Carrots or Sticks?
Short-Time Work vs. Lay-off Taxes*

Preliminary and incomplete - do not circulate

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Abstract

While unemployment insurance systems are widely used to insure workers against income losses after lay-offs, it is well known that they can inefficiently increase separations in the labor market. There are two common policy instruments that can counter this known problem: lay-off taxes and short-time work schemes. This study provides a search-and-matching framework to evaluate which of the two is the better policy tool. We show that if only a few firms are financially constrained, lay-off taxes are better because they do not distort working hours in the economy. With a large share of financially constrained firms, short-time work emerges as the superior tool, as lay-off taxes have trouble deterring separations in financially constrained firms. Additionally, short-time work can help provide insurance against income losses to risk-averse workers that constrained firms cannot afford to provide in their wage contracts. Calibrating the model to the U.S economy, we find that short-time work is the superior policy instrument if 40% of firms in the economy or more are financially constrained.

Keywords: Short-time Work, Lay-off Taxes, Unemployment Insurance, Search and Matching, Optimal Policy

JEL: J63, J64, J65, J68

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1 Introduction

It has long been understood that unemployment insurance systems can inefficiently increase separations in the labor market. To counter this inefficiency, governments commonly have two main policy instruments at their disposal. On the one hand, there are lay-off taxes that, in effect, punish firms for firing workers. On the other, there is the short-time work system that rewards the retainment of endangered workers through subsidizing and enabling hours reductions[1].

This raises the natural question, which of the two governments should employ. Existing literature highlights that lay-off taxes have many desirable properties. (Cahuc and Zylberberg, 2008) and Blanchard and Tirole (2008) show in implicit contract frameworks that lay-off taxes can implement the planner solution. Short-time work on the other hand, has the problem of introducing new inefficiencies into the economy through the distortion of working hours (Stiepelmann (2024)). However, these results crucially hinge on the absence of financial constraints for firms. This assumption is widespread in the search and matching literature, but clearly is at odds with reality.

In this study, we relax this assumption in a rich yet analytically tractable DMP framework and allow for a share of firms to be financially constrained. Using the Ramsey policy approach, we then determine the welfare consequences of optimally set short-time work benefits and optimally set lay-off taxes.

Our main result is that lay-off taxes are indeed superior when the share of financially constrained firms is small. Short-time work emerges as the superior policy instrument if the share of constrained firms becomes sufficiently large. This is a consequence of two main channels. Firstly, lay-off taxes have trouble deterring separations in financially constrained firms, while short-time work can still operate as effectively as before. Secondly, with financial constraints, firms lose their ability to insure risk-averse workers against negative income shocks. Short-time work can then partially mitigate this and provide insurance against income shocks in the firms' stead.

Quantitatively, after calibrating the model to the U.S. economy, we find that short-time work is the superior policy instrument if 40% of firms in the economy or more are finan-

[1]Short-time work systems are pervasive, e.g., in European countries where they were utilized during the Great Recession and the Covid period. Lay-off taxes are implemented, e.g., in the U.S. through an experience-rated unemployment insurance system.

cially constrained.

The backbone of our model is a canonical Diamond-Mortensen-Pissaridis type search and matching model (Mortensen and Pissarides (1993), Mortensen and Pissarides (1994)) with idiosyncratic productivity shocks, endogenous separations, and generalized Nash-Bargaining between workers and firms. We augment the standard model with three realistic key ingredients: Risk aversion on the worker side, financial constraints on the firm side, and flexibly adjustable working hours. Workers and firms are randomly matched, form expectations over their match productivity, and bargain over wages, working hours, and separation productivity thresholds. Productivity shocks are i.i.d. and realize after firms and workers complete bargaining.

Government implements an unemployment insurance system under which unemployed workers are paid lump sum benefits and either a lay-off tax system or a short-time work scheme. Further, it collects taxes on firms to finance the systems.

In the presence of lay-off taxes, firms have to pay lump-sum lay-off taxes once worker and firm agree to separate after productivity falls below the separation threshold. Importantly, firms pay no lay-off taxes if the worker unilaterally leaves the match.

The Short-time work scheme consists of two main components: eligibility conditions and benefit payments. Workers become eligible for short-time work once they agree to reduce their working hours below a threshold specified by the government. Short-time work benefits compensate for lost income by providing fixed payments for each hour not worked relative to normal working hours. Short-time work effectively acts as a subsidy paid directly to workers. Working hours under short-time work are also determined through bargaining between firms and workers. In this setup, firms and workers have an incentive to reduce working hours inefficiently to attract more government support.

We assume that a share of firms is financially constrained. Specifically, we assume that these firms can never borrow more than the expected discounted value of the firm. These constraints have direct welfare implications. With risk-averse workers and risk-neutral firms, firms would like to offer workers insurance against low productivity shocks and commit to paying the worker a fixed (consumption equivalent) wage, no matter how low or high productivity turns out to be. The worker is then willing to accept a slightly lower wage in return for the insurance. However, if the firm is financially constrained and the borrowing constraint binds, shocks pass through to the worker's wage fully. Workers are not committed to staying in the match and quit once they hit their participation constraint (i.e. once the value from quitting, becoming unemployed, and looking for a new job is greater than staying). Therefore, the welfare effect of financial constraints is twofold: Firstly, firms cannot provide as much insurance to workers as they would like, and secondly, workers quit sooner.

We derive closed-form expressions for optimal lay-off taxes and optimal short-time work schemes, depending on how many firms are financially constrained. The theoretical results show that the sole purpose of lay-off taxes is to offset the fiscal externality of the unemployment insurance system on financially unconstrained firms. However, lay-off taxes cannot correct inefficient separations or address the lack of insurance for workers in financially constrained firms. The intuition is straightforward: once financial constraints become binding, firms can no longer absorb shocks, which are instead passed on to workers in the form of reduced income. If income falls sufficiently, workers eventually choose to quit unilaterally. In such cases, firms are neither able nor obligated to pay lay-off taxes, rendering them ineffective.

By contrast, short-time work is particularly effective in supporting financially constrained firms. By supplementing the income of workers who experience reduced hours, it incentivizes workers to remain attached to the firm. In addition, it provides income insurance.

When the share of financially constrained firms is low, lay-off taxes are clearly preferable to short-time work, as the distortionary cost of reduced working hours under short-time work outweighs the cost of insufficient support for financially constrained firms under a lay-off tax regime. However, as the prevalence of financially constrained firms increases, the trade-off shifts. Both theoretically and quantitatively, we demonstrate that short-time work becomes the superior policy instrument once a sufficiently large share (more than 40%) of firms face financial constraints.

Related literature We contribute to four branches of the literature. Firstly, we add to the literature about optimal unemployment insurance. The trade-off between the benefits of unemployment insurance and adverse effects on job-search, and separations has been a recurring theme in the economic literature (e.g. Shavell and Weiss (1979), Baily (1978)). How UI should be used optimally has therefore been examined in seminal papers such as Hopenhayn and Nicolini (1997) and Chetty (2006). More recently Landais et al. (2018) how UI should vary with labor market tightness and Kroft and Notowidigdo (2016) show evidence that moral hazard costs of UI are procyclical. We contribute to this literature by evaluating policy tools that can mitigate the adverse externalities of unemployment insurance.

Secondly, there is a branch of the literature that concerns itself with lay-off taxes, often emphasizing their effectiveness in overcoming UI fiscal externalities. Blanchard and Tirole (2008) and Cahuc and Zylberberg (2008) propose frameworks in which lay-off taxes can decentralize the planner solution. Closely related to our work, Jung and Kuester (2015) and Michau (2015) take a Ramsey planner approach and examine optimal unemployment insurance with lay-off taxes and vacancy subsidies in a DMP. Duggan et al. (2023) show that lay-off taxes can act as a stabilizer over the business cycle. Postel-Vinay and Turon (2011), on the other hand, show that employers can use severance packages to coax workers into quitting and avoid lay-off taxes. Ratner (2013) makes the point that the experience-rated UI system in the U.S. reduces layoffs but also hampers hires. Similarly, Johnston (2021) finds that increases in lay-off taxes lead to less hiring. We contribute to this literature by introducing firm borrowing constraints and showing that they reduce the effectiveness of lay-off taxes.

Thirdly, there is a growing body of research on short-time work, though the literature remains divided on its overall usefulness. Some studies emphasize potential inefficiencies. For example, Burdett and Wright (1989) argue that short-time work encourages inefficient reductions in working hours. Cooper et al. (2017) highlight the risk of subsidizing employment at unproductive firms, while Giupponi and Landais (2022) raise concerns about impeding beneficial worker reallocations, though they also note that short-time work may support firms in maintaining efficient levels of labor hoarding. Cahuc et al. (2021) raise concern about the potential windfall effects of short-time work.

Other contributions focus on the potential advantages of short-time work. Balleer et al. (2016) show that it can introduce valuable flexibility at the intensive margin of employment. Giupponi et al. (2021) argue that short-time work complements unemployment insurance by insuring against different types of labor market shocks. Similarly, Braun and Brügemann (2017) analyze optimal unemployment insurance and short-time work jointly within an implicit contract model.

Despite these contributions, the relationship between short-time work and unemployment insurance remains insufficiently understood, as emphasized in a comprehensive review by Cahuc (2024). Addressing this gap, Stiepelmann (2024) introduces the analysis of optimal short-time work policy and optimal unemployment insurance into a search-and-matching framework. Building on this foundation, our paper contributes to a better understanding of the interplay between short-time work and unemployment insurance under financial constraints.

Finally, there is a literature on firms' financial constraints. Drechsel (2023) argues that earnings-based borrowing constraints react more strongly to shocks. A part of the literature shows that financial constraints matter for monetary shock transmissions (e.g. Ottonello and Winberry (2020)) or innovation (e.g. Cascaldi-Garcia et al. (2023)). In the labor market, fewer financial constraints are empirically shown to lead to higher employment (e.g. Duygan-Bump et al. (2015), Fonseca and Van Doornik (2022)). We contribute to this literature by incorporating financial constraints into a search and matching framework with endogenous employment response.

2 Model

2.1 Basic Assumptions

Set-Up Time is discrete. There is a unit mass of workers. Workers can be either employed or unemployed. While unemployed, workers search for vacant jobs and receive unemployment benefits b. Unemployed workers and firms with vacancies are matched randomly. The number of contacts is governed by the Cobb-Douglas matching function $m(v,n) = \bar{m} \cdot v^{(1-\gamma)} \cdot (1-n)^{\gamma}$, where v is the mass of vacancies and n is the mass of employed workers. We denote the job-finding rate by f, the vacancy-filling rate by q, and the separation rate by ρ. Let θ denote labor market tightness, i.e. $\theta = \frac{v}{1-n}$.

A worker-firm match produces output y according to the production function

$$y(\epsilon, h) = A \cdot \epsilon \cdot h^{\alpha} - (\mu_{\epsilon} - \epsilon) \cdot c_f$$

where ϵ is realization of a productivity shocj $\tilde{\epsilon}$ satisfying shock $\log \tilde{\epsilon} \sim \mathcal{N}(\mu, \sigma^2)$ with cdf $G(\epsilon)$ and pdf $g(\epsilon)$. New productivity shocks arrive with probability λ. h is the number of working hours worked by the worker, and A is the total factor productivity. The term $(\mu_{\epsilon} - \epsilon) \cdot c_f$ is a cost shock. While workers are employed they receive a salary $w(\epsilon)$ and suffer dissutility $\phi(h) = \frac{h^{(1+\psi)}}{1+\psi}$ from working h hours.

Firm profits go to firm owners of whom there is a mass of ν_f.

Preferences Workers are risk-averse with a concave flow utility function over consumption, net of work disutility $u(c - \phi(h))$. Because utility is defined over consumption-equivalent units, we introduce notation for production in consumption-equivalent units for use in later expressions and let $z(\epsilon, h) = y(\epsilon, h) - \phi(h)$. Firm owners have the same flow utility function u but draw utility only from their consumption c_f (i.e. $u(c_f)$).

Government Policy The government can choose between two policy regimes: the Short-Time Work (STW) regime and the Lay-Off Tax (LT) regime. Since the chosen policy regime has implications for firm and worker value, bargaining, and equilibrium, the remaining model assumptions are described in two separate sections for each respective policy regime in the following.

2.2 The Model with Lay-off Taxes

Government Under the LT regime, firms have to pay lay-off taxes F once the worker and firm jointly agree to separate. Importantly, firms pay no lay-off taxes if the worker unilaterally leaves the match. In the following, let ρ^F denote the probability that firms and workers separate, and the firm has to pay lay-off taxes. The lay-off tax F and the unemployment insurance benefits b are set by the government. To finance the policy regime, firms pay a lump-sum tax τ whenever their productivity changes. The government budget constraint is

$$n^s \cdot \tau = (1 - n) \cdot b - n^s \cdot \rho^F \cdot F$$

where n^s is the mass of matches that receive a new productivity shock.

Firms There are two types of firms - financially constrained and financially unconstrained firms. Firms are financially constrained with probability p. When constrained, once the productivity shock has realized to ϵ, a firm can borrow no more than its expected value, conditional on its realized productivity. Specifically,

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq -\lambda \cdot \bar{J} - (1 - \lambda) \cdot J^c(\epsilon)$$

must hold. $\bar{J}$ is the expected firm value once a new shock arrives, and $J^c(\epsilon)$ is the value of a constrained firm with productivity draw ϵ. The constrained (monthly) wage function $w^c(\epsilon)$ will be made explicit in the bargaining section below. The bargained-over hours functions will depend on productivity ϵ but not on the financial constraint. Anticipating this, we save on notation and denote $h^u(\epsilon)$ and $h^c(\epsilon)$ as $h(\epsilon)$. Note that with persistent shocks ($\lambda < 1$), a low realization of productivity will lead to a tighter borrowing constraint.

The value of an unconstrained firm *after* productivity realizes to ϵ is

$$J^u(\epsilon) = y(\epsilon, h(\epsilon)) - w^u(\epsilon) + \lambda \bar{J} + (1-\lambda) J^u(\epsilon)$$

and the value of a constrained firm *after* productivity realizes to ϵ that is

$$J^c(\epsilon) = y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \lambda \bar{J} + (1-\lambda) J^c(\epsilon)$$

where $\bar{J}$ is the expected firm value *before* the shock has realized and *before* it becomes known if the firm is constrained. It is given by

$$\bar{J} = -\tau + (1-p)\left(\int_{\epsilon_s^u}^{\infty} J^u(\epsilon)\, dG(\epsilon)\right) + p\left(\int_{\epsilon_s^c}^{\infty} J^u(\epsilon)\, dG(\epsilon)\right) \quad - \rho^F \cdot F - \rho^L \cdot L$$

Firms separate from a worker once the productivity drops below the separation threshold ϵ_s^u or ϵ_s^c for the unconstrained and constrained case, respectively. With probability ρ^F, firms and workers agree on separation, and the firm has to pay a lay-off tax F. When firms become financially constrained, they also lack the means to pay lay-off taxes. In this case, we assume that they become bankrupt. In the event of bankruptcy, firm owners have to pay liquidation costs $0 < L < F$. Let ρ^L denote the probability of this event.

Firms can freely enter the labor market and post vacancies at cost k_v. The mass of vacancies v must therefore solve

$$\frac{k_v}{q} = \bar{J}$$

Workers Employed workers can work at financially constrained and unconstrained firms. The value of a worker at an unconstrained firm *after* productivity realizes to ϵ is

$$V^u(\epsilon) = u(w^u(\epsilon) - \phi(h(\epsilon))) + \lambda \bar{V} + (1-\lambda) V^u(\epsilon).$$

The value of a worker at a constrained firm *after* productivity realizes to ϵ is

$$V^c(\epsilon) = u(w^c(\epsilon) - \phi(h(\epsilon))) + \lambda\bar{V} + (1-\lambda)V^c(\epsilon)$$

where $\bar{V}$ is the expected worker value *before* a new productivity shock has realized It is given by

$$\bar{V} = (1-p)\left(\int_{\epsilon_s^u}^{\infty} V^u(\epsilon)\, dG(\epsilon)\right) + p\left(\int_{\epsilon_s^c}^{\infty} V^c(\epsilon)\, dG(\epsilon)\right) + \rho \cdot U.$$

With probability ρ a worker becomes unemployed. The Value of an unemployed worker U is given by

$$U = u(b) + f \cdot \bar{V} + (1-f) \cdot U$$

Bargaining With financial constraints, bargaining takes place before both - the realization of productivity ϵ and whether the firm is constrained or not - become known. For each possible state $\epsilon \in \mathbb{R}_+$ worker and firm agree on the total monthly wage functions $w^u(\epsilon)$ and $w^c(\epsilon)$, the hours functions $h^u(\epsilon)$ and $h^c(\epsilon)$, on the separation threshold when unconstrained ϵ_s^u, as well as the separation threshold when constrained ϵ_s^c.

Further, there is no commitment to the contract on the workers' side. This means that if the outside option of the worker becomes weakly better than staying with the match, the worker quits unitarily:

$$V^u(\epsilon) \leq U, \qquad V^c(\epsilon) \leq U$$

This will become especially prevalent in the case where firms become financially constrained. Note that when workers quit unitarily the firm does not have to pay the lay-off tax. The commitment problem of workers implies that we can denote the probability that firms and workers decide to separate and the firm does not have to pay the lay-off tax as:

$$\rho^F = (1-p) \cdot G(\epsilon_s^u) \cdot \mathbb{1}(V^u(\epsilon_s^u) > U)$$

Note that financially constrained firms do not have the means to pay lay-off taxes. In this case, they need to pay the liquidation cost L. With the commitment problem of the worker, the probability of this event is:

$$\rho^L = p \cdot G(\epsilon_s^c) \cdot \mathbb{1}(V^c(\epsilon_s^c) > U)$$

Formally, the bargaining outcome is the solution to the maximization problem

$$\max_{w^u(\epsilon), w^c(\epsilon), h^u(\epsilon), h^c(\epsilon), \epsilon_s^u, \epsilon_s^c} \bar{J}^{(1-\eta)} \left(\bar{V} - U\right)^\eta$$

subject to

$$\begin{aligned} \text{Commitment Problem:} \quad & V^u(\epsilon) > U, \quad V^c(\epsilon) > U, \\ \text{Financial Constraints:} \quad & y(\epsilon, h^c(\epsilon)) - w^c(\epsilon) \geq -\lambda \cdot \bar{J} - (1-\lambda) \cdot J^c(\epsilon) \end{aligned}$$

where η denotes the bargaining power of workers. Note that firms and workers have to take the commitment problem of the worker and the financial constraints of the firm into account when writing their contracts. One concern raised by Postel-Vinay and Turon (2011) is that firms may encourage workers to quit using severance packages, in order to circumvent lay-off taxes. To isolate the core trade-off, we abstract from such strategic behavior, effectively comparing short-time work to an idealized benchmark of a lay-off tax. Appendix B.1 derives the bargaining outcomes in detail.

Since workers are risk-averse and firms are risk-neutral, the bargaining outcome is that firms insure workers against low-productivity states. Workers will accept a lower average wage in exchange for insurance. As long as the firm is not constrained (either because it is an unconstrained firm or because constraints do not bind) the firm will *fully* insure the worker against productivity risk and the total monthly wage $w(\epsilon)$ will be set to a constant consumption equivalent c^w, equating worker utility across all realizations of ϵ that do not lead to binding constraints. c^w is pinned down by the condition

$$u'(c(\epsilon)) = u'(c^w)$$

Once a firm becomes constrained and the constraint binds, it instead pays the maximum monthly wage $c^w(\epsilon)$ it can still afford:

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \bar{J}$$

The hours function that worker and firm agree on is, as already mentioned, the same for constrained and unconstrained firms. It is pinned down by

$$A \cdot \alpha \cdot \epsilon \cdot h(\epsilon)^{\alpha-1} = h(\epsilon)^{\psi}$$

both, in the constrained and the unconstrained case. The equilibrium, therefore, only contains one general hours Function $h(\epsilon)$. Since $h(\epsilon)$ equates the marginal disutility of work and the marginal product of labor, working hours are always set efficiently.

Note that because under bargaining, each productivity level ϵ will imply a working hours level $h(\epsilon)$, we write $z(\epsilon, h(\epsilon))$ simply as $z(\epsilon)$. The job-destruction equations pin down the bargained-over separation thresholds of constrained and unconstrained matches

$$z(\epsilon_s^u) + F + \frac{u(c^v) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$
$$\frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that F enters the separation condition for unconstrained firms only. In the unconstrained case, firms insure workers' income against idiosyncratic productivity shocks such that a low idiosyncratic productivity level is not passed on to the worker. As a result, the worker has no incentive to quit unilaterally $V^u(\epsilon_s^u) > U$, and firms have to pay the lay-off tax. A low productivity level in a financially constrained firm is passed onto the income of the worker. At some point, the income received from the firm is so low that the worker quits unilaterally $V^c(\epsilon_s^c) \leq U$. Thus, the firm does not have to pay a lay-off tax in this case.

Labor Markets The separation rate ρ is given by $\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$ where $\rho^u = G(\epsilon_s^u)$ and $\rho^c = G(\epsilon_c^c)$. The steady-state law of motion for employment is

$$n = (1 - \lambda) \cdot n + (1 - \rho) \cdot n^s,$$

where n^s denotes the number of matches that received a new shock

$$n^s = f \cdot (1 - n) + \lambda \cdot n$$

The mass of unemployment workers can be expressed as $u = 1 - n$. The mass of workers who work for unconstrained firms n^u, and the mass of workers who work at constrained firms n^c are given by:

$$n^u = \frac{1-p}{\lambda} \cdot (1 - \rho^u) \cdot n^s$$
$$n^c = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

Equilibrium A steady state Equilibrium consists of the working hours function $h(\epsilon)$, the consumption equivalent c^w paid as the monthly wage to workers without binding constraints, the consumption-equivalent monthly wage $c^w(\epsilon)$ paid when financial constraints bind, the separation thresholds ϵ_s^u and ϵ_s^c, the productivity level at which the borrowing constraint becomes binding ϵ^p and labor market flows, i.e the job-finding rate f, the vacancy-filling rate q and the separation rate ρ^u and ρ^c as well as n. The exact equations pinning down equilibrium and their derivation are delegated to Section B in the appendix.

2.3 The Model with Short-Time Work

Government Under the STW regime, firms become eligible for short-time work benefits if they set their working hours on short-time work $h_{\text{stw}}(\epsilon)$ below a threshold D. In this case, the worker receives $\tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))$ worth of benefits. $\bar{h}$ is a parameter that reflects the normal working hour level. In essence, for every hour the worker works less than she would under normal circumstances, she receives compensation τ_{stw}. Choosing an hours threshold D is a close implementation of what short-time work rules mandate in e.g. Germany.[2] Enforcing working hours reduction can help screen out firms that are actually productive from receiving subsidies because only unproductive firms will find hours reductions optimal (Teichgräber et al. (2022)).

In principle, government chooses the policy parameters D, τ_{stw} and b. However, since the hours functions $h^u(\epsilon)$, $h^c(\epsilon)$, $h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$ will always be strictly decreasing in ϵ, choosing D is equivalent to setting the eligibility productivity threshold ϵ_{stw} that will

[2] In practice, this is implemented as a minimum reduction of hours threshold - which in our model is given by $\frac{D-h}{h}$ and is implicitly chosen by the government through D.

induce the same hours threshold D. The resulting government budget constraint is

$$\begin{aligned} n^s \cdot \tau = & \frac{1-p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^u}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^u\}} \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\ & + \frac{p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\ & + (1-n) \cdot b \end{aligned}$$

where n^s is the mass of matches that receive a new productivity shock. ϵ_s^u and ϵ_s^c are the separation thresholds of matches with access to short-time work of unconstrained and constrained matches, respectively. The max operators in the integral limits are necessary because the government can choose an eligibility condition that is so tight that all matches separate without gaining access to short-time work first.

Firms Again, firms can be either financially constrained or unconstrained. With short-time work, they can also currently be on short-time work or not. Firms are financially constrained with probability p. When constrained, once the productivity shock has realized to ϵ, a firm can borrow no more than its expected value, conditional on its realized productivity. Specifically, without access to short-time work

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) \geq -\lambda \cdot \bar{J} - (1-\lambda) \cdot J^c(\epsilon)$$

and with short-time work

$$y(\epsilon, h_{\text{stw}}(\epsilon)) - w_{\text{stw}}^c(\epsilon) \geq -\lambda \cdot \bar{J} - (1-\lambda) \cdot J_{\text{stw}}^c(\epsilon)$$

must hold. $\bar{J}$ is the expected firm value once a new shock arrives and $J^c(\epsilon)$ is the value of a constrained firm without short-time work and $J_{\text{stw}}^c(\epsilon)$ is the value of a constrained firm on short-time work. The constrained monthly wage function $w^c(\epsilon)$ will be made explicit in the bargaining section below. Like in the model with lay-off taxes, the bargained-over hours functions will depend on productivity ϵ but not on the financial constraint. Anticipating this, we save on notation and denote $h^u(\epsilon)$ and $h^u(\epsilon)$ as $h(\epsilon)$ and $h_{\text{stw}}^u(\epsilon)$ and $h_{\text{stw}}^c(\epsilon)$ as $h_{\text{stw}}(\epsilon)$. Further we write $z(\epsilon)$ instead of $z(\epsilon, h(\epsilon))$ and $z_{\text{stw}}(\epsilon)$ instead of $z(\epsilon, h_{\text{stw}(\epsilon)})$. The value of firm *without* short-time work *after* productivity realizes to ϵ that is unconstrained is

$$J^u(\epsilon) = y(\epsilon, h(\epsilon)) - w^u(\epsilon) + \lambda \bar{J} + (1-\lambda) J^u(\epsilon)$$

and the value of an unconstrained firm *with* short-time work *after* productivity realizes is

$$J^u_{\text{stw}}(\epsilon) = y(\epsilon, h_{stw}(\epsilon)) - w^u_{\text{stw}}(\epsilon) + \lambda \bar{J} + (1-\lambda) J^u_{\text{stw}}(\epsilon).$$

The value of firm *without* short-time work *after* productivity realizes to ϵ that is constrained is

$$J^c(\epsilon) = y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \lambda \bar{J} + (1-\lambda) J^c(\epsilon)$$

and the value of a constrained firm *with* short-time work *after* productivity realizes is

$$J^c_{\text{stw}}(\epsilon) = y(\epsilon, h_{stw}(\epsilon)) - w^c_{\text{stw}}(\epsilon) + \lambda \bar{J} + (1-\lambda) J^c_{\text{stw}}(\epsilon)$$

where $\bar{J}$ is the expected firm value *before* the shock has realized. It is given by

$$\begin{aligned}\bar{J} = -\tau + (1-p) & \left(\int_{\max\{\xi^u_s, \epsilon_{\text{stw}}\}}^{\infty} J^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\epsilon_{\max\{\epsilon^u_s, \text{stw}\}}} J^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\ + p & \left(\int_{\max\{\xi^c_s, \epsilon_{\text{stw}}\}}^{\infty} J^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon^c_s, \epsilon_{\text{stw}}\}} J^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \right)\end{aligned}$$

The max operators in the integral bounds $\max\{\xi^i_s, \epsilon_{\text{stw}}\}$ reflect that a sufficiently strict eligibility threshold ϵ_{stw} can exclude certain matches from accessing short-time work, even if their productivity would allow them to survive with such support but not without it. In this case, otherwise, viable matches are denied access to the short-time work system, leading to unnecessary separations. Firms can freely enter the labor market and post vacancies at cost k_v. The mass of vacancies v must therefore solve

$$\frac{k_v}{q} = \bar{J}$$

Workers The value of a worker at an unconstrained firm *without* short-time work *after* productivity realizes to ϵ is

$$V^u(\epsilon) = u(w^u(\epsilon) - \phi(h(\epsilon))) + \lambda \bar{V} + (1-\lambda) V^u(\epsilon)$$

and the value of a worker at an unconstrained firm *with* short-time work *after* productivity realizes is

$$V^u_{\text{stw}}(\epsilon) = u(w^u_{\text{stw}}(\epsilon) + \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) - \phi(h_{\text{stw}}(\epsilon))) + \lambda \bar{V} + (1-\lambda) V^u_{\text{stw}}(\epsilon)$$

The value of worker *without* short-time work *after* productivity realizes to ϵ who works at a constrained firm is

$$V^c(\epsilon) = u(w^c(\epsilon) - \phi(h(\epsilon))) + \lambda \bar{V} + (1-\lambda) V^c(\epsilon)$$

and the value of a worker *with* short-time work *after* productivity realizes who works at a constrained firm is

$$V^c_{\text{stw}}(\epsilon) = u(w^c_{\text{stw}}(\epsilon) + \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) - \phi(h^c_{\text{stw}}(\epsilon))) + \lambda \bar{V} + (1-\lambda) V^c_{\text{stw}}(\epsilon)$$

where $\bar{V}$ is the expected worker value *before* a new productivity shock has realized. It is given by

$$\begin{aligned}
\bar{V} = (1-p) &\left(\int_{\max_{\{\epsilon_{\text{stw}}, \xi^u_s\}}}^{\infty} V^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon^u_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^u_s\}} V^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\
&+ p \left(\int_{\max\{\epsilon_{\text{stw}}, \xi^c_s\}}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}}, \epsilon^c_s\}} V^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \right) \\
&+ \rho \cdot U
\end{aligned}$$

where ρ is the separation rate. The Value of an unemployed worker U is given by

$$U = u(b) + f \cdot \bar{V} + (1-f) \cdot U$$

Bargaining Bargaining takes place before both - the realization of productivity ϵ and whether the firm is constrained or not - become known. With short-time work, there are a few more items that workers and firms bargain about than in the lay-off tax regime.

Further, there is no commitment to the contract on the workers' side. This implies that workers will unilaterally choose to leave the match once the outside option of becoming unemployed offers a weakly higher expected utility than remaining employed at the firm.

$$V^u(\epsilon) \leq U, \qquad V^u_{stw}(\epsilon) \leq U$$
$$V^c(\epsilon) \leq U, \qquad V^c_{stw}(\epsilon) \leq U$$

In the case with and without short-time work, respectively. For each possible state $\epsilon \in \mathbb{R}_+$ worker and firm agree on the total monthly wage functions $w^u(\epsilon)$ and $w^c(\epsilon)$, as well as the total monthly wages paid in both cases while on short-time work $w^u_{\text{stw}}(\epsilon)$ and $w^c_{\text{stw}}(\epsilon)$. The hours functions, likewise, are bargained over separately - for times in which the firm has no access to short-time work ($h^u(\epsilon)$ and $h^c(\epsilon)$) and for times on short-time work ($h^u_{\text{stw}}(\epsilon)$ and $h^c_{\text{stw}}(\epsilon)$). Lastly, the separation thresholds are also agreed upon for both cases: access to short-time work and no access to short-time work. We denote the separation thresholds without short-time work by ξ^u_s and ξ^c_s. ϵ^u_s, and ϵ^c_s denote the separation thresholds while on short-time work.

Formally, the bargaining outcome is the solution to the maximization problem

$$\max_{\substack{w^u(\epsilon), w^c(\epsilon), w^u_{\text{stw}}(\epsilon), w^c_{\text{stw}}(\epsilon), h^u(\epsilon), \\ h^c(\epsilon), h^u_{\text{stw}}(\epsilon), h^c_{\text{stw}}(\epsilon), \epsilon^u_s, \epsilon^c_s, \xi^u_s, \xi^c_s}} \bar{J}^{(1-\eta)} \left(\bar{V} - U\right)^{\eta}$$

subject to

$$\begin{aligned}
\text{Commitment Problem:} \quad & V^u(\epsilon) > U, \;\; V^c(\epsilon) > U, \;\; V^u_{stw}(\epsilon) > U, \;\; V^c_{stw}(\epsilon) > U, \\
\text{Financial Constraints:} \quad & y(\epsilon, h^c(\epsilon)) - w^c(\epsilon) \geq -\lambda \cdot \bar{J} - (1-\lambda) \cdot J^c(\epsilon) \\
& y(\epsilon, h^c_{\text{stw}}(\epsilon)) - w^c_{\text{stw}}(\epsilon) \geq -\lambda \cdot J^c_{\text{stw}}(\epsilon) - (1-\lambda) \cdot \bar{J}
\end{aligned}$$

where η denotes the bargaining power of workers. Again, firms and workers need to take the commitment problem of the worker and the financial constraints of the firm into account when writing contracts. Appendix C.1 derives the bargaining outcomes.

Since workers are risk-averse and firms are risk-neutral, the bargaining outcome is still that firms insure workers against low-productivity states, while workers will accept a lower average wage in exchange for insurance. Much like in the lay-off tax model, as long as the firm is not constrained (either because it is an unconstrained firm or because constraints do not bind) the firm will *fully* insure the worker against productivity risk and the total monthly wage $w(\epsilon)$ will be set to a constant consumption equivalent c^w, equating worker utility across all realizations of ϵ that do not lead to binding constraints. This also includes the state where workers are on STW:

$$u'(c^w(\epsilon)) = u'(c^w_{stw}(\epsilon)) = u'(c^w)$$

Once a firm becomes constrained and the constraint binds, it instead pays the maximum monthly wage $c^w(\epsilon)$ it can still afford. This will depend on whether the firm has access to short-time work or not:

$$\begin{aligned} c^w(\epsilon) &= z(\epsilon) + \lambda \cdot \frac{k_v}{q} \\ c^w_{\text{stw}}(\epsilon) &= z_{\text{stw}}(\epsilon) + \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \frac{k_v}{q}. \end{aligned}$$

When the firm goes on short-time work, the worker is compensated for the reduced working hours by the government. This means that when the firm has a binding borrowing constraint and can only afford to pay the worker $c_{\text{stw}}(\epsilon)$ in monthly wages, going on short-time work can still increase the worker's income. The hours function that worker and firm agree on also differs depending on whether the firm is on short-time work or not. The two cases are pinned down by the conditions

$$\begin{aligned} A \cdot \alpha \cdot \epsilon \cdot h(\epsilon)^{\alpha-1} &= h(\epsilon)^{\psi} \\ A \cdot \alpha \cdot \epsilon \cdot h_{\text{stw}}(\epsilon)^{\alpha-1} &= h_{\text{stw}}(\epsilon)^{\psi} + \tau_{\text{stw}}. \end{aligned}$$

Again, the hours functions turn out to be independent of financial constraints and equilibrium; therefore, they only contain the two general hours functions $h(\epsilon)$ and $h_{\text{stw}(\epsilon)}$. Note that without short-time work, the bargaining outcome for working hours equates to the marginal product of labor and the marginal disutility from labor and thus sets working hours efficiently. With short-time work, working hours take the subsidy τ_{stw} into account, and hours are set lower than the efficient number of working hours. This means that through inefficiently low working hours, the short-time work scheme introduces a distortion into the economy. The bargained-over separation thresholds are pinned down by the job-destruction equations

$$
\begin{aligned}
z(\xi_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} &= 0 \\
z(\epsilon_s^u) + \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} &= 0 \\
\frac{u(c(\xi_s^u)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1 - \eta} \cdot \frac{k_v}{q} &= 0 \\
\frac{u(c_{stw}(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1 - \eta} \cdot \frac{k_v}{q} &= 0.
\end{aligned}
$$

In unconstrained firms, the separation decision is determined as a bargaining outcome: firms and workers separate once their joint surplus becomes negative. Importantly, short-time work benefits are not included in the worker's utility function. The rationale is straightforward—under STW, the firm continues to insure the worker's income against idiosyncratic productivity shocks. The worker receives a constant consumption equivalent c^w, regardless of the realization of ϵ. As a result, the worker has no incentive to quit unilaterally. STW operates by reducing the wage burden on the firm required to sustain this consumption level. In effect, STW lowers the cost of providing income insurance, thereby reducing the firm's incentive to initiate separations.

In unconstrained firms, the worker's commitment problem becomes binding: low productivity realizations are passed through to the worker's income. If income falls sufficiently, the worker chooses to quit the match. Short-time work increases the worker's available income in such cases, thereby increasing the worker's incentive to remain with the firm.

Labor Markets The separation rate ρ is given by $\rho = (1-p)\cdot\rho^u + p\cdot\rho^c$ where $\rho^u = G(\epsilon_s^u) + G(\max\{\xi_s^u, \epsilon_{\text{stw}}\}) - G(\max\{\epsilon_s^u, \epsilon_{\text{stw}}\})$ and $\rho^c = G(\epsilon_c^c) + G(\max\{\xi_s^c, \epsilon_{\text{stw}}\}) - G(\max\{\epsilon_s^c, \epsilon_{\text{stw}}\})$. The remaining worker flows are given by $f = \frac{m}{1-n}$ and $q = \frac{m}{v}$. The steady-state law of motion for employment is

$$n = (1 - \lambda) \cdot n + (1 - \rho) \cdot n^s,$$

where n^s denotes the number of matches that received a new shock

$$n^s = f \cdot (1 - n) + \lambda \cdot n$$

Unemployment can be denoted as $u = 1 - n$. The number of workers who work for unconstrained firms n^u, and the number of workers who work at constrained firms n^c are given by:

$$n^u = \frac{1-p}{\lambda} \cdot (1 - \rho^u) \cdot n^s$$
$$n^c = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

Equilibrium A steady state Equilibrium consists of the working hours functions $h(\epsilon)$, $h_{\text{stw}}(\epsilon)$, the consumption equivalent c^w paid as the monthly wage to workers without binding constraints, the consumption-equivalent wage $c^w(\epsilon)$ and c^w_{stw} paid when financial constraints bind, the separation thresholds ξ^u_s, ξ^c_s, ϵ^u_s and ϵ^c_s and labor market flows, i.e the job-finding rate f, the vacancy-filling rate q and the separation rates ρ^u and ρ^c as well as n. The exact equations pinning down equilibrium and their derivation are delegated to Section C in the appendix.

3 Optimal Policy

3.1 The Ramsey Problem

To show the differences in how short-time work and lay-off taxes act against the fiscal externality of UI on separations, we take a Ramsey planner approach. We set up separate Ramsey problems for the lay-off tax regime and the short-time work regime and proceed to compare optimal policies and their implications. In both cases, the Ramsey planner maximizes welfare, subject to the equilibrium constraints stated in Sections B and C in the appendix.

To state the welfare function formerly, recall that there is a mass of firm owners v^f. Firm owners have the same preferences over consumption as workers and receive equal shares of firm profits in the economy. Further, let Ω denote the *average* loss of production in consumption equivalent units due to hours distortions:

$$\Omega = \frac{1}{1-\rho}\left((1-p)\cdot \underbrace{\int_{\epsilon_s^u}^{\max\{\epsilon_{\text{stw}},\epsilon_s^u\}} \Omega(\epsilon)\, dG(\epsilon)}_{:=(1-\rho^u)\Omega^u} + p\cdot \underbrace{\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \Omega(\epsilon)\, dG(\epsilon)}_{:=(1-\rho^c)\Omega^c}\right)$$

where

$$\Omega(\epsilon) = z(\epsilon) - z_{\text{stw}}(\epsilon).$$

Stiepelmann (2024) shows that $\Omega \geq 0$, $\frac{\partial \Omega}{\partial \tau_{\text{stw}}}$ and $\frac{\partial \Omega}{\partial \epsilon_{\text{stw}}}$.

The welfare function is given by

$$\begin{aligned} W(\pi) &= n^u \cdot u(c^w) + n^c \cdot u^c + (1-n)\cdot u(b) \\ &\quad + v^f \cdot u((n\cdot(z-\Omega) - n^u \cdot c^w - n^c \cdot e^c - \tau(b) - \theta\cdot(1-n)\cdot k_v)/v^f) \end{aligned} \tag{1}$$

where n^u is the mass of workers employed at firms that do not hit their constraint and n^c is the mass of workers whose employers have a binding borrowing constraint. u^c is the *average* utility of a worker at a firm that has hit its borrowing constraint. z is the *average* production, net of work-disutility, and Ω is the *average* loss of production in consumption equivalent units due to hours distortions: Under the lay-off tax regime, $\Omega = 0$ will always hold. The *average* total monthly wage of a worker at a firm that has hit its borrowing constraint is e^c. The UI-tax $\tau(b) = (1-n)\cdot b$ is used to finance the UI system.

π is the vector of policy parameters the planner can choose, i.e. $\pi = (b, F)$ in the lay-off tax regime and $\pi = (b, \epsilon_{\text{stw}}, \tau_{\text{stw}})$ under the short-time work regime. The problem of the Ramsey planner is given by

$$\max_{\pi} W(\pi) \quad \text{s.t. equilibrium conditions are fulfilled}$$

The full problem is stated in Section D in the appendix. Since the analysis does not focus on distributional effects between firm owners and workers, we set the mass of firm owners ν^f such that the consumption equivalent of workers always equals the consumption of firm owners.

3.2 The Optimal Lay-Off Tax Regime

Before we turn our attention to how F is set optimally, we first look at how the planner sets b in the presence of a lay-off tax. This allows us to illustrate how a lay-off tax interacts with the fiscal externalities caused by unemployment insurance.

Setting the unemployment insurance level b must balance a trade-off. On the one hand, the Ramsey planner would like to insure the risk-averse worker against income losses after a separation. If there were no welfare costs to increasing b, the Ramsey planner would like to set b to a level that fully equates the utility of employed and unemployed workers. However, as is well known, unemployment insurance leads to fiscal externalities.

Increasing b will lead to more separations as the workers' outside option (i.e., U) becomes more attractive. This will be true for workers at unconstrained and constrained firms. This decreases the expected firm value, and vacancy posting falls as well. On top of that, with risk-averse workers, constrained firms will lose some of their ability to insure workers against bad productivity shocks because with higher b, the continuation value of a firm will be smaller, further tightening the borrowing constraint. The resulting trade-off between more worker insurance and greater fiscal externalities is balanced by the Ramsey planner's first order condition.

Proposition 1. *The optimal unemployment insurance benefit b given lay-off tax F must fulfill the following first-order condition:*

$$\underbrace{(1-n)\cdot\left(\frac{u'(b)-u'(c^w)}{u'(c^w)}\right)}_{MUIB} = \underbrace{\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge}\cdot u\cdot L_v(b)}_{MLV} + \underbrace{\left(\frac{\mathrm{d}\epsilon_s^u}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^u(p)}{\partial\epsilon_s^u}\cdot L_s^u(b,F)}_{MLS^u}$$
$$+\underbrace{\left(\frac{\mathrm{d}\epsilon_s^c}{\mathrm{d}b}\right)^{ge}\cdot\frac{\partial n^s(p)}{\partial\epsilon_s^c}\cdot L_s^c(b)}_{MLS^c} + \underbrace{n^c(p)\cdot\left(-\left(\frac{\mathrm{d}\theta}{\mathrm{d}b}\right)^{ge}\right)\cdot\hat{IE}_\theta}_{MIE^c}$$

where

$$L_v = \left(\frac{\eta - \gamma}{(\eta - \gamma)(1 - \eta)} \cdot \bar{J} + \frac{1}{f} \cdot b \right)$$

$$L_s^u = \lambda \cdot \left(\frac{1}{f} \cdot b - F \right)$$

$$L_s^c = \frac{\lambda}{f} \cdot b$$

$$\hat{IE}_\theta = \frac{1}{1 - \rho^c} \left(\int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right) \cdot k_v$$

$$\frac{\partial n^u(p)}{\partial \epsilon_s^u} = \frac{1 - p}{\lambda} \cdot n^s \cdot g(\epsilon_s^u)$$

$$\frac{\partial n^c(p)}{\partial \epsilon_s^c} = \frac{p}{\lambda} \cdot n^s \cdot g(\epsilon_s^c)$$

PROOF: See Section E.6 in the appendix.

The term labeled $MUIB$ represents the marginal social benefit of a higher unemployment benefits level b, stemming from improved income insurance for unemployed workers.

The term labeled MLV captures the marginal social loss associated with a decline in hiring induced by an increase in unemployment insurance benefits. The term comprises two components. First, L_v denotes the social value of hiring one additional worker. Second, $\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge} \cdot u$ represents the reduction in the number of new hires. Intuitively, higher unemployment benefits raise the worker's outside option, which puts upward pressure on wages and reduces the value of a worker for the firm. As a result, firms find it less profitable to post vacancies, leading to a decline in job creation.

The MLS terms denote the marginal social loss resulting from increased separations caused by UI benefits. Specifically, MLS^u captures the social cost arising from increased separations in unconstrained firms, while MLS^c captures the corresponding cost in constrained firms. Intuitively, higher unemployment insurance benefits raise workers' outside options, exerting upward pressure on wages. As a result, unconstrained firms want to initiate separations early. In constrained firms, workers are less willing to accept income reductions and thus quit into unemployment sooner.

The term MLS^u consists of two components. First, L_s^u denotes the social value of the marginal match at an unconstrained firm. Second, $\left(\frac{\mathrm{d}\epsilon_s^u}{\mathrm{d}b}\right)^{ge} \cdot \frac{\partial n^u(p)}{\partial \epsilon_s^u}$ represents the number of additional separations in unconstrained firms.

Similarly, MLS^c consists of L_s^c, the social value of the marginal match at a constrained firm, and $\left(\frac{\mathrm{d}\epsilon_s^c}{\mathrm{d}b}\right)^{ge} \cdot \frac{\partial n^c(p)}{\partial \epsilon_s^c}$ which denotes the number of additional constrained matches lost due to higher UI benefits.

Finally, M^c labels the social loss of worker income insurance in financially constrained firms consisting of two parts: $\left(-\left(\frac{\mathrm{d}\theta}{\mathrm{d}b}\right)^{ge}\right) \cdot \hat{IE}_\theta$ denotes the average social loss arising from financially constrained firms' reduced capacity to provide income insurance[3] while n^c denotes the number of constrained firms. Intuitively, higher UI benefits reduce the continuation value of a worker for a firm. Firms can thus borrow less in bad periods, reducing their ability to insure the worker.

Note that, in general, all these loss terms consist of the marginal social effect of b on the respective threshold (or labor market tightness in the case of MIE^c) multiplied by the mass of affected workers.

The lay-off tax F enters the policy trade-off exclusively through the MLS^u term. Importantly, it does not affect any terms associated with financially *constrained* firms. The intuition is straightforward: once firms become financially constrained, they lose their ability to insure workers against adverse productivity shocks. As described in Section 2, low productivity is then fully passed through to the worker's income. If the income falls sufficiently, the value of remaining employed falls below the value of unemployment ($V^c(\epsilon) < U$) and the worker will choose to quit into unemployment. Since lay-off taxes are paid only in cases of mutual separation, they do not apply to separations initiated unilaterally by workers who leave due to insufficient total wage offers from financially constrained firms.

As a result, lay-off taxes can only mitigate the negative fiscal externalities of UI by stabilizing the number of separations in *unconstrained* firms. This means that when $p = 0$, lay-off taxes can induce the efficient number of separations in the economy. As p increases, the lay-off taxes gradually lose their effectiveness until they lose their bite completely once $p = 1$.

On the upside, the planner can choose to completely eliminate this part of fiscal UI externalities. This can be done with the optimal Lay-off tax stated in Proposition 2.

[3] Note that ϵ^p in IE_θ denotes the productivity level at which financial constraints become binding for a constrained firm.

Proposition 2. *The optimal level of F is determined by its FOC:*

$$F = \frac{1}{f} \cdot b + BE_{lt}(b) \tag{2}$$

PROOF: See Section E.5 in the appendix.

The first component captures the fiscal externality of the unemployment insurance system. It reflects the expected UI benefits a worker receives upon entering unemployment, and thus represents the fiscal cost that each separation imposes on the UI system. BE_{lt} is a bargaining effect - an expression acknowledging that setting F will have general equilibrium effects on other equilibrium variables such as worker flows via the bargaining process. Note that the optimal lay-off tax implements the efficient number of separations in unconstrained firms as shown in Corollary 1.

Corollary 1. *(i) Optimal Lay-off taxes fully eliminate socially inefficient separations in* unconstrained *firms.*
(ii) If $p = 0$, then all inefficient separations are eliminated in the economy

PROOF: See Section E in the appendix.

The proof shows that the optimal lay-off tax sets the Lagrange multiplier for the separation condition of unconstrained firms equal to zero.

Note that our model extends the insights of Cahuc and Zylberberg (2008) and Blanchard and Tirole (2008) by embedding optimal lay-off taxes within a modern search-and-matching framework of the labor market. Both of these earlier studies show that lay-off taxes can decentralize the social planner's allocation in settings that focus solely on the separation margin. By contrast, search-and-matching models feature both a hiring margin and a separation margin. In the absence of financial constraints (i.e., when $p = 0$), our model confirms that the logic underlying lay-off taxes - namely, their ability to correct the fiscal externality associated with unemployment insurance - carries over to this richer framework by eliminating inefficient separations.

3.3 The Optimal Short-Time Work Regime

Under the short-time work regime the Ramsey planner chooses τ_{stw} and ϵ_{stw} (through D). Again, we first turn to how the planner sets the level of UI benefits b in the presence of the short-time work scheme. Setting the unemployment insurance level b must balance the same trade-off as before:

Proposition 3. *The optimal unemployment insurance benefit b given a short-time work scheme $(\tau_{stw}, \epsilon_{stw})$ must fulfill the following first-order condition:*

$$
\begin{aligned}
\underbrace{(1-n)\cdot\left(\frac{u'(b)-u'(c^w)}{u'(c^w)}\right)}_{MUIB} = &\underbrace{\left(-\frac{\mathrm{d}f}{\mathrm{d}b}\right)^{ge}\cdot u\cdot L_v(b)}_{MLV} + \underbrace{\frac{\partial n^u(p)}{\partial \epsilon_s^u}\cdot L_s^u(b,\tau_{stw})}_{MLS^u} \\
&+ \underbrace{\mathbb{1}(\epsilon_{stw}\geq \epsilon_s^c)\cdot\frac{\partial n^s(p)}{\partial \epsilon_s^c}\cdot L_{s,\epsilon}^c(b,\tau_{stw})}_{MLS^c} \\
&+ \underbrace{\mathbb{1}(\epsilon_{stw}\leq \xi_s^c)\cdot\frac{\partial n^s(p)}{\partial \xi_s^c}\cdot L_{s,\xi}^c(b,\tau_{stw})}_{MLS^c} \\
&+ n^c(p)\cdot\underbrace{\left(-\left(\frac{\mathrm{d}\theta}{\mathrm{d}b}\right)^{ge}\right)\cdot \hat{IE}_\theta(\tau_{stw},\epsilon_{stw})}_{MIE^c}
\end{aligned}
$$

where

$$
\begin{aligned}
L_v &= \left(\frac{\eta-\gamma}{(\eta-\gamma)(1-n)} \cdot \bar{J} + \frac{\lambda}{f} \cdot b \right) \\
L_s^u &= \left(\frac{\lambda}{f} \cdot b - \tau_{stw} \cdot \left(\bar{h} - h_{stw}(\xi_s^u) \right) \right) \\
L_{s,\epsilon}^c &= \left(\frac{\lambda}{f} \cdot b - \tau_{stw} \cdot \left(\bar{h} - h_{stw}(\xi^c) \right) \right) \\
L_{s,\xi}^c &= \frac{\lambda}{f} \cdot b \\
\hat{IE}_\theta &= \frac{1}{1-\rho^c} \cdot \left(\int_{\max\{\epsilon_{stw},\xi_s^c\}}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right. \\
&\quad \left. + \int_{\epsilon_s^c}^{\max\{\epsilon_{stw},\epsilon_s^c\}} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{stw}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right) \cdot k_v \\
\frac{\partial n^u(p)}{\partial \epsilon_s^u} &= \frac{1-p}{\lambda} \cdot n^s \cdot g(\epsilon_s^u) \\
\frac{\partial n^c(p)}{\partial \epsilon_s^c} &= \frac{p}{\lambda} \cdot n^s \cdot g(\epsilon_s^c)
\end{aligned}
$$

PROOF: See Section F.6 in the appendix.

Again, L_v is the social value of hiring one additional worker. L_s^u is the social value of the marginal match at an unconstrained firm. $L_{s,i}^c$ denotes the social value of the marginal match in a financially constrained firm. Importantly, constrained firms may feature two distinct separation margins: one at the threshold where matches separate with short-time work support, denoted by $i = \epsilon$, and another where they separate without STW support, denoted by $i = \xi$. The eligibility condition for STW determines which of these separation thresholds is operative. Finally, $\hat{IE}_\theta$ captures the social loss through constrained firms' reduced ability to provide workers with insurance. Each of the terms is weighted by the mass of workers it affects.

Like with the lay-off tax regime, the marginal social benefit of more generous unemployment benefits ($MUIB$) has to equal the marginal welfare losses, i.e. the marginal loss through too few vacancies being posted (MLV), too many separations in both unconstrained and constrained firms (MLS^u and MLS^c) and loss of insurance by constrained firms (MIE^c). However, there is a key difference between the FOCs under the lay-off tax regime (Proposition 1) and the FOCs under the short-time work regime. The lay-off tax parameter F entered only the MLS^u term, and lay-off tax could only counteract the adverse effect of unemployment benefits on separations in unconstrained firms. By contrast, the short-time work parameters $(\tau_{\text{stw}}, \epsilon_{\text{stw}})$ appear in all the terms except the MLV term. Unlike lay-off taxes, short-time work has an effect not only on separations in unconstrained firms, but can also act on constrained firms and counteract inefficient separations.

The core problem for financially constrained firms is their inability to absorb negative productivity shocks, which forces them to pass the resulting income loss directly onto workers. If the decline in income is large enough, workers may choose to quit. Short-time work functions as a subsidy that partially replaces this lost income, thereby incentivizing workers to remain with the firm. Through this channel, STW not only reduces inefficient separations, captured by the marginal social loss (MLS) terms but also enhances income insurance by acting on the MIE^s term.

A second difference is that the eligibility threshold ϵ_{stw} determines which firms have access to STW, leading to the MLS^c-term being split into two parts. If the eligibility condition is strict, i.e., $\epsilon_{\text{stw}} < \xi_s^c$, then some firms that could otherwise survive with STW support are excluded from the system. Naturally, UI benefits also influence the separation threshold without STW support. Moreover, if ϵ_{srw} is set even strictly to $\epsilon_{\text{stw}} < \epsilon_s^c$, then constrained firms are entirely excluded from the STW system. As a result, unemployment insurance benefits can no longer influence separations under the STW regime, eliminating the corresponding term from the equation.

It is clear from Proposition 3 that short-time work has the advantage of acting on the loss terms of constrained firms. On the downside, as argued in the bargaining paragraph of Section 2.3, firms on short-time work will distort their working hours, in turn leading to welfare losses. The Ramsey planner has to trade off the benefits of short-time work as a tool that can counteract the fiscal externalities of UI and the adverse effects of hours distortions. This is reflected in how optimal short-time work is set.

We begin by examining how the eligibility of STW should be determined. The following Proposition describes the optimal eligibility condition:

Proposition 4. *Assume that the Ramsey planner never wants to use short-time work to* *impede vacancy posting by destroying the efficiency of the match.*[4] *Depending on where the optimal eligibility threshold* $\epsilon^{\star}_{stw}$ *is set, relative to the different separation thresholds, its optimality condition differs. There are three cases to distinguish. For each case, the following condition will pin down a candidate for* $\epsilon^{\star}_{stw}$*. Which of the candidates is the optimum can then be determined by evaluating the welfare function at the three candidate values.*

Case 1: $\epsilon^c_s \geq \epsilon_{stw} \geq \xi^u_s$

In this case $\epsilon_{stw} = \xi^u_s$ *is the candidate.*

Case 2: let $I = [\max\{\epsilon^c_s, \xi^u_s\}, \xi^c_s]$ *and* $\epsilon_{stw} \in I$

In this case if

$$\underbrace{\frac{\partial n^c(p)}{\partial \epsilon_{stw}} \cdot L^c_{s,\epsilon}(\epsilon)}_{\substack{\textit{preserve matches in}\\ \textit{constrained firms}}} \quad > \quad \underbrace{n^u \cdot \frac{\partial \Omega^u}{\partial \epsilon_{stw}}}_{\substack{\textit{distort working hours}\\ \textit{in unconstrained firms}}} \quad + BE_{stw,2} \quad \forall \epsilon \in I, \tag{3}$$

then the candidate is $\epsilon_{stw} = \xi^c_s$*.* $BE_{stw,2}$ *is a term that captures general equilibrium effects through the generalized Nash bargaining process. If*

$$\frac{\partial n^c(p)}{\partial \epsilon_{stw}} \cdot L^c_{s,\epsilon}(\epsilon) \quad < \quad n^u \cdot \frac{\partial \Omega^u}{\partial \epsilon_{stw}} + \; BE_{stw,2} \quad \forall \epsilon \in I, \tag{4}$$

then the candidate is $\epsilon_{stw} = \xi^u_s$*. In the remaining case, i.e., the above inequality holds as an equality, this equality pins down* ϵ_{stw}*. In this case* $\epsilon_{stw} \in [\xi^u_s, \xi^c_s]$*.*

Case 3: $\epsilon_{stw} \geq \xi^c_s$

[4] In principle, this could happen in extreme cases of deviations from the Hosios condition. In this case, too many vacancies could introduce inefficiencies to such an extent that the Ramsey planner would use short-time work to destroy vacancies. Since this is neither realistic nor the focus of our analysis, we exclude this case. The formal corresponding assumption is stated as Inequality in Section F.5 in the appendix.

In this case, if

$$\underbrace{n \cdot \frac{\partial \Omega}{\partial \epsilon_{stw}}}_{distortion} + BE_{stw,2}$$

$$> \quad \underbrace{\frac{\partial n^c(p)}{\partial \epsilon_{stw}} \cdot \frac{\frac{u(c_{stw}(\epsilon_{stw})) - u(c(\epsilon_{stw}))}{c_{stw}(\epsilon_{stw}) - c(\epsilon_{stw})} - u'(c^w)}{u'(c^w)} \cdot [c_{stw}(\epsilon_{stw}) - c(\epsilon_{stw})]}_{additional\ insurance}$$

then the candidate is $\epsilon_{stw} = \xi_s^c$*. Otherwise, the above inequality holds as an equality and pins down* $\epsilon_{stw} \geq \xi_s^c$*.*

PROOF: See Section F.5 in the appendix.

To explain Proposition 4, one intuitive additional result is crucial, namely that constrained firms will always separate at even higher productivity levels than their unconstrained counterparts. The following lemma confirms this:

Lemma 1.

$$\epsilon_s^c < \epsilon_s^u \quad and \quad \xi_s^c < \xi_s^u$$

PROOF: See Section G in the appendix.

This presents the planner with a trade-off. Remember that any mass of firms on short-time work introduces socially sub-optimal distortion of working hours into the economy. This makes it socially costly to allow firms that would not have separated even without short-time work benefits to access short-time work.

When choosing ϵ_{stw}, the planner can choose to set the threshold leniently enough to allow constrained firms that would otherwise separate onto short-time work - i.e to $\epsilon_{\text{stw}} \geq \xi_s^c$. In this case, however, there will be a mass of unconstrained firms that can access short-time work, even though it would not separate even without it, introducing costly working hours distortions.

Alternatively, the planner can choose to ignore constrained firms and set the eligibility threshold to $\epsilon_{\text{stw}} \in [\epsilon_s^u, \xi_s^c)$ where unconstrained firms separate without access to short-time work. In this case, some constrained firms are cut off from the STW system even though they are in even more need of support than the unconstrained firms, not because of their productivity draw, but because of financial constraints that force them into separation before they become eligible for short-time work.

Proposition 4 reflects this problem. First, let us consider case 3. In this case, the eligibility condition is so loose that all firms in need of support can access short-time work. To achieve this $\epsilon_{\text{stw}} = \xi_s^c$ is sufficient. The question remains whether an even looser eligibility condition can be optimal. Because short-time work can provide income insurance to workers in financially constrained firms even before the match reaches the separation margin, it may be socially efficient to adopt a more generous eligibility condition. However, this effect could be outweighed by the additional working hours distortion introduced to both constrained and unconstrained firms, in which case the threshold remains at ξ_s^c. Our numerical results presented in Section 4.2, indeed, show that quantitatively, the distortion effect dominates and that $\epsilon_{\text{stw}} = \xi_s^c$ will be optimal.

Case 2 considers the case where the eligibility condition is set so strict that some of the firms with financial constraints are excluded from the STW system, even though they would be in need of support. At the same time all unconstrained firms that are in need of support can enter the STW system. The Ramsey planner must trade off the additional hours distortion in excess unconstrained firms against the social gains from protecting constrained firms. This trade-off is reflected by Inequality 3 and 4 and can go either way, depending on the value of p. In case the distortion effect outweighs the benefits of rescuing additional firms with financial constraints[5], the planner will choose to ignore constrained firms and choose the eligibility thresholds ξ_s^u. Only if 3 holds as an equality and the two effects are in the balance could ϵ_{stw} be between the two separation thresholds.

In case 1, the planner excludes all constrained firms. As argued by Stiepelmann (2024) $\epsilon_{\text{stw}} < \xi_s^u$ cannot be optimal as this would exclude the most productive firms in need of support from short-time work. Increasing the threshold cannot be optimal either, as this would not impede additional separations but introduce additional distortions in working

[5]Here $L_s^c(\epsilon)$ denotes the social value of a financially constrained match with productivity level ϵ.

hours.

Proposition 5. *The optimal short-time work subsidy τ_{stw} is pinned down by the first order condition*

$$\bar{\tau}_{stw} = \underbrace{\frac{\lambda}{f} b}_{\textit{Fiscal Ext.}} - \underbrace{\frac{n}{\varphi(p)} \cdot \frac{\partial \Omega}{\partial \tau_{stw}}}_{\textit{Distortion}} + \underbrace{\frac{n^c(p)}{\varphi(p)} \cdot \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{stw}\}} \left(\frac{u'(c_{stw}(\epsilon) - u'(c^w))}{u'(c^w)} \right) (\bar{h} - h_{stw}(\epsilon))\, dG(\epsilon)}_{\textit{Insurance}} + \underbrace{BE_{stw,3}}_{\textit{Bargaining Effect}}$$

with

$$\bar{\tau}_{stw} = \frac{\varphi^u(p)}{\varphi(p)} \tau_{stw}(\bar{h} - h_{stw}(\epsilon_s^u)) + \frac{\varphi^c(p)}{\varphi(p)} \tau_{stw}(\bar{h} - h_{stw}(\epsilon_s^c))$$

where $BE_{stw,\ 3}$ captures general equilibrium effects through wage setting in the generalized Nash bargaining process. $\varphi(p)$, $\varphi^c(p)$ and $\varphi^u(p)$ are weight-terms with $\varphi(p) = \varphi^u(p) + \varphi^c(p)$ that are explicitly stated in Section F.5 in the appendix.

PROOF: See Section F.5 in the appendix.

Proposition 5 reveals that the Ramsey planner sets the average net subsidy $\bar{\tau}_{\text{stw}}$ to reflect three forces (abstracting from the general equilibrium bargaining effect). The first part of the sum is exactly equal to the fiscal externality of unemployment insurance on separations, i.e. the cost that the marginal worker who separates into unemployment imposes on the UI system. The second term reflects the fact that higher subsidies enable constrained firms to offer more insurance to workers to the extent that they are eligible for short-time work. This increases the optimal level of $\bar{\tau}_{\text{stw}}$. However, larger short-time work benefits will also lead to larger working hours distortions, in turn lowering the optimal level of $\bar{\tau}_{\text{stw}}$. τ_{stw} is then determined by distributing the load of $\bar{\tau}$ on constrained and unconstrained firms, taking into account marginal welfare gains.

It is noteworthy that the Ramsey planner sets the *average net* subsidy $\bar{\tau}_{\text{stw}}$ optimally and then adjusts the actual benefits τ_{stw} to achieve this optimal level. Since constrained firms have higher separation thresholds ($\epsilon_s^c > \epsilon_s^u$), it will hold that $\bar{h} - h_{\text{stw}}(\epsilon_s^c) < \bar{h} - h_{\text{stw}}(\epsilon_s^u)$. Because $\varphi^u(p)$ is decreasing in p and $\varphi^c(p)$ is increasing in p, this means that holding ϵ_{stw} fixed, τ_{stw} will be increasing in p.

Having disentangled how the Ramsey planner uses lay-off taxes and short-time work to counteract the fiscal externalities created by unemployment insurance, it is now time to determine which is the superior policy tool. From a theoretical point of view, this cannot be definitively determined. While lay-off taxes do not introduce any inefficiencies into the economy and can in fact induce the efficient number of separations in unconstrained firms, they cannot counteract fiscal externalities on constrained firms. Short-time work, on the other hand, can act on constrained firms as well. This, though, comes at the price of distorted working hours. To determine which is more effective in practice, we focus on the quantitative analysis of the model in the remainder of the paper.

Corollary 2. *If there are no constrained firms in the economy (i.e., $p = 0$), the optimal lay-off tax increases welfare more than the optimal short-time work scheme.*

PROOF: This follows from the fact that lay-off taxes introduce no distortions but can fully eliminate inefficient separations in unconstrained firms. All inefficiencies that short-time work can act on but lay-off taxes cannot, drop out when $p = 0$.

It is also clear that short-time work is the superior policy instrument if all firms are constrained.

Corollary 3. *If there are no unconstrained firms in the economy (i.e., $p = 1$), the optimal short-time work scheme increases welfare more than the optimal lay-off tax.*

PROOF: This follows from the fact that the only inefficiency that short-time work can act on, i.e., inefficient separations in unconstrained firms, cannot occur anymore. Short-time work can still act against inefficient separations in constrained firms and provide worker insurance.

In all cases with $p \in (0,1)$, it is ambiguous which of the two policy regimes is superior without quantitative analysis, to which we turn next. It is clear, however, that as p increases lay-off taxes gradually lose their advantage over short-time work.

4 Quantitative Results

4.1 Calibration

As the utility function, we choose $u(c) = \log(c)$. We calibrate the model without either a lay-off tax or short-time work to the U.S. economy at monthly frequency. In order to evaluate the implications of short-time work and lay-off taxes in the current U.S. economy, we fix the unemployment insurance system and target the wage replacement estimate reported by Engen and Gruber (2001) for the U.S. Up to small numerical imprecisions, this can be done by reproducing the targeted moments exactly by solving a system of equations stated in Section I in the appendix. Table 1 gives an overview of our selected targets.

Target	**Description**	**Value**
q	Vacancy-filling rate	0.3381
f	Job-finding rate	0.40
ρ	Separation rate	0.1
b^{rep}	UI replacement rate	0.45

Table 1: Model Calibration Targets

Table 2: Model Parameters: Calibrated and Set Values

Parameter	Description	Value
Calibrated Parameters		
$\bar{m}$	Matching efficiency parameter	0.3832
c_f	Cost shock parameter	3.3276
k_v	Vacancy posting cost	0.3315
b	Unemployment benefit level	0.9972
$\bar{h}$	Regular working hours level	1.1616
A	Total Factor Productivity (TFP)	1.8
Set Parameters		
λ	Shock arrival probability	0.3
α	Labor elasticity in production function	0.65
μ	Location of productivity shocks	0.094
σ	Scale of productivity shocks	0.12
ψ	Disutility of labor parameter	1.5
γ	Elasticity of matching w.r.t. unemployment	0.65
η	Worker bargaining power	0.65

Notes: The set parameters are based on the following sources: α: Christoffel and Linzert (2010). ψ: targeting Frisch elasticity of 0.65 as in Domeij and Floden (2006). σ: Krause and Lubik (2007). μ: normalizing wage to 1. γ: standard value. η: Hosios condition (Hosios (1990)), reasonable set of parameters in Petrongolo and Pissarides (2001).

The first part of Table 2 shows the calibrated parameters set to achieve the targets. As shown by Costain and Reiter (2008), large-surplus calibrations are needed not to overestimate the elasticity of worker flows to policy changes. We therefore set TFP A to 1.8.

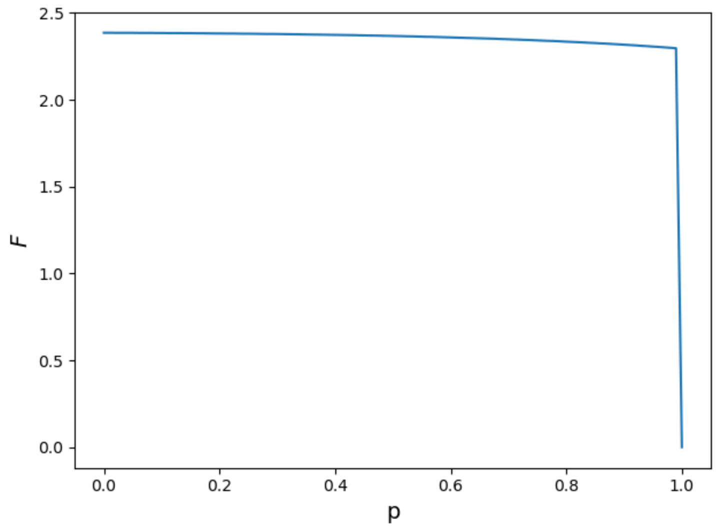
The remaining parameters are set to standard values shown in the second part of Table 2. We set $\lambda = 0.3$, so that a firm will remain at any drawn productivity level for $1/0.3 = 3.33$ months on average.

4.2 Results

To calculate the Ramsey optimal policy parameters, we maximize the welfare function (Equation 1) numerically. The details are described in Section H in the appendix.

Figure 1 shows the results for the optimal lay-off tax F across $p \in [0, 1]$. The result is what its FOC (Equation 2) lets us expect. The planner exactly offsets the fiscal externality on

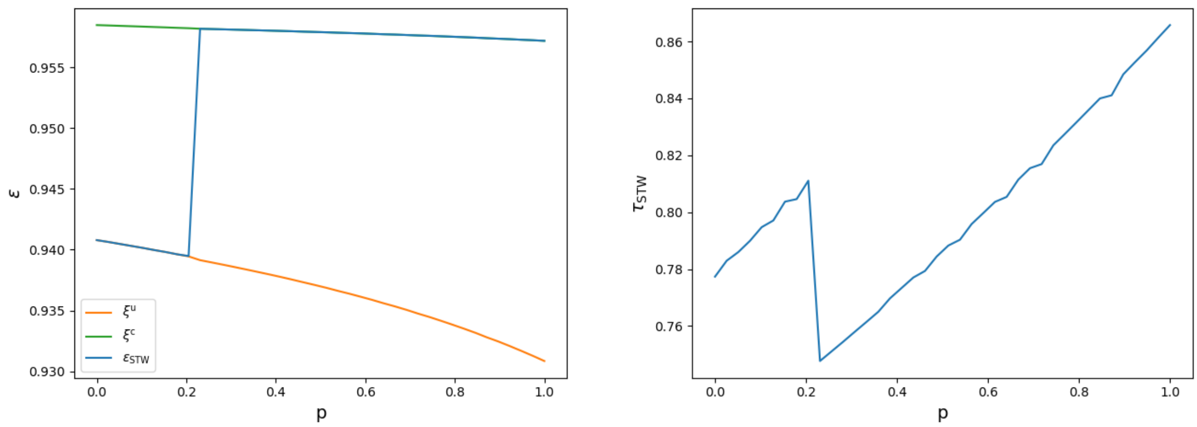
Figure 1: Optimal Lay-off Tax



Notes: Optimal lay-off tax F set by the Ramsey planner across $p \in [0, 1]$.

unconstrained firms $\frac{\lambda}{f} \cdot b$. Since this does not depend on p in any way, the level of the optimal lay-off tax F hardly changes across p. The slight downward slope is explained by general-equilibrium feedback effects in $BE_{\text{lt}}(b)$. At $p = 1$, lay-off taxes lose their bite completely as they can no longer target any inefficiencies, and in fact, no firm will have to pay them. To illustrate this, we show the optimal lay-off tax to be $F = 0$. In fact, any lay-off tax $F \in \mathbb{R}_+$ is optimal.

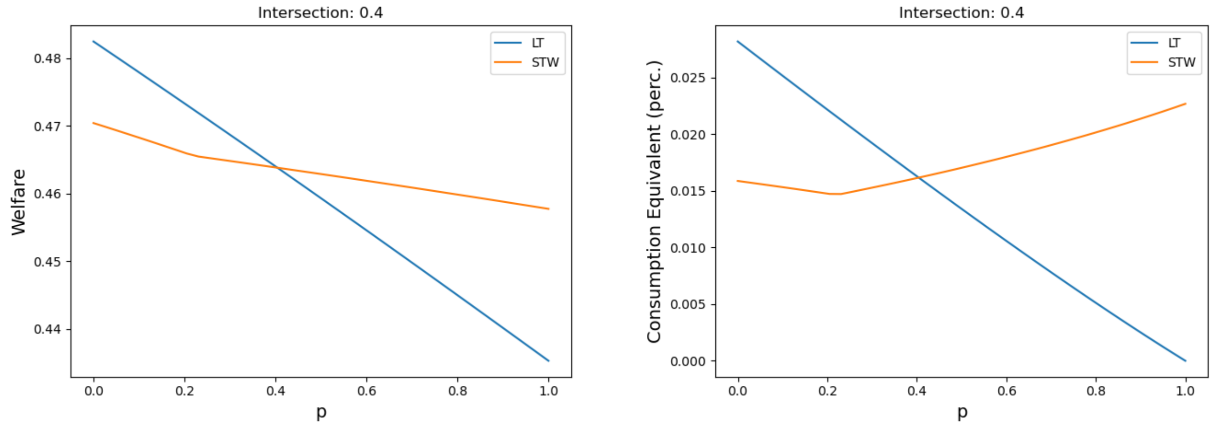
Figure 2: Optimal Short-time Work



(a) Optimal ϵ_{stw} **(b)** Optimal τ_{stw}

Notes: Optimal ϵ_{stw} and τ_{stw} set by the Ramsey planner across $p \in [0, 1]$. ξ_s^u is the separation threshold of an unconstrained firm without access to short-time work. ξ_s^c is the separation threshold of a constrained firm without access to short-time work.

Figure 2 shows the Ramsey optimal short-time work parameters across p. The way that the Ramsey planner sets the eligibility threshold reflects the trade-off described in Section 3.3 under Proposition 4: While there are only a few constrained firms in the economy, it is very costly to protect all firms, including constrained ones, from premature separations. Setting the eligibility constraint high enough to allow all constraint firms onto short-time work before they hit their separation threshold ξ_s^c will allow many unconstrained firms onto short-time work and collect windfall profits, even though they would not separate without it. As discussed, this introduces inefficient working hours distortions, and as long as only a few firms are constrained, these distortions outweigh the benefit of protecting these few constrained firms. The optimal eligibility threshold is therefore ξ_s^u. However, as the mass of constrained firms grows along the x-axis of Figure 2, the benefits from protecting constrained firms eventually outweigh the added distortion from protecting excess unconstrained firms. At this point ($p = 0.205$), the eligibility condition jumps to the threshold ξ_s^c at which constrained firms without short-time work separate.



(a) Welfare (b) Consumption Equivalent

Figure 3: Welfare Comparison

Notes: Plot (a) on the left shows welfare across $p \in [0, 1]$ of the economy with the optimal lay-off tax and the optimal short-time work regimes, respectively. Plot (b) on the right shows improvements of optimal short-time work and optimal lay-off taxes over the economy without either of these two policies in terms of consumption equivalent variation.

The optimal generosity parameter τ_{stw} is increasing in p except at the kink at $p = 0.205$. This is because, as discussed in the context of Proposition 5 in Section 3.3, the planner targets the *average* net subsidy. As constrained firms have less margin to adjust their hours, this means that τ_{stw} has to rise in p as long as ϵ_{stw} does not change much. However, when ϵ_{stw} jumps to ξ_s^c, many more firms are suddenly on short-time work, introducing greater hours distortion to the economy. To counteract that distortion, the planner reduces the τ_{stw} as the more lenient eligibility threshold is introduced.

Having established how the Ramsey planner sets both lay-off taxes and short-time work, it is now time to discuss which is the superior policy tool. Figure 2 shows the respective welfare induced by the optimal lay-off tax and the optimal short-time work schemes across p on the left. Unsurprisingly, welfare is decreasing in p in both cases as higher shares of firms become constrained. As expected, when there are no or only a few constrained firms, the lay-off tax is the superior policy tool. Crucially, however, the two curves have different slopes and intersect at $p = 0.40$. The interpretation is simple: If more than 40% of firms are financially constrained, short-time work becomes the superior policy instrument; for smaller shares of constrained firms, lay-off taxes are the better tool.

For better interpretation, Figure 3 shows consumption equivalent variation, i.e., by what share consumption would need to be increased in the economy without either short-time work or lay-off taxes to induce the same welfare level as the economy with the respective optimal policy, on the right. To calculate consumption equivalent variation Δ^{policy} we solve

$$u(c_0 \cdot (1 + \Delta^{\text{policy}})) = u(c_{\text{policy}})$$

where $c_0 = u^{-1}(W(0))$ and $c_{\text{policy}} = u^{-1}(W(\pi^\star))$. $W(0)$ is welfare without either short-time work or lay-off taxes and $W(\pi^\star)$ is welfare under the respective optimal policy. While welfare improvements in terms of consumption equivalent variation intersect at the exact same point as raw welfare functions, they carry two additional messages. Firstly, the resulting improvements of the respective superior policy scheme are substantial at every p, ranging around 2%. Secondly, as p increases, short-time work becomes not only preferable to lay-off taxes, but improvements in terms of consumption equivalent variation increase. This stems from the effect that short-time work helps firms provide insurance, which becomes increasingly important for higher p.

5 Conclusion

We set out by asking the simple question, which of the two policy tools that policy makers in many countries already employ - lay-off taxes or short-time work - is better at counteracting the fiscal externalities of unemployment insurance. While existing literature emphasizes desirable properties of lay-off taxes, we show that their effectiveness is highly sensitive to the assumption that firms are unconstrained in their ability to smooth shocks.

By introducing firm-level financial constraints into a rich yet tractable Diamond-Mortensen-Pissaridis search-and-matching framework (DMP) with endogenous separations, risk-averse workers, and flexible working hours, we show that lay-off taxes struggle with counteracting these externalities when firms are constrained. Short-time work schemes, on the other hand, can act on constrained firms but have the disadvantage of introducing new inefficiencies into the economy through distorting working hours.

Our theoretical analysis, grounded in a Ramsey policy approach, delivers closed-form expressions for optimal lay-off taxes and STW subsidies as a function of the share of financially constrained firms. We show that lay-off taxes are the superior policy instrument with no or only a few financially constrained firms, but that short-time work benefits are better if sufficient firms are financially constrained. Quantitatively, we find that when 40% or more of firms are financially constrained, short-time work dominates lay-off taxes in welfare terms.

Policy recommendations must therefore depend on the prevalence of financial constraints in an economy. Anecdotally, it seems highly plausible that in the U.S., where a lay-off tax is implemented through experience-rated unemployment insurance, financial constraints play a smaller role than in countries like Germany or France with less developed financial sectors and smaller firms where short-time work is widely used. Empirically determining the exact extent of financial constraints in different countries and the implications for optimal policy is therefore key.

Another interesting way forward is to explore how our results change over the business cycle. If the extent to which firms are borrowing constrained or the share of constrained firms changes in downturns, there could be a case for relying more on short-time work during downturns and more on lay-off taxes during good times.

We view this as a fruitful ground for future research.

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A Table of Model Parameters, Variables and Functions

Symbol	Description
n	Mass of employed workers
$u = 1 - n$	Mass of unemployed workers
v	Mass of posted vacancies
$\theta = \frac{v}{1-n}$	Labor market tightness
f	Job-finding rate
q	Vacancy-filling rate
ρ	Separation rate
ρ^u, ρ^c	Separation rates (unconstrained/ constrained)
ρ^F	Separation with lay-off tax
ρ^L	Separation via bankruptcy
p	Probability firm is constrained
λ	Probability of productivity shock
$\tilde{\epsilon}$	Productivity shock (lognormal)
$G(\epsilon), g(\epsilon)$	CDF and PDF of productivity shocks
$\bar{m}, \gamma$	Matching function parameters
A	Total factor productivity
α	Output elasticity (hours)
ψ	Inverse Frisch elasticity
ν_f	Mass of firm owners
$\bar{h}$	Reference hours under STW
D	Eligibility threshold for STW
ϵ_{stw}	Productivity threshold for STW
b	Unemployment benefits
F	Lay-off tax
L	Liquidation cost
τ	Lump-sum tax per shock
τ_{stw}	STW subsidy per hour gap
k_v	Cost of posting a vacancy
η	Bargaining power of workers

Table 3: Model Parameters, Variables and Functions Part 1

Symbol	Description
u/ c	unconstrained/ constrained
$m(v,n)$	Matching function
$\phi(h)$	Disutility of labor: $\frac{h^{1+\psi}}{1+\psi}$
$u(c)$	Worker utility from consumption net of disutility
$y(\epsilon,h)$	Output
$z(\epsilon,h)$	Output net of disutility (cons.-eq. units)
$z(\epsilon)$	Shorthand for $z(\epsilon,h(\epsilon))$
$z_{\text{stw}}(\epsilon)$	Output net of disutility under STW hours
u^c	Average utility of a worker at a constrained firm
e^c	Average total wage net of disutility paid at a constrained firm
$\Omega(\epsilon)$	Welfare Costs STW in match at productivity ϵ
Ω^u	Average welfare loss STW in unconstrained firms
Ω^c	Average welfare loss STW in constrained firms
Ω	Average welfare loss net of disutility of all firms
$h(\epsilon)$	Hours worked (non-STW)
$h_{\text{stw}}(\epsilon)$	Hours worked under STW
c^w	Constant consumption-equivalent wage
$c^w(\epsilon)$	Constrained consumption wage
$c^w_{\text{stw}}(\epsilon)$	STW-constrained wage
$w^u(\epsilon), w^c(\epsilon)$	(Monthly) Wage functions (u/ c, no STW)
$w^u_{\text{stw}}(\epsilon), w^c_{\text{stw}}(\epsilon)$	(Monthly) Wage (u/ c) under STW
$V^u(\epsilon), V^c(\epsilon)$	Worker value (u/ c, no STW)
$V^u_{\text{stw}}(\epsilon), V^c_{\text{stw}}(\epsilon)$	Worker value (STW)
$\bar{V}$	Expected worker value
U	Value of unemployment
$J^u(\epsilon), J^c(\epsilon)$	Firm value (u/ c, no STW)
$J^u_{\text{stw}}(\epsilon), J^c_{\text{stw}}(\epsilon)$	Firm value (u/ c) under STW
$\bar{J}$	Expected firm value
$\epsilon^u_s, \epsilon^c_s$	Separation thresholds (u/ c, under STW)
ξ^u_s, ξ^c_s	Separation thresholds (u/ c, no STW)
ϵ^p	Productivity where constraint binds
n^s	Mass of matches receiving new shock
n^u, n^c	Mass of workers at u/c firms

Table 4: Model Parameters, Variables and Functions Part 2

B Equilibrium Lay-off Tax

B.1 Bargaining

Nash-Bargaining Problem:

$$\max_{w^u(\epsilon),w^c(\epsilon),h^u(\epsilon),h^c(\epsilon),\epsilon_s^u,\epsilon_s^c} \bar{J}^{(1-\eta)} \left(\bar{V} - U\right)^\eta$$

subject to

1. Financial constraint of financially constrained firms:

$$y(\epsilon, h(\epsilon)) - w^c(\epsilon) + \lambda \cdot J^c(\epsilon) + (1-\lambda) \cdot \bar{J} \geq 0$$

2. Commitment problem (Worker):

$$V^c(\epsilon) > U, \quad V^u(\epsilon) > 0$$

First of all, note that we can rewrite the financial constraint as

$$z(\epsilon) - c(\epsilon) + \lambda J^c(\epsilon) + (1-\lambda)\bar{J} > 0$$

Second, we can integrate the commitment problem of the worker into the value functions for the firm and worker surplus. For $i \in \{c, u\}$, we can rearrange the condition as:

$$u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot (\bar{V} - U) \geq 0$$

Let $\epsilon_s^{i,\text{cp}}$ denote the threshold at which the worker wants to separate on his own terms:

$$u(c(\epsilon_s^{i,\text{cp}})) - u(b) + (\lambda - f) \cdot (\bar{V} - U) = 0$$

We can rewrite the value functions of the firm and the surplus of the worker as:

$$
\begin{aligned}
\bar{J}^B = & -\bar{\tau}^c + (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{1}{\lambda} \cdot \big(z(\epsilon) - c^w(\epsilon) + \lambda \bar{J}\big)\, dG(\epsilon) \\
& + (1-p) \cdot G(\epsilon_s^{u,B}) \cdot \mathbb{1}(\epsilon_s^{u,B} > \epsilon_s^{u,\text{cp}}) \cdot F \\
& + p \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{1}{\lambda} \cdot \big(z(\epsilon) - c^c(\epsilon) + \lambda \bar{J}\big)\, dG(\epsilon) \\
& + p \cdot G(\epsilon_s^{u,B}) \cdot \mathbb{1}(\epsilon_s^{u,B} > \epsilon_s^{u,\text{cp}}) \cdot L \\
(\bar{V} - U)^B = & (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{1}{\lambda} \cdot \big(u(c^u(\epsilon)) - u(b) + \lambda(\bar{V} - U)\big)\, dG(\epsilon) \\
& + p \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{1}{\lambda} \cdot \big(u(c^c(\epsilon)) - u(b) + \lambda(\bar{V} - U)\big)\, dG(\epsilon)
\end{aligned}
$$

Note that $\epsilon_s^{i,B}$ denotes the separation threshold in the bargaining outcome. In the following, we rule out the case where the consumption of the worker can be set so low that the worker quits on their own terms to avoid the lay-off tax. This means that ϵ_s^{cp} is seen as exogenous to the bargaining participants. Several lawsuits in the US show that this might, in reality, actually be a problem of lay-off taxes that we abstract from.

Therefore, lay-off taxes in this model can be interpreted as marginally effective! Further, note that financially constrained firms cannot pay lay-off taxes and go bankrupt when they are constrained and have to pay the lay-off tax. In this case, we assume some liquidation costs $0 \leq L \leq F$, borne by the firm owners. Finally, note that it is equivalent to maximize over $w^i(\epsilon)$ and $c^i(\epsilon)$ as

$$
c^i(z) = w^i(\epsilon) - \phi^i(h(\epsilon)).
$$

Set up Kuhn-Tucker Problem:

$$
\max_{c^c(\epsilon), c^u(\epsilon), h^c(\epsilon), h^u(\epsilon), \epsilon_s^{c,B}, \epsilon_s^{u,B}} \mathcal{B}
$$

where

$$
\mathcal{B} = \left(\bar{J}^B\right)^{\eta} \cdot \left(\bar{V} - U\right)^{1-\eta} - \int_0^{\infty} \mathcal{M}(\epsilon) \left[z(\epsilon) - c(\epsilon) + \lambda \cdot \bar{J} + (\lambda - f) \cdot \bar{J}(\epsilon)\right] d\epsilon
$$

With Kuhn-Tucker conditions for a maximum:

$$\begin{aligned}(I) \quad & \mathcal{M}(\epsilon) \leq 0 \\ (II) \quad & \mathcal{M}(\epsilon) \cdot \left[z(\epsilon) - c(\epsilon) + \lambda \cdot \bar{J} + (\lambda - f) \cdot \bar{J}(\epsilon)\right] = 0\end{aligned}$$

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^u(\epsilon)} &= -(1-\eta) \cdot (1-p) \cdot g(\epsilon) \cdot \left(\frac{\bar{V} - U}{\bar{J}}\right)^{\eta} \\ &\quad + \eta \cdot (1-p) \cdot g(\epsilon) \cdot u'\left(c^u(\epsilon)\right) \cdot \left(\frac{\bar{J}}{\bar{V} - U}\right)^{1-\eta} \\ &= 0\end{aligned}$$

$$\iff \frac{1-\eta}{\eta} \cdot \frac{\bar{V} - U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \quad \Rightarrow \quad u'\left(c^u(\epsilon)\right) = u'(c^w)$$

Note that the firm wants to perfectly insure workers against idiosyncratic productivity shocks as long as they are unconstrained.

Define the joint surplus as

$$\bar{S} = \bar{J} + \frac{\bar{V} - U}{u'(c^w)}$$

The resulting surplus splitting rule gives:

$$J = (1-\eta) \cdot \bar{S}, \quad \frac{\bar{V} - U}{u'(c^u)} = \eta \cdot \bar{S}$$

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^c(\epsilon)} &= -(1-p) \cdot g(\epsilon) \cdot (1-\eta) \cdot \left(\frac{\bar{V} - U}{\bar{S}}\right)^{n} \\ &\quad + (1-p) \cdot g(\epsilon) \cdot u'\left(c^u(\epsilon)\right) \cdot U \cdot \left(\frac{J}{\bar{V} - U}\right)^{1-\eta} \\ &\quad + \mathcal{M}(\epsilon) \overset{!}{=} 0\end{aligned}$$

Insert the surplus splitting rule:

$$\Rightarrow \quad -(1-p)\cdot g(\epsilon)\cdot\left(\frac{\eta}{1-\eta}\cdot u'(c^w)\right)^{\eta}$$
$$+(1-p)\cdot g(\epsilon)\cdot u'\left(c^c(\epsilon)\right)\cdot\eta$$
$$+\mathcal{M}(\epsilon)\cdot\left(u'(c^w)\cdot\frac{1-\eta}{\eta}\right)^{1-\eta}=0$$

$$\Leftrightarrow (1-p)\cdot g(\epsilon)\cdot\eta\cdot\left(u'(c^c(\epsilon))-u'(c^w)\right)=-\mathcal{M}(\epsilon)\cdot\left(u'(c^u)\cdot\frac{\eta}{1-\eta}\right)^{1-\eta}$$

Note that if $c^c(\epsilon)<c^w$, then the RHS of the equation is positive due to risk aversion. Thus, firms and workers gain joint surplus until $c^c(\epsilon)=c^w$.

If

$$z(\epsilon)-c^w+\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\geq 0$$

then the constraint is non-binding and $\mathcal{M}(\epsilon)=0$. ✓

If

$$z(\epsilon)-c^w+\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)<0$$

then the constraint is binding and $\mathcal{M}(\epsilon)<0$. ✓
Thus, it is optimal for the firm to pay as much as it can:

$$J^c(\epsilon)=z(\epsilon)-c^c(\epsilon)+\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)=0$$

$$\Leftrightarrow \quad c^c(\epsilon)=z(\epsilon)+\lambda\cdot\bar{J}$$

Using the surplus splitting rule, the Nash-Bargaining problem can be simplified to

$$\max_{w^u(\epsilon),w^c(\epsilon),h^u(\epsilon),h^c(\epsilon),\epsilon_s^u,\epsilon_s^c}(1-\eta)^{(1-\eta)}\cdot(u'(c^w)\eta)^{\eta}\cdot\bar{S}$$

That is, we just have to maximize the joint surplus with the remaining contracts:

$$
\begin{aligned}
\bar{S} &= -\bar{\tau}^c \\
&\quad + (1-p) \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\infty} \frac{1}{\lambda} \cdot \left(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (1 - \eta \cdot f) \cdot \bar{S} \right) dG(\epsilon) \\
&\quad - (1-p) \cdot G(\epsilon_s^{u,B}) \cdot \mathbb{1}(\epsilon_s^{u,B} > \epsilon_s^{u,\text{cp}}) \cdot F \\
&\quad + p \cdot \int_{\epsilon^p}^{\infty} \frac{1}{\lambda} \cdot \left(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (1 - \eta \cdot f) \cdot \bar{S} \right) dG(\epsilon) \\
&\quad + p \cdot \int_{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\text{cp}}\}}^{\epsilon_p} \frac{1}{\lambda} \cdot \left(z(\epsilon) + \frac{u(c^c(\epsilon)) - u(b)}{u'(c^w)} - c^w + (1 - \eta \cdot f) \cdot \bar{S} \right) dG(\epsilon) \\
&\quad - p \cdot G(\epsilon_s^{u,B}) \cdot \mathbb{1}(\epsilon_s^{u,B} > \epsilon_s^{u,\text{cp}}) \cdot L
\end{aligned}
$$

The FOC for working hours in unconstrained firms is:

$$
\frac{\partial \mathcal{B}}{\partial h^u(\epsilon)} = (1-p) \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} - \phi'(h^u(\epsilon)) \right) = 0
$$

$$
\iff \quad \frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} = \phi'(h^u(\epsilon))
$$

The FOC for working hours in constrained firms is:

$$
\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} = p \cdot g(\epsilon) \cdot \left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) \right) \cdot \frac{u'(c^c(\epsilon))}{u'(c^w)} = 0
$$

$$
\iff \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} = \phi'(h^c(\epsilon))
$$

Suppose that $\epsilon_s^{u^b} > \epsilon_s^{u^{cp}}$

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial \epsilon_s^{u,b}} = -(1-p) \cdot g(\epsilon_s^{u,b}) \cdot \Bigg[\frac{1}{\lambda} \cdot \Bigg(z(\epsilon_s^{u,b}) + \frac{u(c^w) - u(b)}{u'(c^w)} \\
- c^w + (\lambda - \eta f) \cdot \bar{S} \Bigg) + F \Bigg] = 0
\end{aligned}
$$

$$
\iff z(\epsilon_s^{u,b}) + \lambda \cdot F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (\lambda - \eta f) \cdot \bar{S} = 0
$$

Insert free-entry Condition:

$$z(\epsilon_s^{u,b}) + \lambda \cdot F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (\lambda - \eta f) \cdot \left(\frac{k_v}{q}\right) = 0$$

Note that $\epsilon_s^{u^b} > \epsilon_s^{u^{cp}}$ must hold, as workers would never quit under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f)(\bar{V} - U) > 0$$

It always gets positive surplus!

Suppose that $\epsilon_s^{c,B} \geq \epsilon_s^{u,cp}$

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{c,B}} = -(1-p) \cdot g(\epsilon_s^{c,B}) \cdot \Bigg[\frac{1}{\lambda} \cdot \Bigg(z(\epsilon_s^{c,B}) + \frac{u\left(c^c(\epsilon_s^{c,B})\right) - u(b)}{u'(c^w)}$$
$$- c^c(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \bar{S}\Bigg) + L\Bigg] = 0$$

$$\iff z(\epsilon_s^{c,B}) + \lambda \cdot L + \frac{u(c^c(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \bar{S} = 0$$

Insert free-entry condition:

$$z(\epsilon_s^{c,B}) + \lambda \cdot L + \frac{u(c^c(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c^c(\epsilon_s^{cp}) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

The participation constraint of workers can be written with free-entry condition as:

$$z(\epsilon_s^{c,cp}) + \frac{u(c^c(\epsilon_s^{c,cp})) - u(b)}{u'(c^w)} - c^c(\epsilon_s^{c,cp}) + \frac{(\lambda - \eta f)}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

Note that to avoid liquidation costs, firms could like workers to stay within the firm for negative surplus values. Thus, the worker decides to leave the firm before the contractual separation threshold $\epsilon_s^{c,B} < \epsilon_s^{u,cp}$, avoiding lay-off taxes.

B.2 Job Creation and Wage Equation

The notation of this section follows Appendix D for lay-off taxes.

(I) Value of an unconstrained firm after the idiosyncratic productivity shock has realized:

$$\begin{aligned} J^u(\epsilon) &= z(\epsilon) - c^w + \lambda\,\bar{J} + (1-\lambda)\cdot J^u(\epsilon) \\ \Leftrightarrow \quad J^u(\epsilon) &= \frac{1}{\lambda}\left(z(\epsilon) - c^w + \lambda\,\bar{J}\right) \end{aligned}$$

(II) Value of a constrained firm without binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} J^c(\epsilon) &= z(\epsilon) - c^w + \lambda\,\bar{J} + (1-\lambda)\cdot J^c(\epsilon) \\ \Leftrightarrow \quad J^c(\epsilon) &= \frac{1}{\lambda}\left(z(\epsilon) - c^w + \lambda\,\bar{J}\right) \end{aligned}$$

(III) Value of a constrained firm with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} J^c(\epsilon) &= z(\epsilon) - c^w(\epsilon) + \lambda\,\bar{J} + (1-\lambda)\cdot J^c(\epsilon) \\ \Leftrightarrow \quad J^c(\epsilon) &= \frac{1}{\lambda}\left(z(\epsilon) - c^w(\epsilon) + \lambda\,\bar{J}\right) \end{aligned}$$

(IV) Value of a firm before the idiosyncratic productivity shock has realized:

$$\bar{J} = -\tau + (1-p)\cdot\int_{\epsilon_s^u}^{\infty} J^u(\epsilon)\,dG(\epsilon) + p\cdot\int_{\epsilon_s^c}^{\infty} J^c(\epsilon)\,dG(\epsilon) - (1-p)\cdot\rho^u\cdot F$$

Inserting (I)-(III) into (IV) gives:

$$\begin{aligned} \Leftrightarrow \quad \bar{J} = -\tau + \frac{1}{\lambda}\Big[&(1-\rho)\cdot z - (1-p)\cdot(1-\rho^u)\cdot c^w - p\cdot(1-\rho^c)\cdot e^c \\ &+ (1-p)\cdot\lambda\cdot\bar{J}\Big] - (1-p)\cdot\rho^u\cdot F \end{aligned}$$

(V) Value of an unconstrained worker after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^u(\epsilon) &= u(c^w) + \lambda\,\bar{V} + (1-\lambda)\cdot V^u(\epsilon) \\ \Leftrightarrow \quad V^u(\epsilon) &= \frac{1}{\lambda}\left(u(c^w) + \lambda\,\bar{V}\right) \end{aligned}$$

(VI) Value of a constrained worker without binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & V^c(\epsilon) = u(c^w) + \lambda \bar{V} + (1-\lambda) \cdot V^c(\epsilon) \\ \Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{\lambda} \left(u(c^w) + \lambda \bar{V} \right) \end{aligned}$$

(VII) Value of a constrained worker with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & V^c(\epsilon) = u(c^w(\epsilon)) + \lambda \bar{V} + (1-\lambda) \cdot V^c(\epsilon) \\ \Leftrightarrow \quad & V^c(\epsilon) = \frac{1}{\lambda} \left(u(c^w(\epsilon)) + \lambda \bar{V} \right) \end{aligned}$$

(VIII) Value of a worker before the idiosyncratic productivity shock has realized:

$$\bar{V} = (1-p) \cdot \int_{\epsilon_s^u}^{\infty} V^u(\epsilon)\, dG(\epsilon) + p \cdot \int_{\xi^c}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \rho \cdot U$$

Inserting (V)-(VII) into (VIII) gives:

$$\Leftrightarrow \quad \bar{V} = \frac{1}{\lambda} \Big[(1-p)(1-\rho^u) \cdot u(c^w) + p \cdot (1-\rho^c) \cdot u^c + (1-\rho) \cdot \lambda \cdot \bar{V} + \rho \cdot U \Big]$$

(IX) Unemployment:

$$U = u(b) + f \cdot \bar{V} + (1-f) \cdot U$$

Next, we turn to calculating the joint surplus.

A) First, let us calculate the surplus of the worker after the idiosyncratic productivity shock has been realized:

$$\begin{aligned} & V^i(\epsilon) - U = u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot V^i(\epsilon) + (\lambda - f) \cdot \bar{V} - (\lambda - f) \cdot U \\ \Leftrightarrow \quad & V^i(\epsilon) - U = u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot (V^i(\epsilon) - U) + (\lambda - f) \cdot (\bar{V} - U) \\ \Leftrightarrow \quad & V^i(\epsilon) - U = \frac{1}{\lambda} \left(u(c^i(\epsilon)) - u(b) + (\lambda - f) \cdot (\bar{V} - U) \right) \end{aligned}$$

B) Next, we calculate the expected surplus of the worker, before the shocks have been realized:

$$\bar{V} - U = (1-\rho) \cdot \int_{\epsilon_s^u}^{\infty} \left(V^u(\epsilon) - U\right)\, dG(\epsilon) + p \cdot \int_{\epsilon_s^c}^{\infty} \left(V^c(\epsilon) - U\right)\, dG(\epsilon)$$

$$= \frac{1}{\lambda}\Bigg[(1-\rho)\cdot(1-\rho^u)\cdot(u(c^u) - u(b))$$

$$+ p \cdot (1-\rho^c)\cdot(u(c^c) - u(b))$$

$$+ (1-\rho)\cdot(\lambda - f)\cdot(\bar{V} - U)\Bigg]$$

C) Next, we can calculate the joint surplus from the expected value of a worker for a firm and the expected surplus of the worker:

$$S = \bar{J} + \frac{\bar{V} - U}{u'(c^u)}$$

$$= -\tau + \frac{1}{\lambda}\Bigg[(1-\rho)\cdot z$$

$$+ (1-\rho)\cdot(1-\rho^u)\cdot\left(\frac{u(c^u) - u(b)}{u'(c^u)} - c^u\right)$$

$$+ p \cdot (1-\rho^c)\cdot\left(\frac{u(c^c) - u(b)}{u'(c^u)} - e^c\right)$$

$$+ (1-\rho)\cdot\lambda\cdot\bar{J} + (1-\rho)(\lambda - f)\cdot\frac{\bar{V} - U}{u'(c^u)}\Bigg] - (1-p)\cdot\rho^u\cdot F$$

Using the surplus splitting rule, we can rewrite the equation above as:

$$S = -\tau + \frac{1}{\lambda}\Bigg[(1-\rho)\cdot z$$

$$+ (1-\rho)\cdot(1-\rho^u)\cdot\left(\frac{u(c^u) - u(b)}{u'(c^u)} - c^u\right)$$

$$+ p \cdot (1-\rho^c)\cdot\left(\frac{u(c^c) - u(b)}{u'(c^u)} - e^c\right)$$

$$+ (1-\rho)\cdot(\lambda - \eta f)\cdot S\Bigg] + (1-p)\cdot\rho^u\cdot F$$

Next, we want to replace the tax in the surplus equation. We can do this by using the budget constraint of the government:

$$
\begin{aligned}
n^s \cdot \tau &= n^s \cdot (1-p) \cdot \rho^u \cdot F + (1-n) \cdot b \\
\Leftrightarrow \quad \tau &= (1-p) \cdot \rho^u \cdot F + \frac{1-n}{n^s} \cdot b \\
\text{Insert } n^s &= \frac{\lambda}{1-\rho} \cdot n \\
\Rightarrow \quad \tau &= n^s \cdot (1-p) \cdot \rho^u \cdot F + \frac{1}{\lambda} \cdot (1-\rho) \cdot \frac{1-n}{n} \cdot b
\end{aligned}
$$

Inserting it into the equation for the joint surplus gives:

$$
S = \frac{1}{\lambda}\Bigg[(1-\rho) \cdot \left(z - \frac{1-n}{n} \cdot b\right) + (1-p) \cdot (1-\rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w\right)
$$

$$
+ p \cdot (1-\rho^c) \cdot \left(\frac{u(c^c) - u(b)}{u'(c^w)} - e^c\right) + (1-\rho) \cdot (\lambda - \eta f) \cdot S\Bigg]
$$

Next, insert the free-entry condition: $\frac{k_v}{q} = \bar{J}$

$$
\begin{aligned}
\frac{1}{1-\eta} \cdot \frac{k_v}{q} = \frac{1}{\lambda} \cdot \Bigg[& \\
& (1-\rho) \cdot \left(z - \frac{1-n}{n} \cdot b\right) \\
& + (1-p) \cdot (1-\rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w\right) \\
& + p \cdot (1-\rho^c) \cdot \left(\frac{u(c^c) - u(b)}{u'(c^w)} - e^c\right) \\
& + (1-\rho) \cdot \left(\frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q}\right) \Bigg]
\end{aligned}
$$

The subsequent derivation of the wage equation is completely analogous to the derivation under STW regime described in Section C.2. The wage equation thus reads:

$$(1-p)(1-\rho^u)\cdot(1-\eta)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}+\eta\cdot c^w\right)=$$
$$\eta\cdot\left[(1-\rho)\cdot z-(1-\rho)\cdot\frac{1-n}{n}\cdot b+(1-\rho)\cdot\theta\cdot k_v\right]$$
$$-p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}+\eta\cdot e^c\right)$$

C Equilibrium STW

C.1 Bargaining

Nash-Bargaining Problem:

$$\max_{\substack{w^u(\epsilon),w^c(\epsilon),w^u_{\text{stw}}(\epsilon),w^c_{\text{stw}}(\epsilon),\\ h^u(\epsilon),h^c(\epsilon),h^u_{\text{stw}}(\epsilon),h^c_{\text{stw}}(\epsilon),\\ \epsilon^u_s,\epsilon^c_s,\xi^u_s,\xi^c_s}} \bar{J}^{1-\eta}\cdot(\bar{V}-U)^\eta$$

Subject to:

1. Financial constraints of financially constrained firms:

$$y(\epsilon,h(\epsilon))-w^c(\epsilon)\geq\lambda\cdot J^c(\epsilon)+(1-\lambda)\cdot\bar{J}$$
$$y(\epsilon,h_{\text{stw}}(\epsilon))-w^c_{\text{stw}}(\epsilon)\geq\lambda\cdot J^c_{\text{stw}}(\epsilon)+(1-\lambda)\cdot\bar{J}$$

2. Workers have a commitment problem:

$$U>V^u(\epsilon),\quad U>V^u_{\text{stw}}(\epsilon)$$
$$U>V^c(\epsilon),\quad U>V^c_{\text{stw}}(\epsilon)$$

Second, we can integrate the commitment problem of the worker into the value functions for the firm and worker surplus. For $i\in\{\text{stw},\text{no stw}\}$ and $j\in\{u,c\}$, we can reformulate the condition as:

$$u(c^j_i(\epsilon))-u(b)+(\lambda-f)\cdot(\bar{V}-U)\geq 0$$

Let $\epsilon^{j,\text{cp}}_s$ denote the threshold at which the worker wants to separate from a firm with STW support:

$$u(c_i^j(\epsilon_s^{j,\text{stw,cp}})) - u(b) + (\lambda - f)(\bar{V} - U) = 0$$

Likewise, we can determine the threshold at which workers leave firms without STW support, $\xi_s^{j,\text{cp}}$, as:

$$u(c_i^j(\xi_s^{j,\text{cp}})) - u(b) + (\lambda - f)(\bar{V} - U) = 0$$

We can rewrite the value functions of the firm and the surplus of the worker as:

$$\begin{aligned}
\bar{J}^B = -\tau + (1-p)\cdot \Bigg[& \int_{\max\{\xi_s^{u,B},\, \xi_s^{u,\text{cp}},\, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{\lambda}\left(z(\epsilon) - c^u(\epsilon) + \lambda \bar{J}\right) dG(\epsilon) \\
& + \int_{\max\{\epsilon_s^{u,B},\, \epsilon_s^{u,\text{cp}}\}}^{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}, \epsilon_{\text{stw}}\}} \frac{1}{\lambda}\left(z_{\text{stw}}(\epsilon) - c^u_{\text{stw}}(\epsilon) + \lambda \bar{J}\right) dG(\epsilon) \Bigg] \\
+ p \cdot \Bigg[& \int_{\max\{\xi_s^{c,B},\, \xi_s^{c,\text{cp}},\, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{\lambda}\left(z(\epsilon) - c^c(\epsilon) + \lambda \bar{J}\right) dG(\epsilon) \\
& + \int_{\max\{\epsilon_s^{c,B},\, \epsilon_s^{c,\text{cp}}\}}^{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,cp}, \epsilon_{\text{stw}}\}} \frac{1}{\lambda}\left(z_{\text{stw}}(\epsilon) - c^c_{\text{stw}}(\epsilon) + \lambda \bar{J}\right) dG(\epsilon) \Bigg]
\end{aligned}$$

$$\begin{aligned}
& (\bar{V} - U)^B \\
& = (1-p)\cdot \Bigg[\int_{\max\{\xi_s^{u,B},\, \xi_s^{u,\text{cp}},\, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{\lambda}\left(u(c^u(\epsilon)) - u(b) + \lambda(\bar{V} - U)\right) dG(\epsilon) \\
& \quad + \int_{\max\{\epsilon_s^{u,B},\, \epsilon_s^{u,\text{cp}}\}}^{\max\{\epsilon_s^{u,B}, \epsilon_s^{u,cp}, \epsilon_{\text{stw}}\}} \frac{1}{\lambda}\left(u(c^u_{\text{stw}}(\epsilon)) + \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon)) - u(b) + \lambda(\bar{V} - U)\right) dG(\epsilon) \Bigg] \\
& \quad + p \cdot \Bigg[\int_{\max\{\xi_s^{c,B},\, \xi_s^{c,\text{cp}},\, \epsilon_{\text{stw}}\}}^{\infty} \frac{1}{\lambda}\left(u(c^w(\epsilon)) - u(b) + \lambda(\bar{V} - U)\right) dG(\epsilon) \\
& \quad + \int_{\max\{\epsilon_s^{c,B},\, \epsilon_s^{c,\text{cp}}\}}^{\max\{\epsilon_s^{c,B}, \epsilon_s^{c,cp}, \epsilon_{\text{stw}}\}} \frac{1}{\lambda}\left(u(c^c_{\text{stw}}(\epsilon)) + \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon)) - u(b) + \lambda(\bar{V} - U)\right) dG(\epsilon) \Bigg]
\end{aligned}$$

$$
\begin{aligned}
&(\bar{V}-U)^B \\
&= (1-p)\cdot\Bigg[\int_{\max\{\xi_s^{u,B},\,\xi_s^{u,\text{cp}},\,\epsilon_{\text{stw}}\}}^{\infty}\frac{1}{\lambda}\big(u(c^u(\epsilon))-u(b)+\lambda(\bar{V}-U)\big)\,dG(\epsilon) \\
&\quad+\int_{\max\{\epsilon_s^{u,B},\,\epsilon_s^{u,\text{cp}}\}}^{\max\{\epsilon_s^{u,B},\,\epsilon_s^{u,\text{cp}},\,\epsilon_{\text{stw}}\}}\frac{1}{\lambda}\big(u(c^u_{\text{stw}}(\epsilon))+\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))-u(b)+\lambda(\bar{V}-U)\big)\,dG(\epsilon)\Bigg] \\
&\quad+p\cdot\Bigg[\int_{\max\{\xi_s^{c,B},\,\xi_s^{c,\text{cp}},\,\epsilon_{\text{stw}}\}}^{\infty}\frac{1}{\lambda}\big(u(c^w(\epsilon))-u(b)+\lambda(\bar{V}-U)\big)\,dG(\epsilon) \\
&\quad+\int_{\max\{\epsilon_s^{c,B},\,\epsilon_s^{c,\text{cp}}\}}^{\max\{\epsilon_s^{c,B},\,\epsilon_s^{c,\text{cp}},\,\epsilon_{\text{stw}}\}}\frac{1}{\lambda}\big(u(c^c_{\text{stw}}(\epsilon))+\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))-u(b)+\lambda(\bar{V}-U)\big)\,dG(\epsilon)\Bigg]
\end{aligned}
$$

Note that $\xi_s^{u,B}$, $\xi_s^{c,\text{B}}$, $\epsilon_s^{u,B}$ and $\epsilon_s^{c,\text{B}}$ denote the separation thresholds at bargaining outcome. To be consistent with the bargaining problem under lay-off taxes, we assume that $\epsilon_s^{i,\text{cp}}$ and $\xi_s^{i,\text{cp}}$ are exogenous to the bargaining problem.

Note that we can rewrite the problem so that we optimize over consumption equivalents instead of salaries. $c^i_{\text{stw}}(\epsilon)$ denotes the consumption equivalent paid from the firm to the worker on STW. $c^i(\epsilon)$ is the consumption equivalent consumed by the worker.

$$
\max_{\substack{c^u(\epsilon),\,c^c(\epsilon),\,c^u_{\text{stw}}(\epsilon),\,c^c_{\text{stw}}(\epsilon),\\ h^u(\epsilon),\,h^c(\epsilon),\,h^u_{\text{stw}}(\epsilon),\,h^c_{\text{stw}}(\epsilon),\\ \xi_s^{u,B},\,\xi_s^{c,B},\,\epsilon_s^{u,B},\,\epsilon_s^{c,B}}} \left(\bar{J}^B\right)^{1-\eta}\cdot\left(\bar{V}-U\right)^{\eta}
$$

1. Financial constraints of financially constrained firms

$$
z(\epsilon)-c^c(\epsilon)\geq-\lambda\cdot J^c(\epsilon)-(1-\lambda)\cdot\bar{J}
$$

$$
z_{\text{stw}}(\epsilon)-c^c_{\text{stw}}(\epsilon)\geq-\lambda\cdot J^c_{\text{stw}}(\epsilon)-(1-\lambda)\cdot\bar{J}
$$

Set up Kuhn-Tucker Conditions:

$$
\begin{aligned}
&\left(\bar{J}^B\right)^{\eta}\cdot\left(\bar{V}-U\right)^{1-\eta}-\int_0^{\infty}\mathcal{M}(\epsilon)\left[z(\epsilon)-c^c(\epsilon)+\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c(\epsilon)\right]dG(\epsilon) \\
&\quad-\int_0^{\infty}\mathcal{M}_{\text{stw}}(\epsilon)\left[z_{stw}(\epsilon)-c^c_{\text{stw}}(\epsilon)+\lambda\cdot\bar{J}+(1-\lambda)\cdot J^c_{\text{stw}}(\epsilon)\right]dG(\epsilon)
\end{aligned}
$$

With Kuhn-Tucker conditions for a maximum:

(A) (i) $\mathcal{M}(\epsilon) \leq 0$

(ii) $\mathcal{M}(\epsilon)\left[z(\epsilon) - c^c(\epsilon) + \lambda \cdot \bar{J} + (1-\lambda) \cdot J^c(\epsilon)\right] = 0$

(B) (i) $\mathcal{M}_{\text{stw}}(\epsilon) \leq 0$

(ii) $\mathcal{M}_{\text{stw}}(\epsilon)\left[z_{stw}(\epsilon) - c^c_{\text{stw}}(\epsilon) + \lambda \cdot \bar{J} + (1-\lambda) \cdot J^c_{\text{stw}}(\epsilon)\right] = 0$

FOC for $c^u(\epsilon)$:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^u(\epsilon)} &= -(1-\eta)(1-p) \cdot g(\epsilon) \left(\frac{\bar{V} - U}{\bar{J}}\right)^{\eta} \\ &\quad + \eta \cdot (1-p) \cdot g(\epsilon) \cdot u'(c^u(\epsilon)) \left(\frac{\bar{J}}{\bar{V} - U}\right)^{1-\eta} = 0\end{aligned}$$

Unconstrained firms want to insure workers against idiosyncratic productivity shocks, outside STW:

$$\iff \quad \frac{1-\eta}{\eta} \cdot \frac{\bar{V} - U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \quad \Rightarrow \quad u'(c^u(\epsilon)) = u'(c^u(\epsilon')) = u'(c^u)$$

FOC for $c^u_{\text{stw}}(\epsilon)$:

$$\begin{aligned}\frac{\partial \mathcal{B}}{\partial c^u_{\text{stw}}(\epsilon)} &= -(1-\eta)(1-p) \cdot g(\epsilon) \left(\frac{\bar{V} - U}{\bar{J}}\right)^{\eta} \\ &\quad + \eta \cdot (1-p) \cdot g(\epsilon) \cdot u'\left(c^u_{\text{stw}}(\epsilon)\right) \cdot \eta \cdot \left(\frac{\bar{J}}{\bar{V} - U}\right)^{1-\eta} = 0\end{aligned}$$

Unconstrained firms want to insure workers against idiosyncratic productivity shocks, also on STW:

$$\begin{aligned}\iff \quad & \frac{1-\eta}{\eta} \cdot \frac{\bar{V} - U}{\bar{J}} = u'\left(c^u(\epsilon)\right) \\ \Rightarrow \quad & u'\left(c^{u,\text{f}}_{\text{stw}}(\epsilon)\right) = u'\left(c^u_{\text{stw}}(\epsilon')\right) = u'(c^w)\end{aligned}$$

FOC for $c^c(\epsilon)$:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial c^c(\epsilon)} &= -(1-p)\cdot g(\epsilon)\cdot(1-\eta)\cdot\left(\frac{\bar{V}-U}{\bar{J}}\right)^{\eta} \\
&\quad + (1-p)\cdot g(\epsilon)\cdot u'(c^c(\epsilon))\cdot \bar{N}\cdot\left(\frac{\bar{J}}{\bar{V}-U}\right)^{1-\eta} + \mathcal{M}(\epsilon) \\
&\overset{!}{=} 0
\end{aligned}
$$

Insert the surplus splitting rule:

$$
\begin{aligned}
&-(1-p)\cdot g(\epsilon)\cdot\left(\frac{\eta}{1-\eta}\cdot u'(c^w)\right)^{\eta} \\
&+(1-p)\cdot g(\epsilon)\cdot u'(c^c(\epsilon))\cdot\eta\cdot\left(\frac{1-\eta}{\eta}\cdot\frac{1}{u'(c^w)}\right)+\mathcal{M}(\epsilon)\overset{!}{=}0
\end{aligned}
$$

$$
\iff \quad (1-p)\cdot g(\epsilon)\cdot\eta\cdot(u'(c^c(\epsilon))-u'(c^w)) = -\mathcal{M}(\epsilon)\cdot\left(u'(c^w)\cdot\frac{\eta}{1-\eta}\right)^{1-\eta}
$$

Note: If $c^c(\epsilon) < u'(c^w)$, then the RHS of the equation is positive. Thus, firms and workers gain joint surplus until $c^c(\epsilon) = c^w$.

If

$$
z(\epsilon) - c^w + \lambda\cdot\bar{J} + (1-\lambda)\cdot J^c(\epsilon) \geq 0,
$$

then the constraint is non-binding and $\mathcal{M}(\epsilon) = 0$. ✓

If

$$
z(\epsilon) - c^w + \lambda\cdot\bar{J} + (1-\lambda)\cdot J^c(\epsilon) < 0,
$$

then the constraint is binding and $\mathcal{M}(\epsilon) < 0$. ✓

Thus, it is optimal for the firm to pay as much as it can:

$$
J^c(\epsilon) = z(\epsilon) - c^c(\epsilon) + \lambda\cdot\bar{J} + (1-\lambda)\cdot J^c(\epsilon) = 0
$$

$$
\iff \quad c^c(\epsilon) = z(\epsilon) + \lambda\cdot\bar{J}
$$

Using the same arguments, we get that under STW

$$c_{\text{stw}}(\epsilon) = c^w \quad \text{if the financial constraint is non-binding,}$$

and

$$c_{\text{stw}}(\epsilon) = \epsilon_{\text{stw}}(\epsilon) + \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon)) + \lambda \cdot \bar{J} \quad \text{when it is binding.}$$

Using the surplus splitting rule, we can simplify the Nash-Bargaining problem to:

$$\max_{h^u(\epsilon), h^c(\epsilon), h^u_{\text{stw}}(\epsilon), h^c_{\text{stw}}(\epsilon),\, \xi^u_s, \xi^c_s, \xi^{u^c}_s, \xi^{c,c}_s} (1-\eta)^{1-\eta} \cdot (u'(c^w) \cdot \eta)^{\eta} \cdot \bar{S}$$

That is, we just have to maximize the joint surplus:

$$\begin{aligned}
\bar{S} = &-\tau \\
&+ (1-p) \cdot \int_{\max\{\xi^{u,B}_s, \xi^{u,cp}_s, \epsilon_{stw}\}}^{\infty} \frac{1}{\lambda}\Big(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (1 - \eta \cdot f) \cdot \bar{S}\Big) dG(\epsilon) \\
&+ (1-p) \cdot \int_{\max\{\epsilon^{u,B}_s, \epsilon^{u,cp}_s\}}^{\max\{\epsilon^{u,B}_s, \epsilon^{u,cp}_s, \epsilon_{stw}\}} \frac{1}{\lambda}\Big(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w \\
&\qquad + \tau_{stw}(\bar{h} - h_{stw}(\epsilon)) + (1 - \eta \cdot f) \cdot \bar{S}\Big) dG(\epsilon) \\
&+ p \cdot \int_{\epsilon^p}^{\infty} \frac{1}{\lambda}\Big(z(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + (1 - \eta \cdot f) \cdot \bar{S}\Big) dG(\epsilon) \\
&+ p \cdot \int_{\max\{\xi^c_s, \xi^{c,cp}_s, \epsilon_{stw}\}}^{\epsilon^p} \frac{1}{\lambda}\Big(z(\epsilon) + \frac{u(c^c(\epsilon)) - u(b)}{u'(c^w)} - c^c(\epsilon) + (1 - \eta \cdot f) \cdot \bar{S}\Big) dG(\epsilon) \\
&+ p \cdot \int_{\max\{\epsilon^c_s, \epsilon^{c,cp}_s\}}^{\max\{\epsilon^c_s, \epsilon^{c,cp}_s, \epsilon_{stw}\}} \frac{1}{\lambda}\Big(z_{stw}(\epsilon) + \frac{u(c^c_{stw}(\epsilon)) - u(b)}{u'(c^w)} \\
&\qquad - c^c(\epsilon) + (1 - \eta \cdot f) \cdot \bar{S}\Big) dG(\epsilon)
\end{aligned}$$

Note that without financial constraints, the firm smooths the consumption equivalent consumed by the worker:

$$c^w = c^u_{\text{stw}}(\epsilon) = c^u_{\text{stw}}(\epsilon) + \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))$$

$$\Leftrightarrow \quad c^u_{\text{stw}} = c^w - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))$$

STW makes it less expensive to smooth consumption equivalents consumed by the worker.

The FOC for working hours in unconstrained firms outside STW is:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial h^u(\epsilon)} &= (1-p)\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} - \phi'(h^u(\epsilon))\right) = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon, h^u(\epsilon))}{\partial h^u(\epsilon)} &= \phi'(h^u(\epsilon))
\end{aligned}
$$

The FOC for working hours in constrained firms *outside* STW is:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} &= p\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon))\right)\cdot\frac{u'(c^c(\epsilon))}{u'(c^w)} = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} &= \phi'(h^c(\epsilon))
\end{aligned}
$$

The FOC for working hours in unconstrained firms *on* STW is:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial h^u_{\text{stw}}(\epsilon)} &= (1-p)\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon, h^u_{\text{stw}}(\epsilon))}{\partial h^u_{\text{stw}}(\epsilon)} - \phi'(h^u_{\text{stw}}(\epsilon)) - \tau_{\text{stw}}\right) = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon, h^u_{\text{stw}}(\epsilon))}{\partial h^u_{\text{stw}}(\epsilon)} &= \phi'(h^u_{\text{stw}}(\epsilon)) + \tau_{\text{stw}}
\end{aligned}
$$

The FOC for working hours in constrained firms *on* STW is:

$$
\begin{aligned}
\frac{\partial \mathcal{B}}{\partial h^c(\epsilon)} &= p\cdot g(\epsilon)\cdot\left(\frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} - \phi'(h^c(\epsilon)) - \tau_{\text{stw}}\right)\cdot\frac{u'(c^c(\epsilon))}{u'(c^w)} = 0 \\
\Leftrightarrow \quad \frac{\partial y(\epsilon, h^c(\epsilon))}{\partial h^c(\epsilon)} &= \phi'(h^c(\epsilon)) + \tau_{\text{stw}}
\end{aligned}
$$

Suppose that

$$
\max\{\xi_s^{u,B},\, \xi_s^{u,\text{cp}},\, \epsilon_{\text{stw}}\} = \xi_s^{u,B}
$$

Then the FOC for the separation threshold of the unconstrained firm without STW support is:

$$
\frac{\partial \mathcal{B}}{\partial \xi_s^{u,B}} = -(1-p)\cdot g(\xi_s^{u,B})\cdot\frac{1}{\lambda}\left(z(\xi_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\bar{S}\right) = 0
$$

$$
\Longleftrightarrow\ z(\xi_s^{u,B}) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + (\lambda-\eta f)\cdot\bar{S} = 0
$$

Insert free-entry Condition:

$$z(\xi_s^{u,B}) + z(\xi_s^{u,B}) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{(\lambda - \eta f)}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that $\xi_s^{u,B} > \xi_s^{u,\mathrm{cp}}$ must hold, as workers would never quit under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f)(\bar{V} - U) > 0$$

It always gets positive surplus!

Suppose that

$$\max\{\xi_s^{c,B},\, \xi_s^{c,\mathrm{cp}},\, \epsilon_{\mathrm{stw}}\} = \xi_s^{c,B}$$

Then the FOC for the separation threshold of the unconstrained firm without STW support is:

$$\frac{\partial \mathcal{B}}{\partial \xi_s^{c,B}} = -(1-p) \cdot g(\xi_s^{c,B}) \cdot \left[\frac{1}{\lambda} \cdot \left(z(\xi_s^{c,B}) + \frac{u\left(c^c(\xi_s^{c,B})\right) - u(b)}{u'(c^w)} \right.\right.$$
$$\left.\left. - c^c(\xi_s^{c,B}) + (\lambda - \eta f) \cdot \bar{S} \right) + L \right] = 0$$

$$\iff z(\xi_s^{c,B}) + \lambda \cdot F + \frac{u(c^c(\xi_s^{c,B})) - u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + (\lambda - \eta f) \cdot \bar{S} = 0$$

Insert free-entry condition:

$$z(\xi_s^{c,B}) + \frac{u(c^c(\xi_s^{c,B})) - u(b)}{u'(c^w)} - c^c(\xi_s^{c,B}) + \frac{(\lambda - \eta f)}{1-\eta} \cdot \frac{k_v}{q} = 0$$

The participation constraint of workers can be written with free-entry condition as:

$$z(\xi_s^{c,\mathrm{cp}}) + \frac{u(c^c(\xi_s^{c,\mathrm{cp}})) - u(b)}{u'(c^w)} - c^c(\xi_s^{c,\mathrm{cp}}) + \frac{(\lambda - \eta f)}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that these are the same conditions. This implies that given the inability to insure workers against idiosyncratic productivity shocks, separations for a constrained firm without access to STW are efficient.

Suppose that

$$\max\{\epsilon_s^{u,B}, \epsilon_s^{u,\mathrm{cp}}, \epsilon_{\mathrm{stw}}\} = \epsilon_s^{u,B}$$

Then the FOC for the separation threshold of the unconstrained firm with STW support is:

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{u,B}} = -(1-p) \cdot g(\epsilon_s^{u,B}) \cdot \Bigg[\frac{1}{\lambda} \cdot \Big(z(\epsilon_s^{u,B}) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w$$

$$+ \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^{u,B})) + (\lambda - \eta f) \cdot \bar{S} \Big) \Bigg] = 0$$

$$\iff z(\epsilon_s^{u,B}) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^{u,B})) + (\lambda - \eta f) \cdot \bar{S} = 0$$

Insert free-entry Condition:

$$z(\epsilon_s^{u,B}) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that $\epsilon_s^{u,B} > \epsilon_s^{u,\text{cp}}$ must hold, as workers would never quit under the insurance constraint of the firm:

$$V(\epsilon) - U = u(c^w) - u(b) + (\lambda - f)(\bar{V} - U) > 0$$

It always gets positive surplus!

Suppose that

$$\max\{\epsilon_s^{c,B},\, \epsilon_s^{c,\text{cp}},\, \epsilon_{\text{stw}}\} = \epsilon_s^{c,B}$$

Then the FOC for the separation threshold of the constrained firm with STW support is:

$$\frac{\partial \mathcal{B}}{\partial \epsilon_s^{c,B}} = -(1-p) \cdot g(\epsilon_s^{c,B}) \cdot \Bigg[\frac{1}{\lambda} \cdot \Big(z(\epsilon_s^{c,B}) + \frac{u\left(c_{\text{stw}}^c(\epsilon_s^{c,B})\right) - u(b)}{u'(c^w)}$$

$$- c_{\text{stw}}^c(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \bar{S} \Big) + L \Bigg] = 0$$

$$\iff z(\epsilon_s^{c,B}) + \frac{u(c_{\text{stw}}^c(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c_{\text{stw}}^c(\epsilon_s^{c,B}) + (\lambda - \eta f) \cdot \bar{S} = 0$$

Insert free-entry condition:

$$z(\epsilon_s^{c,B}) + \frac{u(c_{\text{stw}}^c(\epsilon_s^{c,B})) - u(b)}{u'(c^w)} - c_{\text{stw}}^c(\epsilon_s^{c,B}) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

The participation constraint of workers can be written with free-entry condition as:

$$z(\epsilon_s^{c,\text{cp}}) + \frac{u(c^c_{\text{stw}}(\epsilon_s^{c,\text{cp}})) - u(b)}{u'(c^w)} - c^c_{\text{stw}}(\epsilon_s^{c,\text{cp}}) + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

Note that these are the same conditions. This implies that, given the inability to insure workers against idiosyncratic productivity shocks, separations for a constrained firm with access to STW are efficient.

C.2 Job Creation and Wage Equation

This section derives the job creation and the equation for STW. It follows its notation part D of the appendix.

(I) Value of an unconstrained firm outside STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^u(\epsilon) = z(\epsilon) - c^w + \lambda \bar{J} + (1-\lambda) J^u(\epsilon) \\ \Leftrightarrow \quad & J^u(\epsilon) = \frac{1}{\lambda} \left(z(\epsilon) - c^w + \lambda \bar{J} \right) \end{aligned}$$

(II) Value of an unconstrained firm on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^u_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) + \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon)) - c^w + \lambda \bar{J} + (1-\lambda) J^u_{\text{stw}}(\epsilon) \\ \Leftrightarrow \quad & J^u_{\text{stw}}(\epsilon) = \frac{1}{\lambda} \left(z_{\text{stw}}(\epsilon) - c^w + \lambda \bar{J} \right) \end{aligned}$$

(III) Value of a constrained firm outside STW with non-binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w + \lambda \bar{J} + (1-\lambda) J^c(\epsilon) \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{\lambda} \left(z(\epsilon) - c^w + \lambda \bar{J} \right) \end{aligned}$$

(IV) Value of a constrained firm outside STW with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} & J^c(\epsilon) = z(\epsilon) - c^w(\epsilon) + \lambda\,\bar{J} + (1-\lambda)\,J^c(\epsilon) \\ \Leftrightarrow \quad & J^c(\epsilon) = \frac{1}{\lambda}\left(z(\epsilon) - c^w(\epsilon) + \lambda\,\bar{J}\right) \end{aligned}$$

(V) Value of a constrained firm on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} J^c_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon) - c^w_{stw}(\epsilon) + \lambda J + (1-\lambda) J^c_{\text{stw}}(\epsilon) \\ \Leftrightarrow \quad J^c_{\text{stw}}(\epsilon) = \frac{1}{\lambda}\left(z_{\text{stw}}(\epsilon) - c^w_{stw}(\epsilon) + \lambda J\right) \end{aligned}$$

(VI) Value of a firm before the idiosyncratic productivity shock has realized:

$$\begin{aligned} \bar{J} = -\tau + (1-p)\cdot\left(\int_{\epsilon_{\text{stw}}}^{\infty} J^u(\epsilon)\,dG(\epsilon) + \int_{\epsilon^u_s}^{\epsilon_{\text{stw}}} J^u_{\text{stw}}(\epsilon)\,dG(\epsilon)\right) \\ + p\cdot\left(\int_{\max\{\epsilon_{\text{stw}},\xi^c_s\}}^{\infty} J^c(\epsilon)\,dG(\epsilon) + \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}},\epsilon^c_s\}} J^c_{\text{stw}}(\epsilon)\,dG(\epsilon)\right) \end{aligned}$$

Inserting (I) - (IV) into (V) gives:

$$\begin{aligned} \bar{J} = & -\tau \\ & + \frac{1}{\lambda}\Bigg[(1-\rho)\cdot(z-\Omega) \\ & - (1-p)\cdot(1-\rho^u)\cdot\left(c^w - \int_{\epsilon^u_s}^{\epsilon_{\text{stw}}} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right) \\ & - p\cdot(1-\rho^c)\cdot\left(e^c - \int_{\epsilon^c_s}^{\max\{\epsilon_{\text{stw}},\epsilon^c_s\}} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\,dG(\epsilon)\right) \\ & + (1-\rho)\cdot\lambda\cdot\bar{J}\Bigg] \end{aligned}$$

(VII) Value of an unconstrained worker outside STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^u(\epsilon) &= u(c^w) + \lambda\,\bar{V} + (1-\lambda)\,V^u(\epsilon) \\ \Leftrightarrow \quad V^u(\epsilon) &= \frac{1}{\lambda}\left(u(c^w) + \lambda\,\bar{V}\right) \end{aligned}$$

(VIII) Value of an unconstrained worker on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^u_{\text{stw}}(\epsilon) &= u(c^w) + \lambda\,\bar{V} + (1-\lambda)\,V^u_{\text{stw}}(\epsilon) \\ \Leftrightarrow \quad V^u_{\text{stw}}(\epsilon) &= \frac{1}{\lambda}\left(u(c^w) + \lambda\,\bar{V}\right) \end{aligned}$$

(IX) Value of a constrained worker outside STW with non-binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^c(\epsilon) &= u(c^w) + \lambda\,\bar{V} + (1-\lambda)\,V^c(\epsilon) \\ \Leftrightarrow \quad V^c(\epsilon) &= \frac{1}{\lambda}\left(u(c^w + \lambda\,\bar{V}\right) \end{aligned}$$

(X) Value of a constrained worker outside STW with binding constraints after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^c(\epsilon) &= u(c^w(\epsilon)) + \lambda\,\bar{V} + (1-\lambda)\,V^c(\epsilon) \\ \Leftrightarrow \quad V^c(\epsilon) &= \frac{1}{\lambda}\left(u(c^w(\epsilon)) + \lambda\,\bar{V}\right) \end{aligned}$$

(XI) Value of a constrained worker on STW after the idiosyncratic productivity shock has realized:

$$\begin{aligned} V^c_{\text{stw}}(\epsilon) &= u(c_{\text{stw}}(\epsilon)) + \lambda\,\bar{V} + (1-\lambda)\,V^c_{\text{stw}}(\epsilon) \\ \Leftrightarrow \quad V^c_{\text{stw}}(\epsilon) &= \frac{1}{\lambda}\left(u(c_{\text{stw}}(\epsilon)) + \lambda\,\bar{V}\right) \end{aligned}$$

(XII) Value of a worker before the idiosyncratic productivity shock has realized:

$$\bar{V} = (1-p) \cdot \Bigg(\int_{\epsilon_{stw}}^{\infty} V^u(\epsilon)\, dG(\epsilon) + \int_{\epsilon_s^u}^{\epsilon_{stw}} V^u_{\text{stw}}(\epsilon)\, dG(\epsilon) \Bigg) + p \cdot \Bigg(\int_{\epsilon_p}^{\infty} V^c(\epsilon)\, dG(\epsilon) + \int_{\max\{\xi_s^c, \epsilon_{stw}\}}^{\epsilon_p} V^c(\epsilon)\, dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{stw}\}} V^c_{\text{stw}}(\epsilon)\, dG(\epsilon) \Bigg) + \rho \cdot U$$

Inserting (VII) - (XI) into (XII) gives:

$$\bar{V} = \frac{1}{\lambda}\Big[(1-p)(1-\rho^u) \cdot u(c^w) + p \cdot (1-\rho^c) \cdot u^c + (1-\rho) \cdot \lambda \cdot \bar{V} + \rho \cdot U\Big]$$

(XIII) Unemployment:

$$U = u(b) + f \cdot \bar{V} + (1-f) \cdot U$$

Next, we want to calculate the joint surplus of firms and workers.

(A) The surplus of the worker after the idiosyncratic productivity shock has realized is:

$$\begin{aligned} & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot V_i^j(\epsilon) + (\lambda - f) \cdot \bar{V} - (\lambda - f) \cdot U \\ \Leftrightarrow \quad & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot (V_i^j(\epsilon) - U) + (\lambda - f) \cdot (\bar{V} - U) \\ \Leftrightarrow \quad & V_i^j(\epsilon) - U = u(c_i^j(\epsilon)) - u(b) + (\lambda - f) \cdot (V_i^j(\epsilon) - U) + (\lambda - f) \cdot (\bar{V} - U) \\ \Leftrightarrow \quad & V_i^j(\epsilon) - U = \frac{1}{\lambda}\big(u(c_i(\epsilon)) - u(b) + (\lambda - f) \cdot (\bar{V} - U)\big) \end{aligned}$$

for $i \in \{\text{stw}, \text{no stw}\}$ and $j \in \{u, c\}$.

(B) The surplus of the worker before shock realizations are known:

$$\bar{V} - U = (1-p) \cdot \left(\int_{\epsilon_{\text{stw}}}^{\infty} V^u(\epsilon) - U \, dG(\epsilon) + \int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} V^u_{\text{stw}}(\epsilon) - U \, dG(\epsilon) \right)$$
$$+ p \cdot \left(\int_{\epsilon_p}^{\infty} V(\epsilon) - U \, dG(\epsilon) + \int_{\max\{\xi_s^c, \epsilon_{\text{stw}}\}}^{\epsilon_p} V^c(\epsilon) - U \, dG(\epsilon) \right.$$
$$\left. + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} V^c_{\text{stw}}(\epsilon) - U \, dG(\epsilon) \right)$$

Inserting A into B gives:

$$\bar{V} - U = \frac{1}{\lambda} \Big[(1-p)(1-\rho^u) \cdot (u(c^w) - u(b))$$
$$+ p \cdot (1-\rho^c) \cdot (u^c - u(b))$$
$$+ (\lambda - f) \cdot (\bar{V} - U) \Big]$$

(C) Finally, we can calculate the joint surplus before shock realizations are known:

$$S = J + \frac{\bar{V} - U}{u'(c^w)}$$
$$= -\tau + \frac{1}{\lambda} \left[(1-\rho)(1-\rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right. \right.$$
$$\left. + \frac{1}{1-\rho^u} \int_{\epsilon_s^u}^{\epsilon_{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \right)$$
$$+ p(1-\rho^c) \cdot \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right.$$
$$\left. + \frac{1}{1-\rho^c} \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \right)$$
$$\left. + (1-\rho)\lambda J + (1-\rho)(\lambda - f) \cdot \frac{\bar{V} - U}{u'(c^w)} \right]$$

Next, we start deriving the job-creation condition. Inserting the surplus splitting rule

$$J = \eta \cdot S, \quad \frac{\bar{V} - U}{u'(c^w)} = (1-\eta) \cdot S$$

into the equation for the joint surplus gives:

$$\begin{aligned}
S &= J + \frac{\bar{V} - U}{u'(c^w)} = \eta S + (1-\eta) S \\
&= -\tau + \frac{1}{\lambda} \Bigg[(1-\rho) \left(z - \Omega - \frac{1-n}{n} b \right) \\
&\quad + (1-p)(1-\rho^u) \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\
&\quad + (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\
&\quad + p(1-\rho^c) \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\
&\quad + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\
&\quad + (1-\rho) \cdot \lambda \cdot (1-\eta) \cdot S + (1-\rho)(\lambda - f)\eta \cdot S \Bigg] \\
\Leftrightarrow \quad S &= -\tau + \frac{1}{\lambda} \Bigg[(1-\rho) \left(z - \Omega - \frac{1-n}{n} b \right) \\
&\quad + (1-p)(1-\rho^u) \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\
&\quad + (1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\
&\quad + p(1-\rho^c) \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\
&\quad + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon)) \, dG(\epsilon) \\
&\quad + (1-\rho)(\lambda - \eta \cdot f) \cdot S \Bigg]
\end{aligned}$$

Next, we want to replace the tax. To do this, we need to know the budget constraint of the government:

$$
\begin{aligned}
n^s \cdot \tau = \frac{n}{1-\rho} \cdot & \Bigg((1-p) \int_{\epsilon_s^u}^{\epsilon_{stw}} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + (1-n) \cdot b
\end{aligned}
$$

Inserting the employment equation

$$
n = (1-\rho) \cdot \frac{n^s}{\lambda}
$$

gives:

$$
\begin{aligned}
\tau = \frac{1}{\lambda} \cdot & \Bigg((1-p) \int_{\epsilon_s^u}^{\infty} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + \frac{(1-\rho)}{n^s} \cdot b
\end{aligned}
$$

Inserting the expression for the number of firms that receive a shock

$$
n^s = \frac{n}{1-\rho} \cdot \lambda
$$

gives:

$$
\begin{aligned}
\tau = \frac{1}{\lambda} \cdot & \Bigg((1-p) \int_{\epsilon_s^u}^{\infty} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \\
& + p \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) \Bigg) \\
& + \frac{1}{\lambda} \cdot (1-\rho) \cdot \left(\frac{1-n}{n} \cdot b \right)
\end{aligned}
$$

Inserting the tax into the surplus equation gives:

$$
\begin{aligned}
S = \frac{1}{\lambda}\Bigg[&(1-\rho)\left(z-\Omega-\frac{1-n}{n}\cdot b\right) \\
&+(1-p)(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right) \\
&+(1-p)\int_{\epsilon_s^u}^{\epsilon_{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
&+p(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right) \\
&+p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon))\,dG(\epsilon) \\
&+(1-\rho)(\lambda-f)\cdot S\Bigg]
\end{aligned}
$$

Inserting the free-entry condition

$$\frac{k_v}{q}=J$$

gives:

$$
\begin{aligned}
\frac{1}{1-\eta}\cdot\frac{k_v}{q}=\frac{1}{\lambda}\Bigg[&(1-\rho)\left(z-\Omega-\frac{1-n}{n}\cdot b\right) \\
&+(1-p)(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right) \\
&+p(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right) \\
&+(1-\rho)\cdot\frac{\lambda-\eta\cdot f}{1-\eta}\cdot\frac{k_v}{q}\Bigg]
\end{aligned}
$$

This is the job-creation condition used in the Ramsey problem for STW in Appendix D.

Next, we want to calculate the wage-equation. Remember the surplus splitting rule from Nash-Bargaining:

$$\eta\cdot\bar{J}=(1-\eta)\cdot\frac{\bar{V}-U}{u'(c^w)}$$

Inserting the tax into the value of a worker for a firm gives:

$$\begin{aligned}\bar{J} = \frac{1}{\lambda}\Big[&(1-\rho)(z-\Omega) - p(1-\rho^u)e^w - (1-p)(1-\rho^c)e \\ &- (1-\rho)\cdot\frac{1-n}{n}b + (1-\rho)\lambda\bar{J}\Big]\end{aligned}$$

Likewise, we can calculate the surplus of a worker:

$$\begin{aligned}\bar{V} - U = \frac{1}{\lambda}\Bigg[&(1-p)(1-\rho^u)\left(u(c^w) - u(b)\right) + p(1-\rho^c)\left(u^c - u(b)\right) \\ &+ (1-\rho)(\lambda - f)\cdot(\bar{V} - U)\Bigg]\end{aligned}$$

Next, we can insert the value of a worker for a firm and the surplus of a worker into the surplus splitting rule:

$$\begin{aligned}\frac{\eta}{\lambda}\cdot\Big[\quad &(1-\rho)(z-\Omega) - p(1-\rho^u)\cdot c^w - (1-p)(1-\rho^c)\cdot e^c \\ &- (1-\rho)\cdot\frac{1-n}{n}\cdot b + (1-\rho)\cdot\lambda\cdot\bar{J} \\ = &\frac{1-\eta}{\lambda}\cdot\frac{1}{u'(c^w)}\cdot\Big[(1-p)(1-\rho^u)\cdot(u(c^w) - u(b)) \\ &+ p(1-\rho^c)\cdot(u^c - u(b)) \\ &+ (1-\rho)(\lambda - f)\cdot(\bar{V} - U)\Big]\Big]\end{aligned}$$

Insert the surplus splitting rule again: $\eta\cdot\bar{J} = (1-\eta)\cdot\frac{\bar{V}-U}{u'(c^w)}$

$$
\begin{aligned}
\frac{\eta}{\lambda}\cdot\Big[\quad & (1-\rho)(z-\Omega) - p(1-\rho^u)\cdot c^w - (1-p)(1-\rho^c)\cdot e^c \\
& - (1-\rho)\cdot\frac{1-n}{n}\cdot b + (1-\rho)\cdot\lambda\cdot\bar{J}\Big] \\
= & \frac{1-\eta}{\lambda}\cdot\frac{1}{u'(c^w)}\cdot\Big[(1-p)(1-\rho^u)\cdot(u(c^w)-u(b)) \\
& + p(1-\rho^c)\cdot(u^c - u(b)) \\
& + (1-\rho)(\lambda - f)\cdot u'(c^w)\cdot\frac{\eta}{1-\eta}\cdot\bar{J}\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad & \frac{\eta}{\lambda}\Big[(1-\rho)(z-\Omega) - p(1-\rho^u)\cdot c^w - (1-p)(1-\rho^c)\cdot e^c \\
& - (1-\rho)\cdot\frac{1-n}{n}\cdot b + (1-\rho)\cdot\lambda\cdot f\cdot\bar{J}\Big] \\
= \ & \frac{1-\eta}{\lambda}\cdot\frac{1}{u'(c^w)}\Big[(1-p)(1-\rho^u)(u(c^w)-u(b)) \\
& + p(1-\rho^c)(u^c - u(b))\Big]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad \eta\cdot & \Big[(1-\rho)(z-\Omega) - (1-\rho)\cdot\frac{1-n}{n}\cdot b + (1-\rho)\cdot f\cdot\bar{J}\Big] \\
= & (1-p)(1-\rho^u)(1-\eta)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)} + \eta\cdot c^w\right) \\
& + p(1-\rho^c)(1-\eta)\cdot\left(\frac{u^c - u(b)}{u'(c^w)} + \eta\cdot e^c\right)
\end{aligned}
$$

Inserting the free-entry condition again, we get the wage equation from Appendix D for STW.

$$
\begin{aligned}
& (1-p)(1-\rho^u)(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)} + \eta\cdot c^w \\
& = \eta\cdot\Big[(1-\rho)(z-\Omega) - (1-\rho)\cdot\frac{1-n}{n}\cdot b + (1-\rho)\cdot\theta\cdot k_v\Big] \\
& \quad - p(1-\rho^c)(1-\eta)\cdot\left(\frac{u^c - u(b)}{u'(c^w)} + \eta\cdot e^c\right)
\end{aligned}
$$

D The full Ramsey Problems

D.1 Lay-off Tax

The Ramsey planner's problem reads:

$$\max_{b,\tau_{\mathrm{stw}},\epsilon_{\mathrm{stw}}} n^u \cdot u(c^w) + n^c \cdot u^c + (1-n) \cdot u(b)$$
$$+ v^f \cdot u((n \cdot z - n^u \cdot c^w - n^c \cdot e^c - \tau(b) - \theta \cdot (1-n) \cdot k_v)/v^f)$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$n^u = \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot n^s$$

(II) Separation rate, unconstrained workers:

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1-\rho^c) \cdot n^s$$

(IV) Separation rate, constrained workers:

$$\rho^c = G(\epsilon_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1-p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1-n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of constrained worker:

$$u^c = \frac{1}{1-\rho^c}\left((1 - G(\epsilon^p)) \cdot u(c^w) + \int_{\epsilon_s^c}^{\epsilon^p} u(c(\epsilon))\, dG(\epsilon)\right)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1-\rho^c}\left[(1 - G(\epsilon^p)) \cdot c^w + \int_{\epsilon_s^c}^{\epsilon^p} c^w(\epsilon)\, dG(\epsilon)\right]$$

(X) Average production (without distortions):

$$z = \frac{1}{1-\rho}\left((1-p)\int_{\epsilon_s^u}^{\infty} z(\epsilon)\, dG(\epsilon) + p\int_{\epsilon_s^c}^{\infty} z(\epsilon)\, dG(\epsilon)\right)$$

(XI) Job-creation condition:

$$\begin{aligned}\frac{1}{1-\eta} \cdot \frac{k_v}{q} = \frac{1}{\lambda}\Bigg[&(1-\rho) \cdot \left(z - \frac{1-n}{n} \cdot b\right) \\ &+ (1-p)(1-\rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w\right) \\ &+ (1-p)(1-\rho^c) \cdot \left(\frac{u^c - u(b)}{u'(c^w)} - e^c\right) \\ &+ (1-\rho) \cdot \frac{\lambda - f \cdot \eta}{1-\eta} \cdot \frac{k_v}{q}\Bigg]\end{aligned}$$

(XII) Wage:

$$\begin{aligned}WE =& (1-p) \cdot (1-\rho^u) \cdot \left(\eta \cdot c^w + (1-n) \cdot \frac{u(c^w) - u(b)}{u'(c^w)}\right) \\ &- \eta \cdot (1-\rho) \cdot \left(z - \frac{1-n}{n} \cdot b + \theta \cdot \frac{k_v}{q}\right) \\ &+ p \cdot (1-\rho^c) \cdot \left[(1-n) \cdot \frac{u^c - u(b)}{u'(c^w)} + \eta \cdot e^c\right] = 0\end{aligned}$$

(XIII) Separation condition unconstrained firm:

$$z(\epsilon_s^u) + F + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

(XIV) Threshold at which the constraint is binding:

$$z(\epsilon^p) + \lambda \cdot \frac{k_v}{q} = c^w$$

(XV) Separation condition constrained firm:

$$\frac{u(c^w(\epsilon_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

(XVI) Consumption of constrained worker:

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \frac{k_v}{q}$$

D.2 Short-time Work

The Ramsey planner's problem reads:

$$\begin{aligned} \max_{b, \tau_{\text{stw}}, \epsilon_{\text{stw}}} \; & n^u \cdot u(c^w) + n^c \cdot u^c + (1 - n) \cdot u(b) \\ & + v^f \cdot u((n \cdot z - n^u \cdot c^w - n^c \cdot e^c - \tau(b) - \theta \cdot (1 - n) \cdot k_v)/v^f) \end{aligned}$$

subject to the following constraints:

(I) Number of unconstrained workers:

$$n^u = \frac{1 - p}{\lambda} \cdot (1 - \rho^u) \cdot n^s$$

(II) Separation rate (unconstrained workers):

$$\rho^u = G(\epsilon_s^u)$$

(III) Number of constrained workers:

$$n^c = \frac{p}{\lambda} \cdot (1 - \rho^c) \cdot n^s$$

(IV) Separation rate (constrained workers):

$$\rho^c = G\left(\max\{\xi_s^c, \epsilon_{\text{stw}}\}\right) - G\left(\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}\right) + G(\xi_s^c)$$

(V) Aggregate separation rate:

$$\rho = (1 - p) \cdot \rho^u + p \cdot \rho^c$$

(VI) Number of firms that are hit by a shock:

$$n^s = \theta \cdot q(\theta) \cdot (1 - n) + \lambda \cdot n$$

(VII) Total employment:

$$n = n^u + n^c$$

(VIII) Average utility of a constrained worker:

$$u^c = \frac{1}{1 - \rho^c} \Bigg((1 - G(\epsilon^p)) \cdot u(c^w) + \int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon_p} u(c^w(\epsilon))\, dG(\epsilon) \\ + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} u(c_{\text{stw}}^w(\epsilon)) dG(\epsilon) \Bigg)$$

(IX) Average cost of a constrained worker for a firm:

$$e^c = \frac{1}{1 - \rho^c} \Bigg[(1 - G(\epsilon^p)) \cdot c^w + \int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon_p} c^w(\epsilon)\, dG(\epsilon) \\ + \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \left(c_{\text{stw}}^w(\epsilon) - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon)) \right) dG(\epsilon) \Bigg]$$

(X) Average production (without distortions):

$$z = \frac{1}{1-\rho}\left((1-p)\int_{\xi_s^u}^{\infty} z(\epsilon)\,dG(\epsilon) + p\int_{\max\{\xi_s^c,\epsilon_{\text{stw}}\}}^{\infty} z(\epsilon)\,dG(\epsilon) + \int_{\epsilon_s^c}^{\max\{\epsilon_s^c,\epsilon_{\text{stw}}\}} z(\epsilon)\,dG(\epsilon)\right)$$

(XI) Average distortion of working hours:

$$\Omega = \frac{1}{1-\rho}\left((1-p)\int_{\epsilon_s^u}^{\epsilon_{\text{stw}}} \Omega(\epsilon)\,dG(\epsilon) + p\int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}} \Omega(\epsilon)\,dG(\epsilon)\right)$$
$$\text{with} \quad \Omega(\epsilon) = \bar{z}(\epsilon) - z_{\text{stw}}(\epsilon)$$

(XII) Job-creation condition:

$$\frac{1}{1-\eta}\cdot\frac{k_v}{q} = \frac{1}{\lambda}\Bigg[(1-\rho)(z-\Omega-\frac{1-n}{n}b) + p(1-\rho^u)\left(\frac{u(c^w)-u(b)}{u'(c^w)} - c^w\right) + (1-p)(1-\rho^c)\left(\frac{u^c-u(b)}{u'(c^w)} - e^c\right) + (1-\rho)\cdot\frac{\lambda-\eta\cdot f}{\eta}\cdot\frac{k_v}{q}\Bigg]$$

(XIII) Wage:

$$WE = (1-p)\cdot(1-\rho^u)\left(\eta\cdot c^w + (1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right) - \eta(1-\rho)\left(z - \frac{1-n}{n}b + \theta\cdot k_v\right) + p\cdot(1-\rho^c)\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)} + \eta\cdot e^c\right] = 0$$

(XIV) Separation condition for unconstrained firm without STW:

$$z(\xi_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XV) Separation condition for unconstrained firm with STW:

$$z_{stw}(\epsilon_s^u) + \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XVI) Threshold at which the constraint is binding:

$$z(\epsilon_p) + \lambda \cdot \frac{k_v}{q} = c^w$$

(XVII) Separation condition, constrained firm without STW:

$$\frac{u(c^w(\xi_s^c)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XVIII) Separation condition, constrained firm with STW:

$$\frac{u(c^w_{stw}(\epsilon_s^c)))}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(XIX) Consumption, constrained worker outside STW

$$c^w(\epsilon) = z(\epsilon) + \lambda \cdot \frac{k_v}{q}$$

(XX) Consumption, constrained worker on STW

$$c^w_{stw}(\epsilon) = z_{stw}(\epsilon) + \tau_{stw}(\bar{h} - h_{stw}(\epsilon)) + \lambda \cdot \frac{k_v}{q}$$

E Ramsey FOCs with Lay-off Tax

In the following, multipliers from the Lagrangian, implied by the maximization problem from the previous section, are denoted by λ_{idx}, the index depending on the constraint. Here, λ_n denotes the Lagrange multiplier for the total employment equation, λ_{n^u} for the number of unconstrained firms, λ_{n^c} for the number of constrained firms, λ_{n^s} for the number of firms that received a shock, λ_θ for the job-creation condition, λ_c for the wage equation, $\lambda_{\epsilon_s^u}$ for the separation condition of unconstrained firms and $\lambda_{\epsilon_s^c}$ for the separation condition of constrained firms. Every other equation listed in the Ramsey problem for the lay-off tax is assumed to be plugged in.

E.1 Employment (lay-off tax)

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n^s} &= -\lambda_{n^s} + \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot (u(c^w) - u'(c^w) \cdot c^u) \\ &+ \frac{p}{\lambda} \cdot (1-\rho^c) \cdot (u(c^w) - u'(c^w) \cdot e^c) \\ &+ \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \lambda_{n^u} + \frac{p}{\lambda} \cdot (1-\rho^c) \cdot \lambda_{n^c} = 0\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial n^u} = -\lambda_{n^u} + \lambda_n = 0$$

$$\frac{\partial \mathcal{L}}{\partial n^c} = -\lambda_{n^c} + \lambda_n = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n} &= -\lambda_n + \lambda_{n^s} \cdot (\lambda - q(\theta)\cdot\theta) + u'(c^w) \cdot [z + b + \theta \cdot c] - u(b) \\ &+ \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \lambda_\theta + \eta(1-\rho) \cdot \frac{b}{n^2} \cdot \lambda_c = 0\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad \frac{\lambda_n}{u'(c^w)} &= (\lambda - q(\theta)\cdot\theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + [z + b + \theta \cdot c] - \frac{u(b)}{u'(c^u)} \\ &+ \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \frac{\lambda_\theta}{u'(c^w)} + \frac{\lambda \cdot b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)}\end{aligned}$$

E.2 Optimal Job Creation Condition (lay-off tax)

Before we begin, let us define the insurance effect as:

$$\tilde{IE}_\theta = \frac{p}{\lambda}\left(\int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)}\, dG(\epsilon)\right) \cdot k_v$$

$$IE_\theta = n^s \cdot u'(c^w) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \tilde{IE}_\theta$$

The FOC for labor market tightness denotes:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \theta} &= -k_v(1-n)u'(c^w) + IE_\theta \cdot k_v + (1-\gamma)q(\theta)(1-n)\lambda_{n^s} \\ &\quad - \frac{\gamma}{1-\eta}k_v\lambda_\theta + \frac{\lambda\gamma - f\eta}{(1-\eta)f}\left(\frac{1-\rho}{\lambda}\lambda_\theta - \lambda_{\epsilon_s^u}\right) \\ &\quad - \left(\lambda \cdot \frac{\gamma}{f}u'(c(\epsilon_s^u)) + \frac{\lambda\gamma - f}{(1-\eta)f}\eta\right)\lambda_{\epsilon_s^c} \\ &\quad - \lambda_c \cdot \frac{\partial WE}{\partial \theta}\end{aligned}$$

$$\Leftrightarrow \quad \frac{\lambda_{n^s}}{u'(c^w)} = \frac{1+\chi}{1-\eta} \cdot \frac{k_v}{q}$$

with

$$\begin{aligned}\chi = \frac{1}{u \cdot u'(c^w)} \cdot &\left[\frac{1}{f} \cdot \frac{1}{1-\eta} \cdot \left(\gamma\lambda_\theta - (\lambda\gamma - f\eta)\left(\frac{1-\rho}{\lambda}\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right)\right. \\ &+ \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w(\epsilon_s^u)) + \frac{\lambda\gamma - f}{(1-\eta)f} \cdot \eta\right)\lambda_{\epsilon_s^c} \\ &\left. + \frac{1}{k_v} \cdot \left(\lambda_c \cdot \frac{\partial WE}{\partial \theta} - IE_\theta\right)\right]\end{aligned}$$

Using the FOCs for employment and labor market tightness gives:

$$\begin{aligned}
u'(c^w) \cdot \frac{1+\chi}{1-\gamma} \cdot \frac{k_v}{q} = &\frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot (u(c^w) - u(b) - u'(c^w) \cdot c^w) \\
&+ \frac{p}{\lambda} \cdot (1-\rho^c) \cdot (u^c - u(b) - u'(c^w) \cdot e^c) \\
&+ u'(c^w) \cdot \frac{1-\rho}{\lambda} \cdot \left(z + b + \frac{\lambda - \gamma f + \chi(\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \right) \\
&+ \frac{1-\rho}{\lambda} \cdot \frac{\lambda_\theta}{n^s} \cdot \frac{b}{n} + \frac{1-\rho}{n^s} \cdot \frac{b}{n} \cdot \lambda_c
\end{aligned}$$

Rearranging gives the *Optimal Job Creation Condition:*

$$\begin{aligned}
\frac{1+\chi}{1-\gamma} \cdot \frac{k_v}{q} = &\frac{1-\rho}{\lambda} \cdot (z+b) + \frac{1-\rho}{\lambda} \cdot \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \frac{b}{n} + \frac{1-\rho}{\lambda} \cdot \frac{\lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \cdot \frac{b}{n} \\
&+ \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \left(\frac{u(c^w) - u(b)}{u'(c^w)} - c^w \right) \\
&+ \frac{p}{\lambda} \cdot (1-\rho^c) \cdot \left(\frac{u^c - u(b)}{u'(c^w)} - e^c \right) \\
&+ \frac{1-\rho}{\lambda} \cdot \frac{\lambda - \gamma f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q}
\end{aligned}$$

Subtracting the decentralized job-creation condition from the optimal gives:

$$\begin{aligned}
\left(\chi - \frac{\eta - \gamma}{1-\eta} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} = &\left(1 + \frac{\lambda_\theta + \eta + \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \right) \cdot \frac{1-\rho}{\lambda} \cdot \frac{b}{n} \\
&+ \frac{(1-\rho) \cdot (\lambda - f)}{\lambda} \cdot \left(\chi - \frac{\eta - \gamma}{1-\gamma} \right) \cdot \frac{k_v}{q}
\end{aligned}$$

Rearranging gives:

$$\begin{aligned}
\left(\chi - \frac{\eta - \gamma}{1-\eta} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} &= \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \right) \cdot \frac{\frac{1-\rho}{\lambda}}{\rho + (1-\rho) \cdot \frac{f}{\lambda}} \cdot \frac{b}{n} \\
&= \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \right) \cdot \frac{b}{f}
\end{aligned}$$

E.3 Optimal separation condition (unconstrained firms)

FOC for the separation threshold of unconstrained firms:

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^u} = & \frac{n}{1-\rho} \cdot (1-p) \cdot g(\epsilon_s^u) \cdot u'(c^w) \cdot \left(z(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w - z \right) \\
& + \lambda_\theta \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
& + \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot \lambda_{n^u} + \lambda_{\epsilon_s^u} \cdot \frac{\partial S_{lt}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

Insert for λ_{n^u} and subtract decentralized separation condition of unconstrained firms:

$$
\begin{aligned}
& g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \cdot \left[z(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w + \frac{\lambda_n}{u'(c^w)} \right] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[\lambda F + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

Insert for λ_n:

$$
\begin{aligned}
& g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \left[z(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w - z \right] \\
& + g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \left[(\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + (z + b + \theta \cdot c) - \frac{u(b)}{u'(c^w)} \right. \\
& \qquad \left. + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \frac{\lambda_\theta}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} \right] \\
& + \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \left[\lambda F + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

$$
\begin{aligned}
\iff \quad & z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(\lambda F + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

Inserting λ_n^s gives the *Optimal Separation Condition*:

$$
\begin{aligned}
& z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot (\lambda \cdot F - b) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) = 0
\end{aligned}
$$

Subtracting the decentralized separation condition and rearranging gives:

$$
\begin{aligned}
\lambda \cdot F = b &+ (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\gamma} \right) \cdot \frac{k_v}{q} \\
&+ \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \\
&- \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot (\lambda F - b) \\
&+ \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)
\end{aligned}
$$

$$
\begin{aligned}
\iff 0 = & \left(1 + \frac{\lambda_\theta}{u'(c^w) \cdot n^s} \right) \cdot \left(b + (\lambda - f) \cdot \frac{\frac{1-\rho}{\lambda}}{\rho + (1-\rho) \cdot \frac{f}{\lambda}} \cdot \frac{b}{n} - \lambda \cdot F \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
& + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\partial S_{\text{lt}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}
\end{aligned}
$$

$$
\iff \quad 0 = \left(1 + \frac{\lambda_\theta}{u'(c^w)\cdot n^s}\right)\cdot\left(b + (\lambda - f)\cdot\frac{b}{f} - \lambda F\right)
+ \frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)}\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}
+ \frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n} + \frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial \epsilon_s^u}\right)
$$

$$
\iff \quad 0 = \left(1 + \frac{\lambda_\theta}{u'(c^w)\cdot n^s}\right)\cdot\left(\frac{\lambda}{f}\cdot b - \lambda F\right)
+ \frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)}\cdot\frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u}
+ \frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n} + \frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial \epsilon_s^u}\right)
$$

Rearranging gives the Lagrange multiplier for unconstrained firms:

$$
-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} = \frac{1-p}{\lambda}\cdot\frac{g(\epsilon_s^u)}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}}\cdot\left(\left(1 + \frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b - \lambda F\right)
+ \frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n} + \frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial \epsilon_s^u}\right)\right)
$$

E.4 Optimal separation condition (constrained firms)

The FOC for the separation threshold in the constrained firm is:

$$
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \frac{n}{1-\rho}\cdot p\cdot g(\epsilon_s^c)\cdot u'(c^w)\left(z(\epsilon_s^c) + \frac{u(c^w(\epsilon_s^c)) - u(c^w)}{u'(c^w)} - c^w(\epsilon_s^c) - z\right)
+ \lambda_\theta\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\left[z(\epsilon_s^c) + \frac{c^w(\epsilon_s^c) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) - \frac{1-n}{n}\cdot b + \frac{\lambda - \eta f}{1-\eta}\cdot\frac{k_v}{q}\right]
+ \frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot\lambda_{n^c}
+ \lambda_c\cdot\frac{\partial WE}{\partial \epsilon_s^c}
+ \lambda_{\epsilon_s^c}\cdot\frac{\partial S^c_{\text{lt}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
$$

Note that we can reformulate the decentralized separation threshold of a worker in a constrained firm as:

$$
\begin{aligned}
& \frac{u(c^w(\epsilon_s^c))-u(b)}{u'(c^w)}+(\lambda-f)\cdot\frac{\eta}{1-\eta}\cdot\frac{k_v}{q}=0 \\
\iff \quad & \frac{u(c^w(\epsilon_s^c))-u(b)}{u'(c^w)}+(\lambda-f)\cdot\frac{\eta}{1-\eta}\cdot\frac{k_v}{q}=c^w(\epsilon_s^c)-z(\epsilon_s^c)-\lambda\cdot\frac{k_v}{q} \\
\iff \quad & z(\epsilon_s^c)+\frac{u(c^w(\epsilon_s^c))-u(b)}{u'(c^w)}-c^w(\epsilon_s^c)+\frac{\lambda-\eta f}{1-\eta}\cdot\frac{k_v}{q}=0
\end{aligned}
$$

This simplifies the equation to:

$$
\begin{aligned}
-\frac{\partial\mathcal{L}}{\partial\epsilon_s^c} &= \frac{n}{1-\rho}\cdot p\cdot g(\epsilon_s^c)\cdot\left(z(\epsilon_s^c)+\frac{u(c^w(\epsilon_s^c))}{u'(c^w)}-c^w(\epsilon_s^c)-z\right) \\
&\quad+\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot\frac{\lambda_{n^c}}{u'(c^w)} \\
&\quad-\frac{\lambda_\theta}{u'(c^w)}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{1-n}{n}\cdot b \\
&\quad+\frac{\lambda_c}{u'(c^w)}\cdot\frac{\partial WE}{\partial\epsilon_s^c}+\frac{\lambda_{\epsilon_s^c}}{u'(c^w)}\cdot\frac{\partial S^c(\epsilon_s^c)}{\partial\epsilon_s^c}\overset{!}{=}0
\end{aligned}
$$

Inserting λ_n^s gives the *Optimal Separation Condition*

$$
\begin{aligned}
-\frac{\partial\mathcal{L}}{\partial\epsilon_s^c} &= z(\epsilon_s^c)+\frac{u(c^w(\epsilon_s^c))}{u'(c^w)}-c^w(\epsilon_s^c)+b+\frac{\lambda-\gamma\cdot f+\chi\cdot(\lambda-f)}{1-\gamma}\cdot\frac{k_v}{q} \\
&\quad+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\cdot b \\
&\quad+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right) \\
&\quad+\frac{\lambda}{p}\cdot\frac{1}{n^s\cdot g(\epsilon_s^c)}\cdot\frac{\lambda_{\epsilon_s^c}}{u'(c^w)}\cdot\frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial\epsilon_s^c}=0
\end{aligned}
$$

Subtracting the decentralized separation condition gives:

$$
\begin{aligned}
0 &= b+(\lambda-\theta q(\theta))\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} \\
&\quad+\frac{\lambda\cdot\epsilon_s^c}{n^s\cdot u'(c^w)}\cdot\frac{1}{\rho}\cdot\frac{1}{\lambda}\cdot\frac{\partial S_{\text{lt}}^c(\epsilon_s^c)}{\partial\epsilon_s^c}\cdot\frac{1}{g(\epsilon_s^c)} \\
&\quad+\frac{\lambda\cdot\theta}{n^s\cdot u'(c^w)}\cdot b+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)
\end{aligned}
$$

Rearranging gives back the Lagrange multiplier:

$$-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \right)$$

E.5 The Optimal Lay-Off Tax

We start from

$$\frac{\partial \mathcal{L}}{\partial F} = -\lambda_{\epsilon_s^u} = 0$$

Note that the optimal lay-off tax sets the Lagrange multiplier for the separation condition equal to zero. This implies that the lay-off tax implements the optimal separation threshold for unconstrained firms. Then

$$-\frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^u)}{\frac{\partial S^u_{\text{lt}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \right.$$
$$\left. + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \right)$$

This leads to:

$$F = \frac{1}{f} \cdot b + BE_{lt}$$
$$BE_{lt} = \frac{1}{\lambda} \frac{1}{\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right)} \cdot \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)$$

E.6 Optimal UI given Lay-off Tax

From the FOC of UI, we can derive the optimality condition for unemployment insurance. With an optimal lay-off tax, the Lagrange multiplier for the separation condition of the unconstrained firm is equal to zero.

$$
\begin{aligned}
(1-\eta)\cdot\left(u'(b)-u'(c^w)\right) = {} & \lambda_\theta\cdot(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)\\
& + \left(\lambda_{\epsilon_s^c}+\lambda_{\xi_s^c}\right)\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)\\
& + \lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)
\end{aligned}
$$

To get better insight, we need to determine the Lagrange multipliers for λ_θ, λ_c. We already know the Lagrange multipliers for the separation conditions of constrained firms:

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} = {} & \frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}}\cdot\Bigg[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right)\\
& + \frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} = {} & \frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{1}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial\epsilon_s^c}}\cdot\Bigg[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\\
& + \frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\Bigg]
\end{aligned}
$$

First, let us find an expression for λ_c:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c^w} = &-\Bigg((1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} \\
&+p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta \\
&-(1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) \\
&\quad -p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_c \\
&+\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^u} \\
&+\frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c}=0
\end{aligned}$$

Next we can solve for the Lagrange multiplier of the wage equation:

$$\begin{aligned}
\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}\Bigg[&\frac{\partial \epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\cdot n^s\cdot u'(c^w)\cdot\left(\frac{\lambda}{f}\cdot b-\lambda\cdot F\right) \\
&+\frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot n^s\cdot u'(c^w)\cdot\frac{\lambda}{f}\cdot b \\
&+\left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right)\cdot\lambda_\theta\Bigg]
\end{aligned}$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \left(n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \right.$$
$$\left. + \lambda_c \cdot \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^u} \right) \right)$$

$$\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \right.$$
$$\left. + \lambda_c \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^c} \right) \right)$$

Inserting λ_c gives:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \left(n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \right.$$
$$+ \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial c^w}{\partial n} \right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u} \right)^{\text{total}} \right)$$
$$\cdot \left[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \right.$$
$$\left.\left. + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w} \right) \cdot \lambda_\theta \right] \right)$$

$$\lambda_{\epsilon_s^c} = \frac{1}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \right.$$
$$+ \left(\left(\frac{\partial n}{\partial \epsilon_s^c} \right) \cdot \left(-\frac{\partial c^w}{\partial n} \right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c} \right)^{\text{total}} \right)$$
$$\cdot \left[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \right.$$
$$\left.\left. + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w} \right) \cdot \lambda_\theta \right] \right)$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(- n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) + \lambda_c \cdot \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^u}\right)\Bigg)$$

$$\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b + \lambda_c \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial WE}{\partial n}\right) + \frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg)$$

Inserting λ_c gives:

$$\begin{aligned}\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}} \cdot \Bigg(& n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(\lambda + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ & + \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right) \\ & \cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ & + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta\Bigg]\Bigg)\end{aligned}$$

$$\begin{aligned}\lambda_{\epsilon_s^c} = -\frac{1}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(& n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \\ & + \left(\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right) \\ & \cdot \Bigg[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda \cdot F\right) \\ & + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta\Bigg]\Bigg)\end{aligned}$$

Now we have everything to calculate λ_θ from the two equations:

$$(1) \quad \left(\chi - \frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}$$

$$\begin{aligned}(2) \quad \chi = & \frac{1}{1-\eta}\cdot\frac{1}{u\cdot f}\cdot\frac{1}{u'(c^w)}\cdot\left[\gamma-(\lambda\gamma-f\eta)\cdot\left(\frac{1}{\lambda}(1-\rho)\lambda_\theta-\lambda_{\epsilon_s^u}\right)\right]\\ & +\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w)+\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot\lambda_{\epsilon_s^c}\\ & +\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot\lambda_c\cdot\frac{\partial WE}{\partial\theta}\cdot\frac{1}{k_v}\\ & -\frac{1}{u}\cdot\frac{1}{u'(c^w)}\cdot IE_\theta\end{aligned}$$

We can rearrange (1) to:

$$\chi\cdot k_v = (1-\gamma)\cdot q\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)$$

Note that

$$(1-\gamma)\cdot q = \frac{\partial f}{\partial\theta}$$

This follows from the fact that:

$$f'(\theta) = q(\theta)+\theta\cdot q'(\theta) = q(\theta)\cdot\left(1+\frac{q'(\theta)\cdot\theta}{q(\theta)}\right) = q(\theta)\cdot(1-\gamma)$$

So we can write:

$$\chi\cdot k_v = \left(\frac{\partial f}{\partial\theta}\right)\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)$$

Inserting χ gives:

$$\begin{aligned}&\left[\gamma-(\lambda\gamma-f\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta\\ = & \, u'(c^w)\cdot\left(\frac{\partial f}{\partial\theta}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right)\\ & +(\lambda\gamma-f\eta)\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot(-\lambda_{\epsilon_s^u})\end{aligned}$$

$$+\left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\epsilon_s^c})$$

$$+\frac{\partial WE}{\partial \theta} \cdot (-\lambda_c) \quad + IE_\theta$$

Inserting $\lambda_{\epsilon_s^u}, \lambda_{\epsilon_s^c}, IE_\theta$ gives

$$\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
&= u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \lambda F\right) \\
&\quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{\lambda}{f} \cdot b \\
&\quad + \lambda_\theta \cdot \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total},2} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + u'(c^w) \cdot n^s \cdot \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \tilde{IE}_\theta
\end{aligned}$$

with

$$\begin{aligned}
\left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} &= \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^u}{\partial c^w}\right) \\
&\quad + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right] \cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w}\right) \\
&\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^u}{\partial c^w}
\end{aligned}$$

$$\begin{aligned}
\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} &= \Bigg[\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^c}{\partial c^w}\right) \\
&\quad + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \cdot \frac{\partial \epsilon_s^c}{\partial c^w} \\
&\quad + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^c}{\partial c^w}\Bigg]
\end{aligned}$$

$$\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} = \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right)$$
$$+ \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right)$$
$$+ f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}$$

This allows us to calculate the Lagrange multiplier for the job-creation condition

$$M \cdot \lambda_\theta = u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right)$$
$$+ u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - \lambda F\right)$$
$$+ u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b$$
$$+ u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta$$

with

$$M = \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f}$$
$$+ \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f}\right)$$
$$+ \left(\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot \left(\frac{\lambda}{f} b - \lambda \cdot F\right)$$
$$+ \left(\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b$$
$$+ \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}}$$
$$+ \tilde{IE}_\theta$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase of the joint surplus on θ:

$$\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}$$

Rearranging for λ_θ gives:

$$\lambda_\theta = u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right)$$
$$+ u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \cdot \frac{\partial n^u}{\partial \epsilon_s^u} \cdot \left(\frac{\lambda}{f}b - \lambda F\right)$$
$$+ u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f}b$$
$$+ u'(c^w) \cdot n^c \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta$$

Note that

$$\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$
$$\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{total}}$$
$$\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}$$

Further define:

$$\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)$$

Finally, we can calculate the Lagrange multiplier for λ_c by inserting into λ_θ:

$$\lambda_c = \frac{-1}{\left(\frac{\partial S}{\partial c^w}\right)^{\text{total}}} \cdot \Bigg\{$$
$$\left(\frac{\partial \epsilon_s^u}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f}b - \lambda F\right)$$
$$+ \left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f}b$$
$$+ \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right)$$
$$- n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{ge}} \cdot \left(\frac{\partial \theta}{\partial S}\right) \cdot \hat{IE}_\theta \Bigg\}.$$

Simplifying:

$$\lambda_c = -\frac{u'(c^w)}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \left[\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^u}{\partial \epsilon_s^u} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b \right.$$
$$\left. + \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + n^c \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \right]$$

Following the same arguments, we can express the Lagrange multiplier for the separation conditions as:

$$\lambda_{\epsilon_s^u} = -\frac{u'(c^w)}{\left(\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}\right)^{\text{total}}} \left[\left(\frac{\partial n^u}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b \right.$$
$$+ \frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right)$$
$$\left. + n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \right]$$

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}} \left[\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c} \cdot \left(\frac{\partial n^u}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) \right.$$
$$+ \left(\frac{\partial n^c}{\partial \epsilon_s^c}\right)^{\text{ge}} \cdot \frac{\lambda}{f} b + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right)$$
$$\left. + n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \right]$$

For convenience, the optimal UI benefits are replicated here:

$$(1-n) \cdot (u'(b) - u'(c^w)) = \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)}\right) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$
$$+ (\lambda_{\epsilon_s^u} + \lambda_{\epsilon_s^c}) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$
$$+ \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} + (1-\eta) \cdot \frac{u'(b)}{u'(c^w)}\right)$$

Inserting the Lagrange multiplier gives:

$$
\begin{aligned}
&(1-n)\cdot(u'(b)-u'(c^w)) \\
&\quad = u'(c^w)\cdot\left[\frac{\partial c^w}{\partial \epsilon_s^u}\cdot\left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right)\right] \\
&\quad + u'(c^w)\cdot\left[\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^u}{\partial \epsilon_s^u}\cdot\left(\frac{\lambda}{f}b-\lambda F\right)\right] \\
&\quad + u'(c^w)\cdot\left[\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}\cdot\frac{\lambda}{f}b\right] \\
&\quad + u'(c^w)\cdot\left[n^c\cdot\left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}}\cdot \hat{IE}_\theta\right].
\end{aligned}
$$

With:

$$
\begin{aligned}
\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^u}{\partial \epsilon_s^u} = {} & \underbrace{\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^u}{\partial \epsilon_s^u}}_{\text{direct effects}} \\
& + \underbrace{\left[\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}+\frac{\partial \epsilon_s^c}{\partial b}\cdot\left(\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\cdot\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n^u}{\partial \epsilon_s^u}}_{\text{indirect effect}}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c} = {} & \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{direct effects}} \\
& + \underbrace{\left[\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}+\frac{\partial \epsilon_s^u}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{indirect effect}}
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{\partial f}{\partial b}\right)^{\text{ge}} = {} & \underbrace{\left(\frac{\partial S}{\partial b}\cdot\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}\right)}_{\text{direct effect}} \\
& + \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^u}+\frac{\partial \epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}
\end{aligned}
$$

$$\left(\frac{\partial\theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\cdot\left(\frac{\partial\theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}}$$
$$+\underbrace{\left[\frac{\partial\epsilon_s^u}{\partial b}\cdot\frac{\partial c^w}{\partial\epsilon_s^u}+\frac{\partial\epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial\epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial\theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

E.7 Optimal UI under Optimal Lay-off Tax

From the FOC of UI, we can derive the optimality condition for unemployment insurance. In contrast to the section where we take lay-off taxes as given, the government implements the optimal lay-off tax. This implies that the Lagrange multiplier of the separation condition in unconstrained firms is equal to zero. This is also fundamental for the optimality condition for the UI benefits:

$$(1-\eta)\cdot(u'(b)-u'(c^w)) = \lambda_\theta\cdot(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right)+\lambda_{\epsilon_s^u}\cdot\left(-\frac{u'(b)}{u'(c^w)}\right)$$
$$+\lambda_c\cdot(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)$$

The Lagrange multiplier for the separation condition of unconstrained firms is equal to

$$-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} = \frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\frac{1}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial\epsilon_s^c}}\cdot\Bigg[\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b$$
$$+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\Bigg]$$

First, let us find an expression for λ_c:

$$\frac{\partial \mathcal{L}}{\partial c^w} = -\Bigg((1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} + p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta - (1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) - p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_c + \frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c} = 0$$

Insert Lagrange multipliers for separation condition:

$$-\Bigg((1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} + p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta - (1-\eta)\cdot(1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\cdot p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_c + \frac{\partial \epsilon_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\epsilon_s^c)\cdot\Bigg((n^s\cdot u'(c^w)+\lambda_\theta)\cdot\left(\frac{\lambda}{f}\cdot b\right) + \lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial \epsilon_s^c}\right)\Bigg) = 0$$

Rearranging gives:

$$
\begin{aligned}
0 = {} & \lambda_\theta \cdot \frac{\partial S^{\text{total}}}{\partial c^w} \\
& + \lambda_c \cdot \frac{\partial WE^{\text{total}}}{\partial c^w} \\
& + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial S^{\text{total}}}{\partial c^w} = & \Bigg[p(1-\rho^u) \frac{u(c^w) - u(b)}{u'(c^w)} \frac{u''(c^w)}{u'(c^w)} \\
& - (1-p)(1-\rho^c) \frac{u^c - u(b)}{u'(c^w)} \frac{u''(c^w)}{u'(c^w)} \\
& + \frac{\partial \epsilon_s^c}{\partial \theta} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\lambda}{f} b \Bigg]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial WE^{\text{total}}}{\partial c^w} = & \Bigg[-(1-\eta)(1-p\rho^u) \left(1 - (1-\eta) \frac{u(c^w) - u(b)}{u'(c^w)} \frac{u''(c^w)}{u'(c^w)} \right) \\
& + (1-(1-p)\rho^c)(1-\eta) \frac{u^c - u(b)}{u'(c^w)} \frac{u''(c^w)}{u'(c^w)} \\
& + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \lambda_c \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg]
\end{aligned}
$$

Next we can solve for the Lagrange multiplier of the wage equation:

$$
\begin{aligned}
\lambda_c = - \frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} & \Bigg[\frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b \\
& + \left(- \frac{\partial S^{\text{total}}}{\partial c^w} \right) \cdot \lambda_\theta \Bigg]
\end{aligned}
$$

Now we have everything to calculate λ_θ from the two equations:

$$(1) \quad \left(\chi - \frac{\eta-\gamma}{1-\eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} = \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}$$

$$\begin{aligned}
(2) \quad \chi = & \frac{1}{1-\eta} \cdot \frac{1}{u \cdot f} \cdot \frac{1}{u'(c^w)} \cdot \left[\gamma - (\lambda\gamma - f\eta) \cdot \left(\frac{1}{\lambda}(1-\rho)\lambda_\theta\right)\right] \\
& + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot \lambda_{\epsilon_s^c} \\
& + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \lambda_c \cdot \frac{\partial WE}{\partial \theta} \cdot \frac{1}{k_v} \\
& - \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot IE_\theta
\end{aligned}$$

We can rearrange (1) to:

$$\chi \cdot k_v = (1-\gamma) \cdot q \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right)$$

Note that

$$(1-\gamma) \cdot q = \frac{\partial f}{\partial \theta}$$

This follows from the fact that:

$$f'(\theta) = q(\theta) + \theta \cdot q'(\theta) = q(\theta) \cdot \left(1 + \frac{q'(\theta) \cdot \theta}{q(\theta)}\right) = q(\theta) \cdot (1-\gamma)$$

So we can write:

$$\chi \cdot k_v = \left(\frac{\partial f}{\partial \theta}\right) \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right)$$

Inserting χ gives:

$$\begin{aligned}
& \left[\gamma - (\lambda\gamma - f\eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \cdot \lambda_\theta \\
= & \, u'(c^w) \cdot \left(\frac{\partial f}{\partial \theta}\right) \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
& + \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot k_v \cdot (-\lambda_{\epsilon_s^c})
\end{aligned}$$

$$+\frac{\partial WE}{\partial \theta}\cdot(-\lambda_c) \quad + IE_\theta$$

Inserting $\lambda_{\epsilon_s^c}, IE_\theta$ gives

$$\begin{aligned}
&\left[\gamma-(\lambda\gamma-f\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta\\
&=u'(c^w)\cdot\frac{\partial f}{\partial\theta}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{b}{f}\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c)\cdot\frac{p}{\lambda}\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\\
&\quad+\lambda_\theta\cdot\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total,2}}\cdot\left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}}\\
&\quad+u'(c^w)\cdot n^s\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\tilde{IE}_\theta
\end{aligned}$$

with

$$\begin{aligned}
\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}&=\left[\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)+\left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^c}{\partial c^w}\right)\right.\\
&\quad\left.+\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)\cdot\left(\frac{\partial n}{\partial\epsilon_s^c}\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial\epsilon_s^c}\right)^{\text{total}}\right)\right]\cdot\frac{\partial\epsilon_s^c}{\partial c^w}\\
&\quad+f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^c}{\partial c^w}
\end{aligned}$$

$$\begin{aligned}
\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total,2}}&=\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\\
&\quad+\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)\cdot\left(\frac{\partial n}{\partial\epsilon_s^c}\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial\epsilon_s^c}\right)^{\text{total}}\right)\\
&\quad+f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}
\end{aligned}$$

This allows us to calculate the Lagrange multiplier for the job-creation condition

$$\begin{aligned}
M\cdot\lambda_\theta&=u'(c^w)\cdot\frac{\partial f}{\partial\theta}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{b}{f}\right)\\
&\quad+u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c)\cdot\frac{p}{\lambda}\cdot\frac{\lambda}{f}b\\
&\quad+u'(c^w)\cdot n^s\cdot\tilde{IE}_\theta
\end{aligned}$$

with

$$
\begin{aligned}
M = & \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \\
& + \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f}\right) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \\
& + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
& + \tilde{IE}_\theta
\end{aligned}
$$

Note: $\frac{1}{M}$ denotes the general equilibrium effect of an increase of the joint surplus on θ:

$$
\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}
$$

Rearranging for λ_θ gives:

$$
\begin{aligned}
\lambda_\theta = & \, u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right) \\
& + u'(c^w) \cdot \left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b \\
& + u'(c^w) \cdot n^c \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta
\end{aligned}
$$

Note that

$$
\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}
$$

$$
\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}
$$

Further define:

$$
\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \int_{\epsilon_s^c}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon)
$$

Finally, we can calculate the Lagrange multiplier for λ_c by inserting into λ_θ:

$$\lambda_c = \frac{-1}{\left(\frac{\partial S}{\partial c^w}\right)^{\text{total}}}\Bigg[\left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} b$$

$$+ \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \cdot \left(-\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right)$$

$$+ n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{ge}} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right) \cdot \hat{IE}_\theta \Bigg]$$

Simplifying:

$$\lambda_c = -\frac{u'(c^w)}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}}\Bigg[\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^u}{\partial \epsilon_s^u} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}} \cdot \frac{\partial n^c}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b$$

$$+ \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + n^c \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

Following the same arguments, we can express the Lagrange multiplier for the separation condition as:

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}}\Bigg[\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c} \cdot \left(\frac{\partial n^u}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \lambda F\right) + \left(\frac{\partial n^c}{\partial \epsilon_s^c}\right)^{\text{ge}} \cdot \frac{\lambda}{f} b$$

$$+ \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right) + \frac{\partial c^w}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

For convenience, the optimal UI benefits are replicated here:

$$(1-n) \cdot (u'(b) - u'(c^w)) = \lambda_\theta \cdot (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)}\right) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$

$$+ \lambda_{\epsilon_s^c} \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$

$$+ \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} + (1-\eta) \cdot \frac{u'(b)}{u'(c^w)}\right)$$

Inserting the Lagrange multiplier gives

$$(1-n)\cdot(u'(b)-u'(c^w)) = u'(c^w)\cdot\Bigg[\left(-\frac{\partial f}{\partial b}\right)^{\text{ge}}\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right)$$
$$+\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}\cdot\frac{\lambda}{f}b$$
$$+n^c\cdot\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot\hat{IE}_\theta\Bigg]$$

with

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c} = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{direct effects}}+\underbrace{\left[\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\cdot\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n^c}{\partial \epsilon_s^c}}_{\text{indirect effect}}$$

$$\left(\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\left(\frac{\partial S}{\partial b}\cdot\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}\right)}_{\text{direct effect}}+\underbrace{\left[\frac{\partial \epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\cdot\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}}+\underbrace{\left[\frac{\partial \epsilon_s^c}{\partial b}\cdot\frac{\partial c^w}{\partial \epsilon_s^c}+\frac{\partial c^w}{\partial b}\right]\cdot\left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

Inserting the Lagrange multiplier gives

$$(1-n)\cdot\frac{(u'(b)-u'(c^w))}{u'(c^w)} = \left(-\frac{\partial f}{\partial b}\right)^{\text{ge}}\cdot u\cdot L_v$$
$$+\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n^c}{\partial \epsilon_s^c}\cdot L_s^c+n^c\cdot\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot\hat{IE}_\theta$$

Note that when we set the lay-off tax optimally, then UI will not impose any distortions on the separation condition of unconstrained firms. In the context of Proposition 1, this means that $MLS^u = 0$, bolstering the fact that optimal lay-off taxes can implement the optimal number of separations.

F Ramsey FOCs with STW

In the following, multipliers from the Lagrangian, implied by the maximization problem from the previous section, are denoted by λ_{idx}, the index depending on the constraint. Here, λ_n denotes the Lagrange multiplier for the total employment equation, λ_{n^u} for the number of unconstrained firms, λ_{n^c} for the number of constrained firms, λ_{n^s} for the number of firms that received a shock, λ_θ for the job-creation condition, λ_c for the wage equation, $\lambda_{\epsilon_s^u}$ for the separation condition of unconstrained firms with STW support, $\lambda_{\xi_s^c}$ for the separation condition of constrained firms without STW support, and $\lambda_{\epsilon_s^c}$ for the separation condition of constrained firms with STW support. Every other equation listed in the Ramsey problem for STW is assumed to be plugged in.

F.1 Employment

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n^s} &= -\lambda_{n^s} + \frac{1-p}{\lambda}(1-\rho^u)\cdot(u(c^w) - u'(c^w)\cdot c^w) \\ &+ \frac{p}{\lambda}(1-\rho^c)\cdot(u^c - u'(c^w)\cdot e^c) \\ &+ \frac{1-p}{\lambda}\cdot(1-\rho^c)\cdot\lambda_{n^u} + \frac{p}{\lambda}(1-\rho^c)\cdot\lambda_{n^c} = 0\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial n^u} = -\lambda_{n^u} + \lambda_n = 0$$

$$\frac{\partial \mathcal{L}}{\partial n^c} = -\lambda_{n^c} + \lambda_n = 0$$

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial n} &= -\lambda_n + \lambda_{n^s}\cdot(\lambda - q(\theta)\cdot\theta) \\ &+ u'(c^w)\cdot[(z-\Omega) + b + \theta\cdot k_v] - u(b) \\ &+ \frac{(1-\rho)}{\lambda}\cdot\frac{b}{n^2}\cdot\lambda_\theta + \eta(1-\rho)\cdot\frac{b}{n^2}\cdot\lambda_c\end{aligned}$$

$$\begin{aligned}\Leftrightarrow \quad \frac{\lambda_n}{u'(c^w)} &= (\lambda - q(\theta)\cdot\theta) + [z - \Omega + b + \theta\cdot k_v] - \frac{u(b)}{u'(c^w)} \\ &+ \frac{1-\rho}{\lambda}\cdot\frac{b}{n^2}\cdot\frac{\lambda_\theta}{u'(c^w)} + \eta\cdot(1-\rho)\cdot\frac{b}{n^2}\cdot\frac{\lambda_c}{u'(c^w)}\end{aligned}$$

F.2 Job Creation

Before we start, let us define the Insurance effect:

$$IE_\theta = n^s\cdot u'(c^w)\cdot\left(1 + \frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\tilde{IE}_\theta$$

$$\begin{aligned}\tilde{IE}_\theta = \frac{p}{\lambda}\Bigg(&\int_{\max\{\xi_s^c,\epsilon_{stw}\}}^{\epsilon_p}\frac{\lambda}{f}\cdot\gamma\cdot\frac{u'(c(\epsilon)) - u'(c^w)}{u'(c^w)}\,dG(\epsilon) \\ &+ \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}},\epsilon_s^c\}}\frac{\lambda}{f}\cdot\gamma\cdot\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\,dG(\epsilon)\Bigg)\cdot k_v\end{aligned}$$

The FOC for the labor market tightness can be written as:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \theta} &= -k_v(1-n)\cdot u'(c^w) + IE_\theta + (1-\gamma)\cdot q(\theta)\cdot(1-n)\cdot\lambda_{n^s} \\
&\quad - \frac{1}{1-\eta}\cdot\gamma\cdot k_v + \frac{\lambda_\theta}{f} + \frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f\eta}{f}\cdot\left(\frac{1}{\lambda}\cdot(1-\rho)\cdot\lambda_\theta - \lambda_{\epsilon_s^u}\right) \\
&\quad - \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w_{stw}(\epsilon_s^c)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\epsilon_s^c} \\
&\quad - \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w(\xi_s^c)) + \frac{1}{\lambda}\cdot\left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\xi_s^c} \\
&\quad - \lambda_c\cdot\frac{\partial WE}{\partial\theta}
\end{aligned}$$

$$\iff \qquad \frac{\lambda_{n^s}}{u'(c^w)} = \frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q}$$

$$\begin{aligned}
\text{with} \quad \chi = &\frac{1}{(1-n)\cdot u'(c^w)}\cdot\frac{1}{f} \\
\cdot\ &\frac{1}{1-\eta}\cdot\left(\gamma\cdot\lambda_\theta - (\lambda\cdot\gamma - f\cdot\eta)\cdot\left(\frac{1}{\lambda}\cdot(1-\rho)\cdot\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right) \\
&+ \frac{1}{(1-n)\cdot u'(c^w)}\left(\lambda\cdot\frac{\gamma}{f}u'(c^w_{stw}(\epsilon_s^c)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\epsilon_s^c} \\
&+ \frac{1}{(1-n)\cdot u'(c^w)}\left(\lambda\cdot\frac{\gamma}{f}u'(c^w(\xi_s^c)) + \left(\frac{1}{1-\eta}\cdot\frac{\lambda\gamma - f}{f}\right)\eta\right)\cdot\lambda_{\xi_s^c} \\
&+ \frac{\lambda_c}{(1-n)\cdot u'(c^w)}\cdot\frac{\partial WE}{\partial\theta}/k_v \\
&- \frac{1}{(1-n)\cdot u'(c^w)}\cdot IE_\theta/k_v
\end{aligned}$$

Collecting terms from FOCs for employment and job-creation conditions gives:

$$\begin{aligned}
u'(c^w)\cdot\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} &= \frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot(u(c^w) - u(b) - u'(c^w)\cdot c^w) \\
&\quad + \frac{p}{\lambda}\cdot(1-\rho^c)\cdot(u^c - u(b) - u'(c^w)\cdot e^c) \\
&\quad + \left(\frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_\theta}{n^s}\right) + \left(\frac{1-\rho}{\lambda}\cdot\frac{\lambda_c}{n^s}\cdot\eta\cdot\lambda_c\cdot\frac{b}{n}\right) \\
&\quad + u'(c^w)\cdot\frac{1-\rho}{\lambda}\cdot\left(z - \Omega + b + \frac{\lambda - \gamma f + \chi(1-f)}{1-\gamma}\cdot\frac{k_v}{q}\right)
\end{aligned}$$

Optimal Job Creation Condition:

$$
\begin{aligned}
\frac{1+\chi}{1-\gamma}\cdot\frac{k_v}{q} = {} & \frac{1-\rho}{\lambda}\cdot(z-\Omega+b) \\
& + \frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_\theta}{n^s\cdot u'(c^w)} \\
& + \frac{1-\rho}{\lambda}\cdot\frac{b}{n}\cdot\frac{\lambda_c\cdot\lambda}{n^s\cdot u'(c^w)} \\
& + \frac{1-p}{\lambda}\cdot(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right) \\
& + \frac{p}{\lambda}\cdot(1-\rho^c)\cdot\left(\frac{u^c-u(b)}{u'(c^w)}-e^c\right) \\
& + \frac{1-\rho}{\lambda}\cdot\frac{\lambda-\gamma f+\chi(\lambda-f)}{1-\gamma}\cdot\frac{k_v}{q}
\end{aligned}
$$

Subtracting the decentralized job-creation condition from the optimal gives:

$$
\begin{aligned}
\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} = {} & \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{1-\rho}{\lambda}\cdot\frac{b}{n} \\
& + \frac{(1-\rho)(\lambda-f)}{\lambda}\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}
\end{aligned}
$$

Rearranging gives:

$$
\begin{aligned}
\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q} &= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{\frac{1-\rho}{\lambda}}{\rho+(1-\rho)\cdot\frac{f}{\lambda}}\cdot\frac{b}{n} \\
&= \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}
\end{aligned}
$$

F.3 Lagrange Multipliers for Separation Condition

Optimal separation condition unconstrained firms

$$
\begin{aligned}
&-\frac{\partial \mathcal{L}}{\partial \epsilon_s^u} \\
&= \frac{n}{1-\rho} \cdot (1-p) \cdot g(\epsilon_s^u) \cdot u'(c^w) \cdot \left(z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w)}{u'(c^w)} - c^w - (z+\Omega) \right) \\
&\quad + \lambda_\theta \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w \right. \\
&\quad \left. - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
&\quad + \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot \lambda_{n^u} + \lambda_{\epsilon_s^u} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u} \overset{!}{=} 0
\end{aligned}
$$

Insert for λ_{n^u}:

$$
\begin{aligned}
\iff \quad & g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \cdot n^s \\
& \cdot \left[z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w - (z+\Omega) + \frac{\lambda_{n^u}}{u'(c^w)} \right] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left[\tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} + \frac{\lambda_{\epsilon_s^u}}{u'(c^w)} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u} \overset{!}{=} 0
\end{aligned}
$$

$$
\begin{aligned}
\iff \quad & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w)-u(b)}{u'(c^w)} - c^w + b + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \left(\tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\
& + \lambda_c \cdot \frac{1}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s} \right) \\
& + \lambda_{\epsilon_s^u} \cdot \frac{1}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u} = 0
\end{aligned}
$$

Insert for λ_{n^s} :

$$\begin{aligned}\iff \quad & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b \\ & + (\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} + \theta \cdot c \\ & - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(\tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{1-n}{n} \cdot b - \frac{b}{n} \right) \\ & + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\ & + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{1}{n^s} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0\end{aligned}$$

Insert λ_{n^s}

$$\begin{aligned}\iff & z(\epsilon_s^u) - \Omega(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + b \\ & + \frac{\lambda - \gamma f + \chi(\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\ & - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left(\tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) - b \right) \\ & + \lambda \cdot \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\ & + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} = 0\end{aligned}$$

Subtract decentralized separation condition:

$$\begin{aligned}\tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) = & \, b + (\lambda - \theta q(\theta)) \cdot \left(\chi - \frac{\eta - \gamma}{1-\eta} \right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} \\ & + \frac{\lambda_{\epsilon_s^u}}{n^s \cdot u'(c^w)} \cdot \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u} \\ & - \frac{\lambda_\theta \cdot \theta}{n^s \cdot u'(c^w)} \cdot \left(\tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u)) - b \right) \\ & + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{\lambda}{1-p} \cdot \frac{1}{g(\epsilon_s^u)} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right)\end{aligned}$$

Hence

$$\begin{aligned}
0 = &\left(1+\frac{\lambda_\theta}{u'(c^w)\cdot n^s}\right)\\
&\cdot\left(b+(\lambda-f)\cdot\frac{(1-\rho/\lambda)}{(\rho+(1-\rho)\cdot f/\lambda)}\cdot\frac{b}{n}-\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)}\cdot\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}\\
&+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\\
\iff\quad 0 = &\left(1+\frac{\lambda_\theta}{u'(c^w)\cdot n^s}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right)\\
&+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)}\cdot\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}\\
&+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)
\end{aligned}$$

And finally:

$$\begin{aligned}
-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} = &\frac{(1-p)\cdot g(\epsilon_s^u)}{\lambda}\cdot\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial\epsilon_s^u}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\right.\\
&\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^u)\right)\right)\\
&\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{\lambda}{1-p}\cdot\frac{1}{g(\epsilon_s^u)}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\right)
\end{aligned}$$

Optimal separation condition constrained firms

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = &\Bigg[\frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \cdot u'(c^w) \left(z(\epsilon_s^c) - \Omega(\epsilon_s^c)\right. \\
&\left. + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) - (z+\Omega)\right) \\
&+ \lambda_\theta \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \Bigg[z(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} - c^w(\epsilon_s^c) \\
&- \frac{1-n}{n} \cdot b + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q} \Bigg] \\
&- \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_s^c} + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \lambda_{n^c} \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \lambda_{\epsilon_s^c}^c \cdot \frac{\partial S_{\text{stw}}^u(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Subtract decentralized separation condition (constrained firms):

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[&\frac{n}{1-\rho} \cdot p \cdot g(\epsilon_s^c) \left(z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(c^w)}{u'(c^w)} - (z+\Omega) \right) \\
&+ \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \frac{\lambda_{n^c}}{u'(c^w)} \\
&- \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left[\tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) + \frac{1-n}{n} \cdot b \right] \\
&+ \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \frac{\lambda_{\epsilon_s^c}^c}{u'(c^w)} \cdot \frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Insert λ_{n^c} and λ_n

$$
\begin{aligned}
\iff -\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& \frac{p}{\lambda} \cdot n^s g(\epsilon_s^c) \left(z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - u(c^w) - (z + \Omega) \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot \Bigg((\lambda - q(\theta) \cdot \theta) \cdot \frac{\lambda_{n^s}}{u'(c^w)} \\
& + [z - \Omega + b + \theta \cdot c] - \frac{u(b)}{u'(c^w)} \\
& \quad + \frac{1-\rho}{\lambda} \cdot \frac{b}{n^2} \cdot \lambda_\theta \cdot \frac{1}{u'(c^w)} + \eta \cdot \frac{b}{n^2} \cdot \frac{\lambda_c}{u'(c^w)} \Bigg) \Bigg] \\
& - \frac{\lambda_\theta}{u'(c^w)} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left[\tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) + \frac{1-n}{n} \cdot b \right] \\
& + \frac{\lambda_c}{u'(c^w)} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) + \frac{\lambda^c_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

Insert λ_{n^s}

$$
\begin{aligned}
-\frac{\partial \mathcal{L}}{\partial \epsilon_s^c} = \Bigg[& z_{\text{stw}}(\epsilon_s^c) - \Omega(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) + b \\
& + \frac{\lambda - \gamma \cdot f + \chi \cdot (\lambda - f)}{1-\gamma} \cdot \frac{k_v}{q} \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \left[\tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) - b \right] \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \xi_s^c) \\
& + \frac{\lambda}{p} \cdot \frac{1}{n^s \cdot g(\epsilon_s^c)} \cdot \frac{\lambda^c_{\epsilon_s^c}}{u'(c^w)} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c} = 0
\end{aligned}
$$

And finally:

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\epsilon_s^c)}{\frac{\partial s^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \cdot \Bigg(& \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \\
& + \frac{\lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \Bigg) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

Analogously, it can be derived that

$$-\frac{\lambda_{\xi_s^c}}{n^s \cdot u'(c^w)} = \frac{p}{\lambda} \cdot \frac{g(\xi_s^c)}{\frac{\partial s^c_{\text{stw}}(\xi_s^c)}{\partial \xi^c}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b\right) + \frac{\lambda \cdot \lambda_c}{n^s \cdot u'(c^w)} \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \xi^c}\right)\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$

F.4 Optimal STW Benefits

$$\frac{\partial \mathcal{L}}{\partial \tau_{\text{stw}}} = \frac{p}{\lambda} \cdot n^s \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}} (u'(c_{\text{stw}}(\epsilon)) - u'(c^w)) \left(\bar{h} - h_{\text{stw}}(\epsilon)\right) dG(\epsilon)$$
$$+ \lambda_\theta \cdot \frac{p}{\lambda} \cdot \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)}\right) \left(\bar{h} - h_{\text{stw}}(\epsilon)\right) dG(\epsilon)$$
$$- \frac{n^s \cdot (1-\rho)}{\lambda} \cdot u'(c^w) \cdot \frac{\partial \Omega}{\partial \tau_{\text{stw}}} - \lambda_\theta \cdot \frac{(1-\rho)}{\lambda} \cdot \frac{\partial \Omega}{\partial \tau_{\text{stw}}} - \lambda_c \cdot \frac{\partial WE}{\partial \tau_{\text{stw}}}$$
$$- \lambda_{\epsilon_s^u} \cdot \frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \tau_{\text{stw}}} - \lambda_{\epsilon_s^c} \cdot \frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \tau_{\text{stw}}} = 0$$

Insert Lagrange multiplier separation condition:

$$\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{p}{\lambda} \int_{\epsilon_s^c}^{\max\{\epsilon_s^c, \epsilon_{\text{stw}}\}} \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \cdot (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon) - (1-\rho) \cdot \frac{\partial \Omega}{\partial \tau_{\text{stw}}}\right) - \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \frac{\partial WE}{\partial \tau_{\text{stw}}}$$
$$+ \frac{(1-p)}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{\partial \epsilon_s^u}{\partial \tau_{\text{stw}}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u}\right)\right)$$
$$+ \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\partial \epsilon_s^c}{\partial \tau_{\text{stw}}} \cdot \left(\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) + \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c}\right)\right) = 0$$

Finally:

$$\bar{\tau}_{\text{stw}} = \underbrace{\frac{\lambda}{f} b}_{\text{Fiscal Ext.}}$$
$$+ \underbrace{\frac{\mathbb{1}\{\epsilon_{\text{stw}} \geq \epsilon_s^c\}}{\varphi(p)} \cdot n^s \cdot \frac{p}{\lambda} \int_{\epsilon_s^c}^{\epsilon_{\text{stw}}} \left(\frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w))}{u'(c^w)} \right) (\bar{h} - h_{\text{stw}}(\epsilon))\, dG(\epsilon)}_{\text{Insurance}}$$
$$- \underbrace{\frac{n^s}{\varphi(p)} \cdot \frac{1}{\lambda} \cdot (1-\rho) \frac{\partial\, \Omega}{\partial\, \tau_{\text{stw}}}}_{\text{Distortion}} + \underbrace{BE_{\text{stw},3}}_{\text{Bargaining Effect}}$$

$$\bar{\tau}_{\text{stw}} = \frac{\varphi^u(p)}{\varphi(p)} \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) + \frac{\varphi^c(p)}{\varphi(p)} \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon_s^c))$$

where

$$\varphi^u = \frac{(1-p)}{\lambda} \cdot g(\epsilon_s^u) \cdot \frac{\partial \epsilon_s^u}{\partial \tau_{\text{stw}}} \cdot n^s, \quad \varphi^c = \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \frac{\partial \epsilon_s^c}{\partial \tau_{\text{stw}}} \cdot n^s,$$

$$\varphi = \varphi^c + \varphi^u$$

and

$$BE_{\text{stw},3} = \frac{1}{\left(1 + \frac{\lambda_\theta}{n^s\, u'(c^w)}\right)}$$
$$\times \left[- \frac{\lambda_c}{\varphi\, n^s\, u'(c^w)} \frac{\partial WE}{\partial \tau_{\text{stw}}} + \frac{\varphi^u}{\varphi} \frac{\lambda_c}{n^s\, u'(c^w)} \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \frac{\lambda}{1-p} \frac{\partial WE}{\partial \epsilon_s^u} \right) \right.$$
$$\left. + \frac{\varphi^c}{\varphi} \frac{\lambda_c}{n^s\, u'(c^w)} \left(\eta \frac{b}{n} + \frac{1}{g(\epsilon_s^c)} \frac{\lambda}{p} \frac{\partial WE}{\partial \epsilon_s^c} \right) \right]$$

F.5 Optimal Eligibility Condition

Assumption throughout: $\epsilon^p \geq \epsilon_{\text{stw}} \geq \epsilon_s^u$

Further assume that:

$$n^s \cdot u'(c^w) + \lambda_\theta - \eta \cdot \lambda_c > 0, \qquad n^s \cdot u'(c^w) + \lambda_\theta > 0$$

These conditions exclude the case that the Planner would want to use STW's hours distortions to destroy production and thus reduce vacancy posting. The Planner might want to do that when the Hosios condition is not fulfilled. For reasonable parameter values, however, the condition should hold anyway. Recall that by Lemma 1 $\epsilon_s^c > \epsilon_s^u$ and $\xi_s^c > \xi_s^u$.

Case I:

FOC for the eligibility threshold:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \epsilon_{stw}} &= -n^s \cdot u'(c^w) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} u'(c^w) \\ &\quad - (\lambda_\theta - \eta \cdot \lambda_c) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \\ &= -\left(n^s \cdot u'(c^w) + \lambda_\theta - \eta \cdot \lambda_c\right) \cdot \frac{1-p}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}}\end{aligned}$$

Under reasonable parameter values, we get:

$$\frac{\partial \mathcal{L}}{\partial \epsilon_{stw}} < 0 \quad \text{thus it is optimal to set} \quad \epsilon_{stw} = \epsilon_s^u.$$

Case II:

FOC for the eligibility threshold:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{stw}} &= -n^u \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \cdot u'(c^w) \\
&+ \frac{n}{1-\rho} \cdot p \cdot g(\epsilon_{stw}) \cdot u'(c^w) \\
&\quad \cdot \left(z_{\text{stw}}(\epsilon_{stw}) + \frac{u(c_{\text{stw}}(\epsilon_{stw}))}{u'(c^w)} - c_{\text{stw}}(\epsilon_{stw}) - (z + \Omega) \right) \\
&+ \lambda_\theta \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{stw}} \\
&+ \frac{p}{\lambda} \cdot g(\epsilon_{stw}) \\
&\quad \cdot \left[z(\epsilon_{\text{stw}}) + \frac{u(c_{stw}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} \right. \\
&\qquad \left. - c_{stw}(\epsilon_{stw}) - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \frac{k_v}{q} \right] \\
&- \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \cdot \lambda_{n^c} \overset{!}{>} 0.
\end{aligned}$$

Insert n^u

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} &= -n^s \cdot (1-\rho^u) \cdot \left(\frac{1-p}{\lambda} \right) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \cdot u'(c^w) \\
&+ n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \cdot \left(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))}{u'(c^w)} \right) \\
&- n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \cdot (c_{\text{stw}}(\epsilon_{\text{stw}}) + z + \Omega) \\
&+ \lambda_\theta \cdot \frac{g(\epsilon_{\text{stw}})}{\lambda} \cdot (1-p)(1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \\
&+ \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot \left(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) - \frac{1-n}{n} \cdot b \right. \\
&\qquad \left. + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \left(\frac{k_v}{q} \right) \right) \\
&- \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s \cdot \lambda_{n^c} \overset{!}{>} 0
\end{aligned}$$

$$
\begin{aligned}
\Leftrightarrow\ & -(n^s \cdot u'(c^w) + \lambda_\theta) \cdot (1-\rho^u) \cdot \left(\frac{1-p}{\lambda}\right) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \\
& + n^s \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot u'(c^w) \\
& \cdot \left(z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) - (z+\Omega) + \frac{\lambda_{n^c}}{u'(c^w)} \right) \\
& + \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{\text{stw}}) \\
& \qquad - \frac{1-n}{n} \cdot b + \frac{\lambda - \eta \cdot f}{1-\eta} \cdot \left(\frac{k_v}{q}\right) \Bigg] \\
& - \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} \overset{!}{>} 0
\end{aligned}
$$

Insert λ_{n^c} and subtract decentralized separation condition for constrained firms:

$$
\begin{aligned}
\Leftrightarrow\quad & -\left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{(1-p)}{\lambda} \cdot (1-\rho^u) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \\
& + \frac{p}{\lambda} \cdot \Bigg[b + z_{\text{stw}}(\epsilon_{stw}) + \frac{u(c_{\text{stw}}(\epsilon_{stw})) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_{stw}) \\
& + \frac{\lambda - \eta \cdot f + \chi \cdot (\lambda - f)}{1-\eta} \cdot \left(\frac{k_v}{q}\right) \Bigg] \\
& - \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \cdot \frac{p}{\lambda} \cdot [\tau_{\text{stw}} \cdot (h - h_{\text{stw}}(\epsilon_{stw})) - b] \\
& - \frac{\lambda_c}{n^s \cdot u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \frac{p}{\lambda} \cdot \eta \cdot \frac{b}{n} \right) \overset{!}{>} 0
\end{aligned}
$$

Subtract decentralized separation condition:

$$\Leftrightarrow \quad -\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{(1-p)}{\lambda}\cdot(1-\rho^u)\cdot\frac{\partial\Omega_u}{\partial\epsilon_{\text{stw}}}\cdot\frac{1}{g(\epsilon_{\text{stw}})}$$

$$+\frac{p}{\lambda}\cdot\Bigg[z_{\text{stw}}(\epsilon_{\text{stw}})-z_{\text{stw}}(\epsilon_s)+\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))-u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)}$$

$$-\tau_{\text{stw}}\cdot(h-h_{\text{stw}}(\epsilon_s^c))+b+(1-\rho)\cdot(\lambda-f)\cdot\left(\chi-\frac{\eta-\gamma}{1-\eta}\right)\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}\Bigg]$$

$$+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\cdot\frac{p}{\lambda}\cdot\Bigg[z_{\text{stw}}(\epsilon_{\text{stw}})-z_{\text{stw}}(\epsilon_s^c)+\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))-u(c_{stw}(\epsilon_s^c))}{u'(c^w)}$$

$$-\tau_{\text{stw}}\cdot(h-h_{\text{stw}}(\epsilon_s^c))+b]$$

$$-\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\frac{\partial WE}{\partial\epsilon_{\text{stw}}}\cdot\frac{1}{g(\epsilon_{\text{stw}})}\cdot\frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial\epsilon_{\text{stw}}}+\frac{p}{\lambda}\cdot\eta\cdot\frac{b}{n}\right)\overset{!}{>}0$$

Inserting for $(\chi-\frac{\eta-\gamma}{1-\eta})\cdot\frac{1}{1-\gamma}\cdot\frac{k_v}{q}$ gives us the final condition. We can distinguish between three cases:

(A)

$$g(\epsilon_{\text{stw}})\cdot\frac{p}{\lambda}\cdot n^s\cdot\Bigg[z_{\text{stw}}(\epsilon_{\text{stw}})-z_{\text{stw}}(\epsilon_s^c)$$

$$+\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))-u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)}+\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot\big(h-h_{\text{stw}}(\epsilon_s^c)\big)\Bigg]$$

$$>n^s\cdot\frac{1-p}{\lambda}\cdot(1-\rho)\cdot\frac{\partial\Omega_u}{\partial\epsilon_{\text{stw}}}+BE$$

$$\forall\,\epsilon_{\text{stw}}\text{ in case 2}\quad\Rightarrow\quad\epsilon_{\text{stw}}=\xi_s^c$$

(B)

$$g(\epsilon_{\text{stw}})\cdot\frac{p}{\lambda}\cdot n^s\cdot\Bigg[z_{\text{stw}}(\epsilon_{\text{stw}})-z_{\text{stw}}(\epsilon_s^c)$$

$$+\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}}))-u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)}+\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot\big(h-h_{\text{stw}}(\epsilon_s^c)\big)\Bigg]$$

$$<n^s\cdot\frac{1-p}{\lambda}\cdot(1-\rho)\cdot\frac{\partial\Omega_u}{\partial\epsilon_{\text{stw}}}+BE$$

$$\forall\,\epsilon_{\text{stw}}\text{ in case 2}\quad\Rightarrow\quad\epsilon_{\text{stw}}=\max\{\xi_s^c,\xi_s^u\}$$

(C)

$$
\begin{aligned}
&g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Bigg[z_{\text{stw}}(\epsilon_{\text{stw}}) - z_{\text{stw}}(\epsilon_s^c) \\
&\quad + \frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c_{\text{stw}}(\epsilon_s^c))}{u'(c^w)} + \frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot \big(h - h_{\text{stw}}(\epsilon_s^c)\big) \Bigg] \\
&= n^s \cdot \frac{1-p}{\lambda} \cdot (1-\rho) \cdot \frac{\partial \Omega_u}{\partial \epsilon_{\text{stw}}} + BE
\end{aligned}
$$

BE is defined as:

$$
\begin{aligned}
BE = g(\epsilon_{\text{stw}}) \cdot \frac{\lambda_c}{u'(c^w)} \cdot \left(\frac{\partial WE}{\partial \epsilon_{\text{stw}}} \cdot \frac{1}{g(\epsilon_{\text{stw}})} \cdot \frac{\partial S_{\text{stw}}(\epsilon_{\text{stw}})}{\partial \epsilon_{\text{stw}}} + \eta \cdot \frac{b}{n} \right) \\
\Bigg/ \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right)
\end{aligned}
$$

Case III:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \epsilon_{\text{stw}}} = {} & g(\epsilon_{\text{stw}}) \cdot \frac{p}{\lambda} \cdot n^s \cdot \Big(u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}})) \\
& - u'(c^w) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})] \Big) \\
& - n \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \\
& + \lambda_\theta \cdot \frac{p}{\lambda} \cdot \Big(u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}})) \\
& - u'(c^w) \cdot [c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})] \Big) \\
& - \lambda_\theta \cdot \frac{1-\rho}{\lambda} \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} \\
& + \lambda_c \cdot \frac{\partial WE}{\partial \epsilon_{\text{stw}}} \overset{!}{\geq} 0
\end{aligned}
$$

$$\Leftrightarrow \quad \left(1+\frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \left(\frac{1-\rho}{\lambda} \cdot n^s \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} + BE\right)$$

$$\stackrel{!}{\geq} \left(1+\frac{\lambda_\theta}{n^s \cdot u'(c^w)}\right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s$$

$$\cdot \left(\frac{\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}}))}{c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})} - u'(c^w)}{u'(c^w)} \right)$$

$$\cdot \left[c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})\right]$$

$$\Leftrightarrow \quad \left(\frac{1-\rho}{\lambda} \cdot n^s \cdot \frac{\partial \Omega}{\partial \epsilon_{\text{stw}}} + BE\right) \stackrel{!}{\geq} \cdot \frac{p}{\lambda} \cdot g(\epsilon_{\text{stw}}) \cdot n^s$$

$$\cdot \left(\frac{\frac{u(c_{\text{stw}}(\epsilon_{\text{stw}})) - u(c(\epsilon_{\text{stw}}))}{c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})} - u'(c^w)}{u'(c^w)} \right)$$

$$\cdot \left[c_{\text{stw}}(\epsilon_{\text{stw}}) - c(\epsilon_{\text{stw}})\right]$$

If the equation holds with strict inequality, the cost of hours distortion exceeds the benefit of providing additional insurance to workers in constrained firms. Consequently, the Ramsey planner chooses not to allow firms and workers to enter short-time work (STW) when they could survive without reaching the threshold $\epsilon_{\text{stw}} = \xi_s^c$. Conversely, if the equation holds with exact equality, the STW threshold is determined by balancing the additional cost of hours distortions against the benefit of providing extra insurance.

This concludes the proof of Proposition 4.

F.6 Optimal Unemployment Insurance with STW

From the FOC of UI, we can derive the optimality condition of unemployment insurance:

$$
\begin{aligned}
(1-n)\cdot(u'(b)-u'(c^w)) = \lambda_\theta(1-\rho)\cdot\left(\frac{1-n}{n}+\frac{u'(b)}{u'(c^w)}\right) \\
+\left(\lambda_{\epsilon_s^u}+\lambda_{\epsilon_s^c}+\lambda_{\epsilon^g}\right)\cdot\left(-\frac{u'(b)}{u'(c^w)}\right) \\
+\lambda_c(1-\rho)\cdot\left(\eta\cdot\frac{1-n}{n}+(1-\eta)\cdot\frac{u'(b)}{u'(c^w)}\right)
\end{aligned}
$$

To get better insight, we need to determine the Lagrange multipliers for λ_θ, λ_c. We already know the Lagrange multipliers for the separation conditions:

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^u}}{n^s\cdot u'(c^w)} &= \frac{1-p}{\lambda}\cdot\frac{g(\epsilon_s^u)}{\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial\epsilon_s^u}} \\
&\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^u)\right)\right)\right. \\
&\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\lambda_{\xi_s^c}}{n^s\cdot u'(c^w)} &= \frac{p}{\lambda}\cdot\frac{g(\xi_s^c)}{\frac{\partial S^c_{\text{stw}}(\xi_s^c)}{\partial\xi_s^c}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\frac{\lambda}{f}\cdot b\right. \\
&\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\leq\xi_s^c)
\end{aligned}
$$

$$
\begin{aligned}
-\frac{\lambda_{\epsilon_s^c}}{n^s\cdot u'(c^w)} &= \frac{p}{\lambda}\cdot\frac{g(\epsilon_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial\epsilon_s^c}}\cdot\left(\left(1+\frac{\lambda_\theta}{n^s\cdot u'(c^w)}\right)\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{stw}\cdot\left(\bar{h}-h_{\text{stw}}(\epsilon_s^c)\right)\right)\right. \\
&\left.+\frac{\lambda_c}{n^s\cdot u'(c^w)}\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\epsilon_s^c}\right)\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\geq\epsilon_s^c)
\end{aligned}
$$

First, let us find an expression for λ_c:

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c^w} = -\Bigg(& (1-p)\cdot(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)} \\
& + p\cdot(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\cdot\lambda_\theta \\
& - (1-\eta)\cdot(1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) \\
& - (1-p)\cdot(1-\cdot\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_c \\
& + \frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^u} \\
& + \frac{u(c(\xi_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\xi_s^c} \\
& + \frac{u(c_{\text{stw}}(\epsilon_s^c))-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\cdot\lambda_{\epsilon_s^c} = 0
\end{aligned}
$$

Insert Lagrange multipliers for separation conditions

$$
\begin{aligned}
& -\left(p(1-\rho^u)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}+(1-p)(1-\rho^c)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right)\lambda_\theta \\
& -(1-\eta)\cdot\Bigg((1-p)\cdot(1-\rho^u)\cdot\left(1-(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\right) \\
& -p\cdot(1-\rho^c)\cdot(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}\cdot\frac{u''(c^w)}{u'(c^w)}\Bigg)\lambda_c \\
& +\frac{\partial\epsilon_s^u}{\partial c^w}\cdot\frac{1-p}{\lambda}\cdot g(\epsilon_s^u)\,(n^s\cdot u'(c^w)+\lambda_\theta)\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot(\bar{h}-h_{stw}(\epsilon_s^u))\right) \\
& +\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\epsilon_s^u)}\cdot\frac{\lambda}{1-p}\cdot\frac{\partial WE}{\partial\epsilon_s^u}\right) \\
& +\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\,(n^s\cdot u'(c^w)+\lambda_\theta)\cdot\left(\frac{\lambda}{f}\cdot b+\lambda_c\cdot\left(\eta\cdot\frac{b}{n}-\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\right) \\
& \cdot\mathbb{1}(\epsilon_{stw}\le\xi_s^c) \\
& +\frac{\partial\xi_s^c}{\partial c^w}\cdot\frac{p}{\lambda}\cdot g(\xi_s^c)\,(n^s\cdot u'(c^w)+\lambda_\theta)\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot(\bar{h}-h_{stw}(\xi_s^c))\right) \\
& +\lambda_c\cdot\left(\eta\cdot\frac{b}{n}+\frac{1}{g(\xi_s^c)}\cdot\frac{\lambda}{p}\cdot\frac{\partial WE}{\partial\xi_s^c}\right)\cdot\mathbb{1}(\epsilon_{stw}\ge\epsilon_s^c)=0
\end{aligned}
$$

Rearranging yields

$$
\begin{aligned}
0 = \lambda_\theta \cdot \frac{\partial S^{\text{total}}}{\partial c^w} + \lambda_c \cdot \frac{\partial WE^{\text{total}}}{\partial c^w} \\
+ \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \tau_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s \cdot u'(c^w) \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \cdot \left(\frac{\lambda}{f} \cdot b - \tau_{stw} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

with

$$
\begin{aligned}
\frac{\partial S^{\text{total}}}{\partial c^w} = (1-p) \cdot (1-\rho^u) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
- p \cdot (1-\rho^c) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
+ \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \left(\frac{\lambda}{f} b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \left(\frac{\lambda}{f} b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial WE^{\text{total}}}{\partial c^w} = -(1-\eta) \cdot (1-p) \cdot (1-\rho^u) \cdot \left(1 - (1-\eta) \cdot \frac{u(c^w) - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \right) \\
+ p \cdot (1-\rho^c) \cdot (1-\eta) \cdot \frac{u^c - u(b)}{u'(c^w)} \cdot \frac{u''(c^w)}{u'(c^w)} \\
+ \frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot \lambda_c \cdot \left(\eta \cdot \frac{b}{n} + \frac{1}{g(\epsilon_s^u)} \cdot \frac{\lambda}{1-p} \cdot \frac{\partial WE}{\partial \epsilon_s^u} \right) \\
+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot \lambda_c \cdot \left(\eta \cdot \frac{b}{n} - \frac{1}{g(\xi_s^c)} \cdot \frac{\lambda}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot \lambda_c \cdot \left(\eta \cdot \frac{b}{n} - \frac{1}{g(\epsilon_s^c)} \cdot \frac{1}{p} \cdot \frac{\partial WE}{\partial \epsilon_s^c} \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)
\end{aligned}
$$

Collecting terms:

$$\lambda_c = \frac{-1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \left[\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^u)\right) \right) \right.$$
$$+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} \cdot b \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$
$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot \left(\bar{h} - h_{\text{stw}}(\epsilon_s^c)\right) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$
$$\left. + \left(-\frac{\partial S_{\text{total}}}{\partial c^w} \right) \cdot \lambda_\theta \right]$$

Insert λ_c into the Lagrange multipliers for the separation conditions. To do that, let us rewrite the Lagrange multipliers of the separation conditions:

$$\lambda_{\epsilon_s^u} = -\frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^u) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \right.$$
$$\cdot \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right)$$
$$\left. + \lambda_c \left(\frac{\partial n}{\partial \epsilon_s^u} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^u} \right) \right)$$

$$\lambda_{\xi_s^c} = \frac{\mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \cdot \frac{\lambda}{f} \cdot b \right.$$
$$\left. + \lambda_c \left(\frac{\partial n}{\partial \xi_s^c} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \xi_s^c} \right) \right)$$

$$\lambda_{\epsilon_s^c} = -\frac{\mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)}{\frac{\partial S_{\text{stw}}^c(\epsilon_s^c)}{\partial \epsilon_s^c}} \left(n^s \cdot u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left(1 + \frac{\lambda_\theta}{n^s \cdot u'(c^w)} \right) \right.$$
$$\cdot \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right)$$
$$\left. + \lambda_c \left(\frac{\partial n}{\partial \epsilon_s^c} \cdot \left(-\frac{\partial WE}{\partial n} \right) + \frac{\partial WE}{\partial \epsilon_s^c} \right) \right)$$

Inserting λ_c gives:

$$\lambda_{\epsilon_s^u} =$$

$$- \frac{1}{\frac{\partial S_{\text{stw}}^u(\epsilon_s^u)}{\partial \epsilon_s^u}} \Bigg[n^s u'(c^w) \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right)$$

$$+ \left(\frac{\partial n}{\partial \epsilon_s^u} \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}} \right)$$

$$\cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \right.$$

$$+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$

$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$

$$\left. + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \right) \Bigg]$$

$$\lambda_{\xi_s^c} =$$

$$\frac{\mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)}{\frac{\partial S_{\text{stw}}^c(\xi_s^c)}{\partial \xi_s^c}} \Bigg[- n^s u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\xi_s^c))\right)$$

$$+ \left(\frac{\partial n}{\partial \xi_s^c} \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{total}} \right)$$

$$\cdot \left(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \right.$$

$$+ \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$

$$+ \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$

$$\left. + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \right) \Bigg]$$

$$
\begin{aligned}
&\lambda_{\epsilon_s^c} = \\
&\quad - \frac{\mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)}{\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}} \Bigg[n^s u'(c^w) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \\
&\quad + \left(\frac{\partial n}{\partial \epsilon_s^c} \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}} \right) \\
&\quad \cdot \Bigg(\frac{\partial \epsilon_s^u}{\partial c^w} \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + \frac{\partial \xi_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
&\quad + \frac{\partial \epsilon_s^c}{\partial c^w} \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
&\quad + \left(-\frac{\partial S^{\text{total}}}{\partial c^w}\right) \cdot \lambda_\theta \Bigg) \Bigg]
\end{aligned}
$$

Now we have everything to calculate λ_θ from two equations:

$$
\text{(1)} \quad \left(\chi - \frac{\eta - \gamma}{1 - \eta}\right) \cdot \frac{1}{1-\gamma} \cdot \frac{k_v}{q} = \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}
$$

$$
\begin{aligned}
\text{(2)} \quad \chi = {} & \frac{1}{1-\eta} \cdot \frac{1}{u \cdot f} \cdot \frac{1}{u'(c^w)} \cdot \left[\gamma - (\lambda\gamma - f\eta) \cdot \left(\frac{1}{\lambda}(1-\rho)\lambda_\theta - \lambda_{\epsilon_s^u}\right)\right] \\
& + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot \lambda_{\xi_s^c} \\
& + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \left(\lambda \cdot \frac{\gamma}{f} \cdot u'(c^w) + \left(\frac{1}{1-\eta} \cdot \frac{\lambda\gamma - f}{f}\right) \cdot \eta\right) \cdot \lambda_{\epsilon_s^c} \\
& + \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot \lambda_c \cdot \frac{\partial WE}{\partial \theta} \cdot \frac{1}{k_v} \\
& - \frac{1}{u} \cdot \frac{1}{u'(c^w)} \cdot IE_\theta
\end{aligned}
$$

We can rearrange (1) to:

$$
\chi \cdot k_v = (1-\gamma) \cdot q \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta + \eta \cdot \lambda \cdot \lambda_c}{n^s \cdot u'(c^w)}\right) \cdot \frac{b}{f}\right)
$$

Note that

$$(1-\gamma)\cdot q = -\frac{\partial f}{\partial \theta}$$

with

$$\begin{aligned} f'(\theta) &= q(\theta) + \theta \cdot q'(\theta) \\ &= q(\theta) \cdot \left(1 + \frac{q'(\theta)\cdot\theta}{q(\theta)}\right) \\ &= q(\theta)\cdot(1-\gamma) \end{aligned}$$

Inserting χ gives:

$$\begin{aligned} &[\gamma - (\lambda\gamma - f\cdot\eta)]\cdot\frac{1}{\lambda}\cdot(1-\rho)\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta \\ &= u'(c^w)\cdot\left(\frac{\partial f}{\partial\theta}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q} + \left(1+\frac{\lambda_\theta+\eta\cdot\lambda\cdot\lambda_c}{n^s\cdot u'(c^w)}\right)\cdot\frac{b}{f}\right) \\ &\quad + \frac{1}{f}\cdot(\lambda\gamma - f\cdot\eta)\cdot k_v\cdot(-\lambda_{\epsilon_s^u}) \\ &\quad + \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c^w(\epsilon_s^c)) + \left(\frac{1}{1-n}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot k_v\cdot(-\lambda_{\xi_s^c}) \\ &\quad + \left(\lambda\cdot\frac{\gamma}{f}\cdot u'(c(\xi_s^c)) + \left(\frac{1}{1-n}\cdot\frac{\lambda\gamma-f}{f}\right)\cdot\eta\right)\cdot k_v\cdot(-\lambda_{\epsilon_s^c}) \\ &\quad + \frac{\partial WE}{\partial\theta}\cdot(-\lambda_c) \\ &\quad + IE_\theta \end{aligned}$$

Insert $\lambda_{\epsilon_s^u}, \lambda_{\epsilon_s^c}, \lambda_{\xi_s^c}, IE_\theta$:

$$
\begin{aligned}
&\left[\gamma - (\lambda\gamma - f\cdot\eta)\cdot\frac{1}{\lambda}\cdot(1-\rho)\right]\cdot\frac{1}{1-\eta}\cdot\frac{k_v}{f}\cdot\lambda_\theta \\
&= u'(c^w)\cdot\left(\frac{\partial f}{\partial\theta}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{b}{f}\right) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^u) \\
&\quad \cdot\frac{1-p}{\lambda}\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\left(\frac{\lambda}{f}b-\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\xi_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\xi_s^c)\cdot\frac{p}{\lambda}\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\frac{\lambda}{f}b\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c) \\
&\quad + u'(c^w)\cdot n^s\cdot\left(-\frac{\partial\epsilon_s^c}{\partial\theta}\right)^{\text{total}}\cdot g(\epsilon_s^c) \\
&\quad \cdot\frac{p}{\lambda}\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\left(\frac{\lambda}{f}b-\tau_{\text{stw}}(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c) \\
&\quad + \lambda_\theta\cdot\left(-\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
&\quad + n^s\cdot u'(c^w)\cdot\left(1+\frac{\lambda_\theta}{n^s u'(c^w)}\right)\cdot\tilde{IE}_\theta
\end{aligned}
$$

Denote:

$$
\begin{aligned}
\left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)^{\text{total}} &= \left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)+\left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\epsilon_s^u}{\partial c^w}\right) \\
&\quad + \left(-\frac{\partial\epsilon_s^u}{\partial\theta}\right)\cdot\left[\left(\frac{\partial n}{\partial\epsilon_s^u}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial\epsilon_s^u}\right)^{\text{total}}\right]\cdot\left(\frac{\partial\epsilon_s^u}{\partial c^w}\right) \\
&\quad + f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\epsilon_s^u}{\partial c^w}
\end{aligned}
$$

$$
\begin{aligned}
\left(-\frac{\partial\xi_s^c}{\partial\theta}\right)^{\text{total}} &= \Bigg[\left(-\frac{\partial\xi_s^c}{\partial\theta}\right)+\left(\frac{\partial c^w}{\partial\theta}\right)^{\text{total}}\cdot\left(-\frac{\partial\xi_s^c}{\partial c^w}\right) \\
&\quad + \left(-\frac{\partial\xi_s^c}{\partial\theta}\right)\cdot\left[\left(\frac{\partial n}{\partial\xi_s^c}\right)\cdot\left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}}+\left(\frac{\partial c^w}{\partial\xi_s^c}\right)^{\text{total}}\right]\cdot\left(\frac{\partial\xi_s^c}{\partial c^w}\right) \\
&\quad + f'(\theta)\cdot\frac{\partial n}{\partial f}\cdot\frac{\partial c^w}{\partial n}\cdot\frac{\partial\xi_s^c}{\partial c^w}\Bigg]\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c)
\end{aligned}
$$

$$
\begin{aligned}
\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} = & \Bigg[\left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial \epsilon_s^c}{\partial c^w}\right) \\
& + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \cdot \frac{\partial \epsilon_s^c}{\partial c^w} \\
& + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n} \cdot \frac{\partial \epsilon_s^c}{\partial c^w}\Bigg] \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
\left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total,2}} = & \left(-\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \\
& + \left(-\frac{\partial \epsilon_s^u}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^u}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{total}}\right] \\
& + \left(-\frac{\partial \xi_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \xi_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{total}}\right] \\
& + \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right) \cdot \left[\left(\frac{\partial n}{\partial \epsilon_s^c}\right) \cdot \left(-\frac{\partial c^w}{\partial n}\right)^{\text{total}} + \left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{total}}\right] \\
& + f'(\theta) \cdot \frac{\partial n}{\partial f} \cdot \frac{\partial c^w}{\partial n}
\end{aligned}
$$

This allows us to calculate the Lagrange multiplier for the job-creation condition:

$$
\begin{aligned}
& M \cdot \lambda_\theta = \\
& u'(c^w) \cdot \frac{\partial f}{\partial \theta} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f}\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \xi_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(-\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \\
& \quad \cdot \frac{p}{\lambda} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& \quad + u'(c^w) \cdot n^s \cdot \tilde{IE}_\theta
\end{aligned}
$$

with

$$
\begin{aligned}
M = & \left[\gamma - (\lambda\gamma - f \cdot \eta) \cdot \frac{1}{\lambda} \cdot (1-\rho)\right] \cdot \frac{1}{1-\eta} \cdot \frac{k_v}{f} \\
& + \left(-\frac{\partial f}{\partial \theta} \cdot u \cdot \frac{b}{f}\right) \\
& + \left(\frac{\partial \epsilon_s^u}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right) \\
& + \left(\frac{\partial \xi_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial \theta}\right)^{\text{total}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& + \left(\frac{\partial c^w}{\partial \theta}\right)^{\text{total}} \cdot \left(-\frac{\partial S}{\partial c^w}\right)^{\text{total}} \\
& + \tilde{IE}_\theta
\end{aligned}
$$

Note. $\frac{1}{M}$ denotes the general equilibrium effect of an increase of the joint surplus on θ:

$$
\left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} = \frac{1}{M}
$$

Rearranging for λ_θ gives:

$$
\begin{aligned}
& M \cdot \lambda_\theta = \\
& u'(c^w) \cdot \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{\lambda}{f} \cdot b\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(\left(-\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \cdot g(\epsilon_s^u) \cdot \frac{1-p}{\lambda} \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^u))\right)\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(\left(-\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} \cdot g(\xi_s^c) \cdot \frac{p}{\lambda} \cdot \frac{\lambda}{f} \cdot b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)\right) \\
& \quad + u'(c^w) \cdot n^s \\
& \quad \cdot \left(\left(-\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \cdot g(\epsilon_s^c) \cdot \frac{p}{\lambda} \cdot \left(\frac{\lambda}{f} \cdot b - \tau_{\text{stw}} \cdot (\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)\right) \\
& \quad + u'(c^w) \cdot n^s \cdot \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \tilde{IE}_\theta
\end{aligned}
$$

Define:

$$\hat{IE}_\theta = \frac{1}{1-\rho^c} \cdot \left[\int_{\max\{\epsilon_{\text{stw}}, \xi_s^c\}}^{\epsilon^p} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c^w(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right.$$
$$\left. - \int_{\epsilon_s^c}^{\max\{\epsilon_{\text{stw}}, \epsilon_s^c\}} \lambda \cdot \frac{\gamma}{f} \cdot \frac{u'(c_{\text{stw}}(\epsilon)) - u'(c^w)}{u'(c^w)} \, dG(\epsilon) \right] \cdot k_v$$

Note:

$$\left(\frac{\partial f}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \frac{\partial f}{\partial S}$$

$$\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{total}}$$

$$\left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{total}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} = \frac{1}{M} \cdot \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{total}}$$

$$
\begin{aligned}
\lambda_c = -\frac{1}{\left(\frac{\partial WE}{\partial c^w}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial \epsilon_s^u}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}} \right) \\
& \cdot \frac{1-p}{\lambda} \cdot g(\epsilon_s^u) \cdot n^s u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon_s^u)) \right) \\
& + \left(\frac{\partial \xi_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} \right) \\
& \cdot \frac{p}{\lambda} \cdot g(\xi_s^c) \cdot n^s u'(c^w) \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial c^w} + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} \right) \cdot \frac{p}{\lambda} \cdot g(\epsilon_s^c) \\
& \cdot n^s \cdot u'(c^w) \left(\frac{\lambda}{f} b - \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& + \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial f}{\partial S}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \left(1 + \frac{\lambda_\theta}{n^s u'(c^w)}\right) \cdot \frac{b}{f} \right) \\
& - n^c \cdot \left(\frac{\partial S}{\partial c^w}\right)^{\text{total}} \left(\frac{\partial \theta}{\partial S}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}
$$

Following the same arguments, we can express the Lagrange multiplier for the separation conditions as:

$$
\begin{aligned}
\lambda_{\epsilon_s^u} = -\frac{u'(c^w)}{\left(\frac{\partial S^u_{\text{stw}}(\epsilon_s^u)}{\partial \epsilon_s^u}\right)^{\text{total}}} \Bigg[& \left(\frac{\partial n}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \left(\frac{\lambda}{f} b - \tau_{\text{stw}} \left(\bar{h} - h_{\text{stw}}(\epsilon_s^u)\right) \right) \\
& + \left(\frac{\partial \xi_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \xi_s^c} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c) \\
& + \left(\frac{\partial \epsilon_s^c}{\partial \epsilon_s^u}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon_s^c} \left(\frac{\lambda}{f} b - \tau_{\text{stw}} (\bar{h} - h_{\text{stw}}(\epsilon_s^c)) \right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c) \\
& + \frac{\partial c^w}{\partial \epsilon_s^u} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f} \right) \\
& + n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^u} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]
\end{aligned}
$$

$$\lambda_{\xi_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\xi_s^c)}{\partial \xi_s^c}\right)^{\text{total}}} \Bigg[\left(\frac{\partial \xi_s^u}{\partial \xi_s^c}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \xi_s^u} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\xi_s^u))\right)$$

$$+ \left(\frac{\partial n}{\partial \xi_s^c}\right)^{\text{ge}} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \xi_s^c)$$

$$+ \left(\frac{\partial \epsilon_s^c}{\partial \xi_s^c}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon_s^c} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$

$$+ \frac{\partial c^w}{\partial \xi_s^c} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right)$$

$$+ n^c \cdot \frac{\partial c^w}{\partial \xi_s^c} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

$$\lambda_{\epsilon_s^c} = -\frac{u'(c^w)}{\left(\frac{\partial S^c_{\text{stw}}(\epsilon_s^c)}{\partial \epsilon_s^c}\right)^{\text{total}}} \Bigg[\left(\frac{\partial \epsilon_s^u}{\partial \epsilon_s^c}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon_s^u} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\xi_s^u))\right)$$

$$+ \left(\frac{\partial \xi_s^c}{\partial \epsilon_s^c}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon_s^c} \cdot \frac{\lambda}{f} b \cdot \mathbb{1}(\epsilon_{\text{stw}} \leq \epsilon_s^c)$$

$$+ \frac{\partial n}{\partial \epsilon_s^c} \left(\frac{\lambda}{f} b - \tau_{\text{stw}}(\bar{h} - h_{\text{stw}}(\epsilon_s^c))\right) \cdot \mathbb{1}(\epsilon_{\text{stw}} \geq \epsilon_s^c)$$

$$+ \frac{\partial c^w}{\partial \epsilon_s^c} \left(\frac{\partial f}{\partial c^w}\right)^{\text{ge}} \cdot u \cdot \left(\frac{\eta - \gamma}{(1-\gamma)(1-\eta)} \cdot \frac{k_v}{q} + \frac{b}{f}\right)$$

$$+ n^c \cdot \frac{\partial c^w}{\partial \epsilon_s^c} \left(\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}} \cdot \hat{IE}_\theta \Bigg]$$

For convenience, the optimal UI benefits are reproduced here:

$$(1-\eta) \cdot (u'(b) - u'(c^w)) = \lambda_\theta (1-\rho) \cdot \left(\frac{1-n}{n} + \frac{u'(b)}{u'(c^w)}\right)$$

$$+ (\lambda_{\epsilon_s^u} + \lambda_{\xi_s^c} + \lambda_{\epsilon_s^c}) \cdot \left(-\frac{u'(b)}{u'(c^w)}\right)$$

$$+ \lambda_c \cdot (1-\rho) \cdot \left(\eta \cdot \frac{1-n}{n} + (1-\eta) \cdot \frac{u'(b)}{u'(c^w)}\right)$$

Inserting the Lagrange multiplier gives:

$$
\begin{aligned}
&(1-n)(u'(b)-u'(c^w)) \\
&= u'(c^w)\Bigg[\left(\frac{\partial c^w}{\partial b}\right)^{\text{ge}}\left(-\frac{\partial f}{\partial c^w}\right)\cdot u\cdot\left(\frac{\eta-\gamma}{(1-\gamma)(1-\eta)}\cdot\frac{k_v}{q}+\frac{b}{f}\right) \\
&\quad+\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \epsilon_s^u}\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^u))\right) \\
&\quad+\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \xi_s^c}\cdot\frac{\lambda}{f}\cdot b\cdot\mathbb{1}(\epsilon_{\text{stw}}\le\xi_s^c) \\
&\quad+\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \epsilon_s^c}\cdot\left(\frac{\lambda}{f}\cdot b-\tau_{\text{stw}}\cdot(\bar{h}-h_{\text{stw}}(\epsilon_s^c))\right)\cdot\mathbb{1}(\epsilon_{\text{stw}}\ge\epsilon_s^c) \\
&\quad+n^c\left(-\frac{\partial\theta}{\partial b}\right)^{\text{ge}}\cdot \hat{IE}_\theta\Bigg]
\end{aligned}
$$

with

$$
\begin{aligned}
\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\cdot\frac{\partial n}{\partial \epsilon_s^u} &= \underbrace{\left(\frac{\partial \epsilon_s^u}{\partial b}\right)^{\text{ge}}\left(\frac{\partial n}{\partial \epsilon_s^u}\right)}_{\text{direct effects}} \\
&+\underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \epsilon_s^u}{\partial S}\right)^{\text{ge}}+\frac{\partial \xi_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{ge}}+\frac{\partial \epsilon_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{ge}}+\frac{\partial c^w}{\partial b}\left(\frac{\partial \epsilon_s^u}{\partial c^w}\right)^{\text{ge}}\right]\cdot\frac{\partial n}{\partial \epsilon_s^u}}_{\text{indirect effect}}
\end{aligned}
$$

$$\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \xi_s^c} = \underbrace{\left(\frac{\partial \xi_s^c}{\partial b}\right)^{\text{ge}} \left(\frac{\partial n}{\partial \xi_s^c}\right)}_{\text{direct effects}}$$

$$+ \underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \xi_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial \epsilon_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b}\left(\frac{\partial \xi_s^c}{\partial c^w}\right)\right] \cdot \frac{\partial n}{\partial \xi_s^c}}_{\text{indirect effect}}$$

$$\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \cdot \frac{\partial n}{\partial \epsilon_s^c} = \underbrace{\left(\frac{\partial \epsilon_s^c}{\partial b}\right)^{\text{ge}} \left(\frac{\partial n}{\partial \epsilon_s^c}\right)}_{\text{direct effects}}$$

$$+ \underbrace{\left[\frac{\partial S}{\partial b}\left(\frac{\partial \epsilon_s^c}{\partial S}\right)^{\text{ge}} + \frac{\partial \epsilon_s^u}{\partial b}\left(\frac{\partial c^w}{\partial \epsilon_s^u}\right)^{\text{ge}} + \frac{\partial \xi_s^c}{\partial b}\left(\frac{\partial c^w}{\partial \xi_s^c}\right)^{\text{ge}} + \frac{\partial c^w}{\partial b}\left(\frac{\partial \epsilon_s^c}{\partial c^w}\right)\right] \cdot \frac{\partial n}{\partial \epsilon_s^c}}_{\text{indirect effect}}$$

$$\left(-\frac{\partial f}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\left(-\frac{\partial f}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}}$$

$$+ \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \xi_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \xi_s^c} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(-\frac{\partial f}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

$$\left(-\frac{\partial \theta}{\partial b}\right)^{\text{ge}} = \underbrace{\frac{\partial S}{\partial b}\left(-\frac{\partial \theta}{\partial S}\right)^{\text{ge}}}_{\text{direct effect}}$$

$$+ \underbrace{\left[\frac{\partial \epsilon_s^u}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^u} + \frac{\partial \xi_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \xi_s^c} + \frac{\partial \epsilon_s^c}{\partial b} \cdot \frac{\partial c^w}{\partial \epsilon_s^c} + \frac{\partial c^w}{\partial b}\right] \cdot \left(-\frac{\partial \theta}{\partial c^w}\right)^{\text{ge}}}_{\text{indirect effect}}$$

G Proof of Lemma 1

Separation condition: unconstrained matches

$$z_{\text{stw}}(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \tau_{\text{stw}}(\bar{h} - h(\epsilon_s^u)) + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

Separation condition: constrained matches

$$\frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} + \frac{\eta}{1 - \eta} \cdot (\lambda - f) \cdot \frac{k_v}{q}$$

Remember:

$$c_{\text{stw}}(\epsilon) = z_{\text{stw}}(\epsilon_s^c) + \tau_{\text{stw}}(\bar{h} - h(\epsilon_s^c)) + \lambda \cdot \frac{k_v}{q}$$

This allows us to rewrite the separation condition of constrained firms on STW as:

$$z_{\text{stw}}(\epsilon_s^c) + \frac{u(c_{\text{stw}}(\epsilon_s^c)) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon_s^c) + \tau_{\text{stw}}(\bar{h} - h(\epsilon_s^c)) + \frac{\lambda - \eta \cdot f}{1 - \eta} \cdot \frac{k_v}{q} = 0$$

T.b.s.: $\epsilon_s^c > \epsilon_s^u$ **(the proof for** $\xi_s^c > \xi_s^u$ **is completely analogous)**

Note: $c^w > c_{\text{stw}}(\epsilon)$

$$\begin{aligned} \epsilon_s^c > \epsilon_s^u \Leftrightarrow \quad & z_{\text{stw}}(\epsilon) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \tau_{\text{stw}}(\bar{h} - h(\epsilon)) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q} \\ & \geq z_{\text{stw}}(\epsilon) + \frac{u(c_{\text{stw}}(\epsilon)) - u(b)}{u'(c^w)} - c_{\text{stw}}(\epsilon) \\ & + \tau_{\text{stw}}(\bar{h} - h(\epsilon)) + \frac{\lambda - \eta f}{1 - \eta} \cdot \frac{k_v}{q} \end{aligned}$$

$$\Leftrightarrow \quad \frac{u(c^w)}{u'(c^w)} - c^w > \frac{u(c_{\text{stw}}(\epsilon))}{u'(c^w)} - c_{\text{stw}}(\epsilon)$$

$$\Leftrightarrow \quad \frac{u(c^w) - u(c_{\text{stw}}(\epsilon))}{u'(c^w)} > c^w - c_{\text{stw}}(\epsilon)$$

$$\Leftrightarrow \quad u(c^w) - u(c_{\text{stw}}(\epsilon)) > (c^w - c_{\text{stw}}(\epsilon)) \cdot u'(c^w)$$

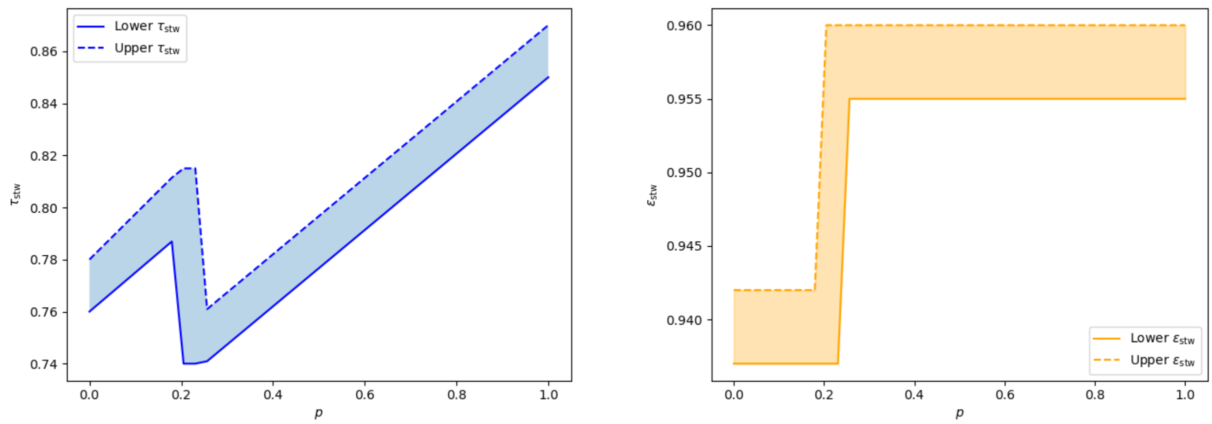
$$\Leftrightarrow \quad \int_{c_{\text{stw}}(\epsilon)}^{c^w} u'(c)\, dc > \int_{c_{\text{stw}}(\epsilon)}^{c^w} u'(c^w)\, dc$$

$$\Leftrightarrow \quad \int_{c_{\text{stw}}(\epsilon)}^{c^w} [u'(c) - u'(c^w)] \; dc > 0 \qquad \checkmark \quad \text{(due to risk aversion)}$$

H Optimization

We rely on gradually refined grid search of welfare, as given by Equation 1. For each grid-point, we compute welfare and search for the maximum with each policy regime. All grids are equidistant. We rely on a grid for p of 40 points between 0 and 1.

For F, τ_{stw}, and ϵ_{stw}, we gradually narrow down the bounds of the intervals in which we search for a new optimum. For F, we start with a grid between 0 and 1.5. The results reported in the paper are based on a grid with limits 0.65 and 0.8. The F grid has 1000 points.



(a) Grid for τ_{stw}

(b) Grid for ε_{stw}

Figure 4: Grid bounds for τ_{stw} and ε_{stw} across $p \in [0, 1]$.

The grids for τ_{stw} and ϵ_{stw} have 400 points each. We start with grids between 0.1 and 1.5 at every point on the p-grid for both parameters. The grids, resulting from sequential narrowing down bounds, on which the results in the paper are based, are shown in Figure 4.

I Calibration

To calibrate the model, we solve the following system of equations for parameters, taking targets as exogenous. Note that we solve for some endogenous model variables, too. This works because the system of equations pins down the model solution for these variables, along with the parameters to be calibrated.

Exogenous:

$$f,\ q, \rho, b_{\text{rep}}, n$$

Endogenous:

$$c_f, \bar{m}, b, \epsilon_s^c, \epsilon_s^u, c^w, \omega$$

(I)

$$\rho = (1-p) \cdot G(\epsilon_s^u) + p \cdot G(\epsilon_s^c)$$

(II)

$$0 = z(\epsilon_s^u) + \frac{u(c^w) - u(b)}{u'(c^w)} - c^w + \frac{\lambda - \eta f}{1-\eta} \cdot \frac{k_v}{q}$$

(III)

$$0 = z(\epsilon_p) + \lambda \cdot \frac{k_v}{q} - c^w$$

(IV)

$$\frac{u(c(\epsilon_s^u)) - u(b)}{u'(c^w)} + (\lambda - f) \cdot \frac{\eta}{1-\eta} \cdot \frac{k_v}{q} = 0$$

(V)

$$\frac{1}{1-\eta}\cdot\frac{k_v}{q}=\frac{1}{\lambda}\Bigg[(1-\rho)\cdot\left(z-\frac{1-\eta}{\eta}b\right)$$
$$+(1-p)(1-\rho^u)\cdot\left(\frac{u(c^w)-u(b)}{u'(c^w)}-c^w\right)$$
$$+p(1-\rho^c)\cdot\left(\frac{u(c^c)-u(b)}{u'(c^w)}-e^c\right)$$
$$+(1-\rho)\cdot\frac{\lambda-f\eta}{1-\eta}\cdot\frac{k_v}{q}\Bigg]$$

(VI)

$$(1-p)\cdot(1-\rho^u)\cdot\left(\eta\cdot c^w+(1-\eta)\cdot\frac{u(c^w)-u(b)}{u'(c^w)}\right)$$
$$=\eta\cdot(1-\rho)\cdot\left(z+\frac{1-n}{n}\cdot b+\theta\cdot k_v\right)$$
$$-p\cdot(1-\rho^c)\cdot\left[(1-\eta)\cdot\frac{u^c-u(b)}{u'(c^w)}+\eta\cdot e^c\right]$$

(VII)

$$\omega=\frac{1}{1-\rho}\cdot\Bigg((1-p)\cdot\int_{\epsilon_s^u}^{\infty}\left[c^w+\phi(h(\epsilon))\right]dG(\epsilon)$$
$$+p\cdot\int_{\epsilon_p}^{\infty}\left[c^w+\phi(h(\epsilon))\right]dG(\epsilon)$$
$$+p\cdot\int_{\epsilon_s^c}^{\epsilon_p}\left[c(\epsilon)+\phi(h(\epsilon))\right]dG(\epsilon)\Bigg)$$

(VIII)

$$b=b_{\text{rep}}\cdot\omega$$