (1-\rho)}{\lambda} \cdot u'(c^w) \cdot \frac{\partial \Omega}{\partial s_{stw}} - \lambda_\theta \cdot \frac{(1-\rho)}{\lambda} \cdot \frac{\partial \Omega}{\partial s_{stw}} - \lambda_c \cdot \frac{\partial WE}{\partial s_{stw}} \\
&\quad - \lambda_{\epsilon_s^u} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial s_{stw}} - \lambda_{\epsilon_s^c} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial s_{stw}} = 0
\end{aligned}
$$

Rearranging gives:

$$MIE_{s_{stw}} - MWL_{s_{stw}} - \frac{\partial WE}{\partial s_{stw}} \cdot \lambda_c = \lambda_{\epsilon_s^u} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial s_{stw}} + \lambda_{\epsilon_s^c} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial s_{stw}}$$

For the subsequent paragraphs, let us introduce the following notation:

$$\mathbb{1}\{\epsilon_{stw} < \xi_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \xi_s^u}\right) = \mathbb{1}\{\epsilon_{stw} < \xi_s^u\} \cdot \frac{1-p}{\lambda} \cdot g(\xi_s^u) \cdot n^s \cdot \frac{1}{\frac{\partial S^u_{\text{stw}}(\xi_s^u)}{\partial \xi_s^u}}$$

$$\mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) = \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^u\} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot \frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}}$$

$$\mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^c\} \cdot \left(-\frac{\partial n^c}{\partial \epsilon_s^c}\right) = \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^c\} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}}$$

$$BE_\xi^u = \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \xi_s^c}\right)$$

$$BE_\epsilon^u = \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)$$

$$BE_\epsilon^c = \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)$$

Insert the Lagrange multiplier of the separation conditions of unconstrained firms with and without STW into the optimality condition for the layoff tax:

$$\begin{aligned}
&\mathbb{1}\{\epsilon_{stw} < \xi_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \xi_s^u}\right) \cdot \left[\left(1 + \frac{\lambda_\theta}{n^u \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + BE_\xi^u\right] \\
&+ \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right) \cdot \left[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F - s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^u)\right)\right) + BE_\epsilon^u\right] = 0
\end{aligned}$$

Rearranging gives the formula for optimal layoff taxes:

$$F = T_{UI} \cdot b - (1 - \omega^\xi) \cdot T_{stw} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^c)\right) + BE_F$$

with

$$BE_F = \omega^\xi \cdot \frac{BE_\xi^u}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + (1 - \omega^\xi) \cdot \frac{BE_\epsilon^u}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)}$$

$$\omega^\xi = \frac{\mathbb{1}\{\epsilon_{stw} < \xi_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \xi_s^u}\right)}{\mathbb{1}\{\epsilon_{stw} < \xi_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \xi_s^u}\right) + \mathbb{1}\{\epsilon_{stw} \geq \epsilon_s^u\} \cdot \left(-\frac{\partial n^u}{\partial \epsilon_s^u}\right)}$$

Next, substitute the Lagrange multipliers of the separation conditions with STW for constrained and unconstrained firms into the first-order condition for STW benefits:

$$\begin{aligned}
& MIE^c_{s_{stw}} - MWL_{s_{stw}} - \frac{\partial WE}{\partial s_{stw}} \cdot \lambda_c \\
= \quad & MMS^u_{s_{stw}} \cdot \left[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F - s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^u_s)\right)\right) + BE^u_\epsilon \right] \\
& + MMS^c_{s_{stw}} \cdot \left[\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^c_s)\right)\right) + BE^c_\epsilon \right]
\end{aligned}$$

Rearrange:

$$\bar{s}_{stw} = \frac{T_{UI}}{T_{stw}} \cdot b - \omega^u \cdot \lambda \cdot F + \frac{MIE^c_{s_{stw}}}{MMS_{s_{stw}}} - \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} + BE_{stw}$$

with

$$\begin{aligned}
\bar{s}_{stw} &= \omega^u \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^u_s)\right) + (1 - \omega^u) \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^c_s)\right) \\
BE_{stw} &= \omega^u \cdot \frac{BE^u_\epsilon}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + (1 - \omega^u) \cdot \frac{BE^c_\epsilon}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + \lambda_c \cdot \frac{\partial WE}{\partial s_{stw}} \\
\omega^u &= \frac{MMS^u_{s_{stw}}}{MMS_{s_{stw}}} \\
MMS_{s_{stw}} &= MMS^u_{s_{stw}} + MMS^c_{s_{stw}}
\end{aligned}$$

Proof Corollary 4 (i) Suppose that all constrained firms on the brink of separation are allowed to enter STW. That is, $\epsilon_{stw} \geq \xi^c_s$. This also implies that all unconstrained firms are able to enter STW so that no worker without access to STW is separated, so that $\omega^\xi = 0$. This simplifies the expression of the optimal layoff tax to:

$$F = T_{UI} \cdot b - T_{stw} \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^c_s)\right) + BE_F$$

We can now insert this expression into the expression for the optimal STW benefits:

$$\begin{aligned}
\bar{s}_{stw} = (1 - \omega^u) \cdot \frac{T_{UI}}{T_{stw}} \cdot b + \omega^u \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon^u_s)\right) + \frac{MIE^c_{s_{stw}}}{MMS_{s_{stw}}} - \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} \\
+ (1 - \omega^u) \cdot \frac{BE^c_\epsilon}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + \lambda_c \cdot \frac{\partial WE}{\partial s_{stw}}
\end{aligned}$$

Rearranging gives:

$$s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^c)\right) = \frac{T_{UI}}{T_{stw}} \cdot b + \frac{1}{1-\omega^u} \cdot \left(\frac{MIE^c_{s_{stw}}}{MMS_{s_{stw}}} - \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}}\right) + \frac{BE^c_\epsilon}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + \frac{\lambda_c}{1-\omega^u} \cdot \frac{\partial WE}{\partial s_{stw}}$$

Define:

$$BE'_{stw} = \frac{BE^c_\epsilon}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + \frac{\lambda_c}{1-\omega^u} \cdot \frac{\partial WE}{\partial s_{stw}}$$

Note that $1 - \omega^u = \frac{MMS^c_{s_{stw}}}{MMM_{s_{stw}}}$. This implies we can reformulate the formula to:

$$s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^c)\right) = \frac{T_{UI}}{T_{stw}} \cdot b + \frac{MIE^c_{s_{stw}}}{MMS^c_{s_{stw}}} - \frac{1}{1-\omega^u} \cdot \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} + BE'_{stw}$$

Further, note that for $p \rightarrow 0$ we must get $\omega^u \rightarrow 1$. Rearranging the formula above then gives

$$\left(\frac{MWL_{s_{stw}}}{MMS_{s_{stw}}}\right) + \lambda_c \cdot \frac{\partial WE}{\partial s_{stw}} = 0$$

as $MIE^c_{s_{stw}} = 0$ when there are no constrained firms in the economy. The equation is fulfilled when $s_{stw} = 0$.

(ii) Suppose that the government screens out all constrained firms: $\epsilon_{stw} \leq \epsilon_s^c$. This implies $\epsilon_s \leq \epsilon_s^c$, and therefore $MMS^c_{s_{stw}} = 0$, which in turn implies $\omega^u = 0$. Further, it implies that $MIE^c_{s_{stw}} = 0$. This simplifies the formula for the optimal STW benefits to:

$$\bar{s}_{stw} = \frac{T_{UI}}{T_{stw}} \cdot b - \lambda \cdot F - \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} + BE_{stw}$$

Inserting the optimal layoff tax gives:

$$s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^u)\right) = (1-\omega^\xi) \cdot s_{stw} \cdot \left(\bar{h} - h_{stw}(\epsilon_s^u)\right) - \frac{MWL_{s_{stw}}}{MMS_{s_{stw}}} + BE^u_\epsilon + \lambda_c \cdot \frac{\partial WE}{\partial s_{stw}} - \omega^\xi \cdot \frac{BE^u_\xi}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} + (1-\omega^\xi) \cdot \frac{BE^u_\epsilon}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)}$$

Guess: $s_{stw} = 0$. This implies $\xi_s^u = \epsilon_s^u$. This implies $\omega^\xi = 0$ and $\frac{\partial WE}{\partial s_{stw}} = 0$. Thus, we get:

$$\frac{MWL_{s_{stw}}}{MMS_{stw}} = 0$$

This holds for $s_{stw} = 0$, proving the guess.